

Detecting Dangerous Queries:

A New Approach for Chosen Ciphertext Security

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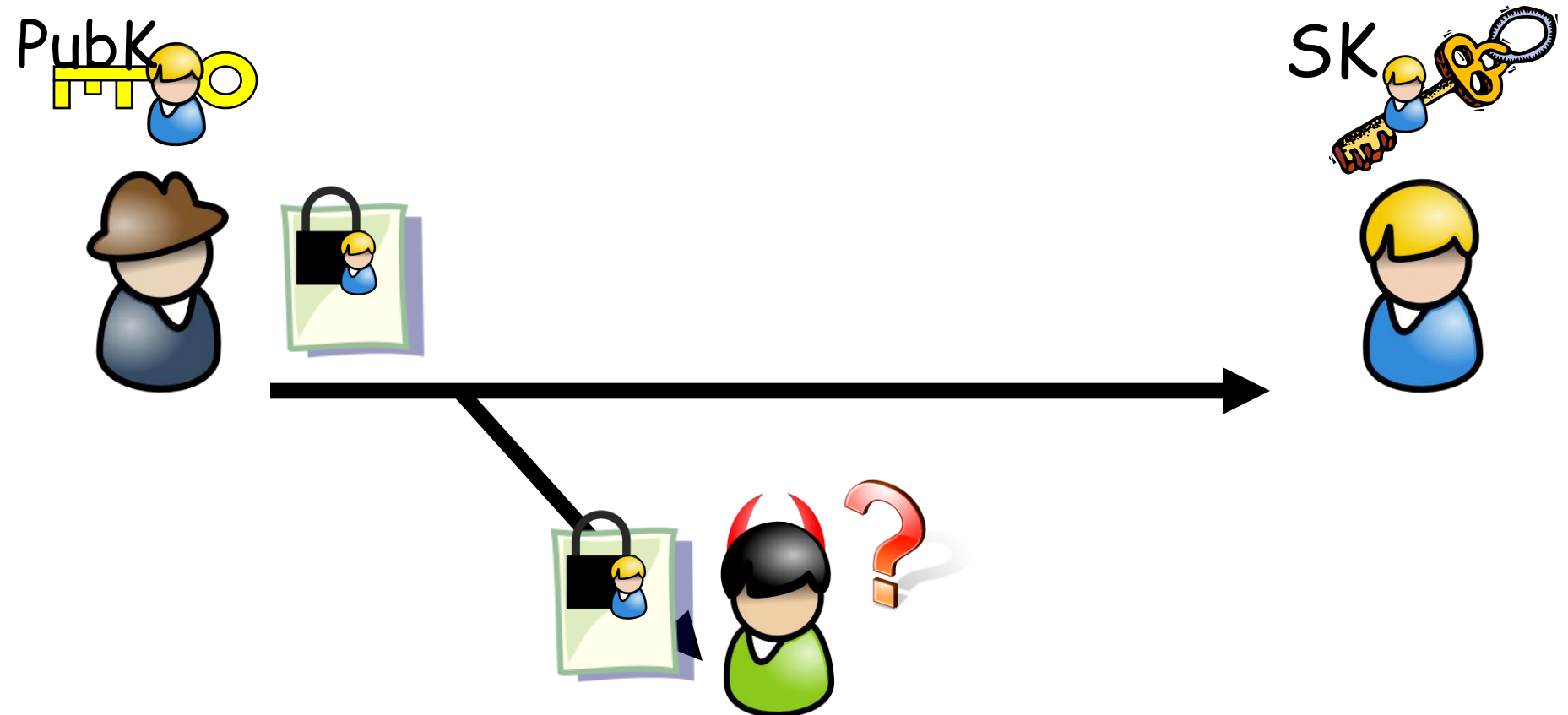
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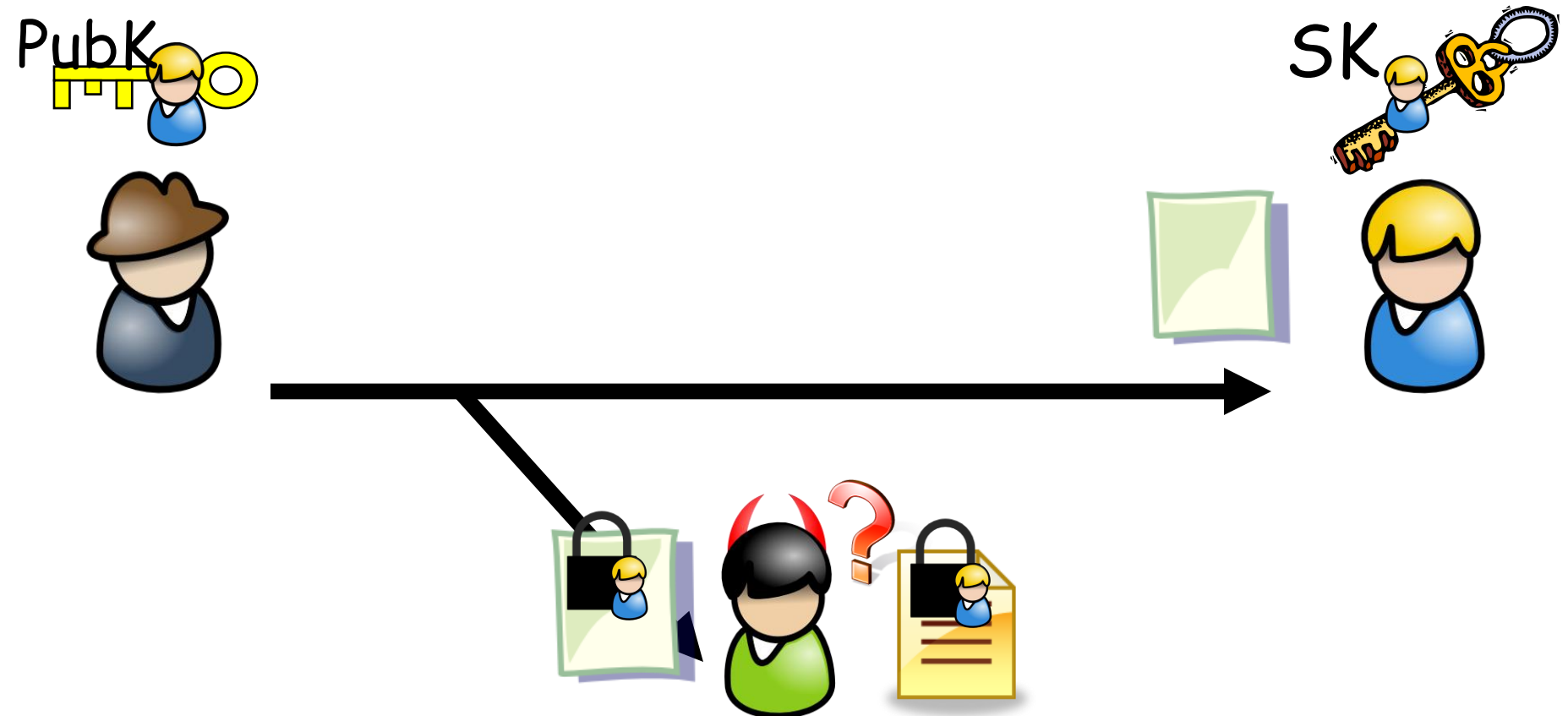
Public Key Encryption [DH76,RSA78,GM84]

Passive Attacker : Chosen Plaintext Attack (CPA)



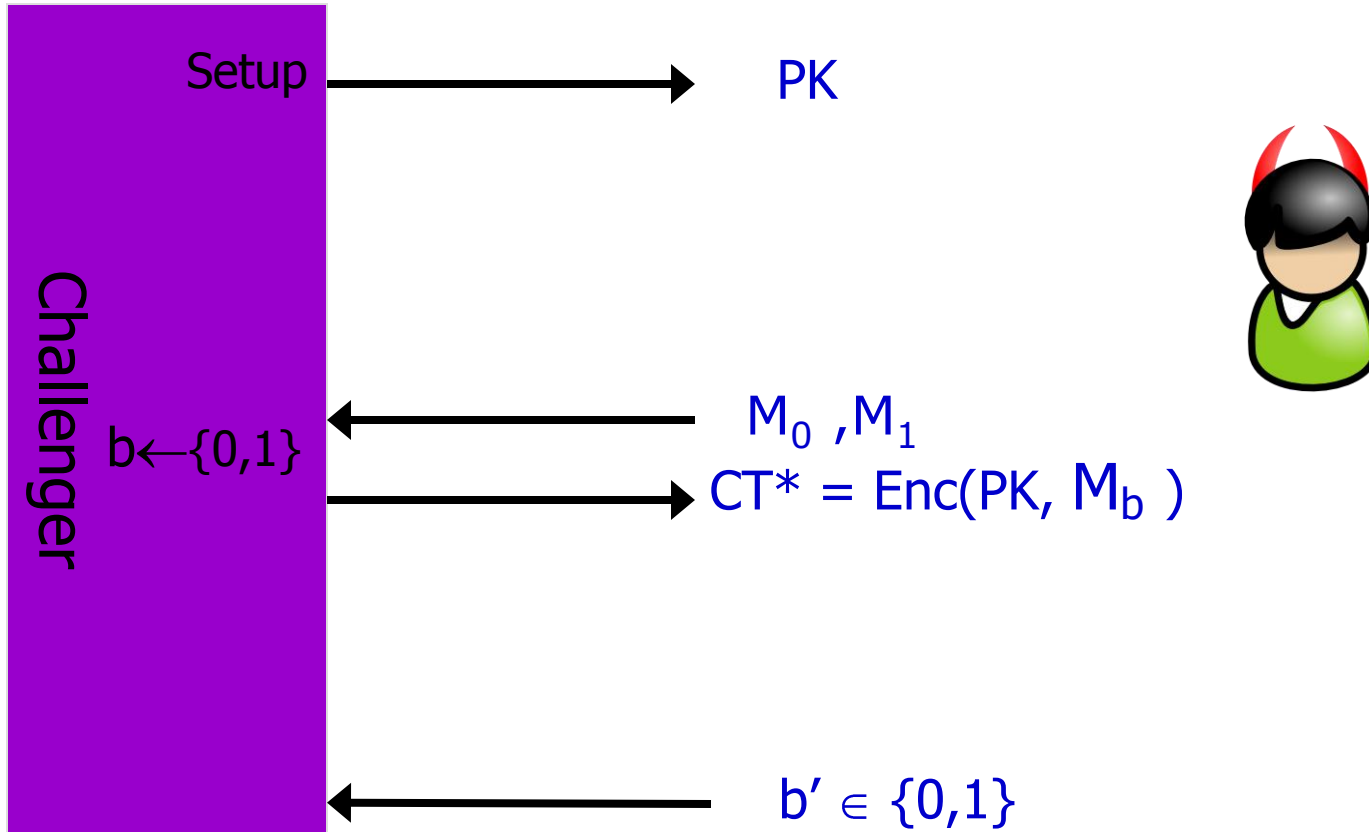
Active Attackers [NY90,DDN91,RS91]

Chosen Ciphertext Attack (CCA)



IND-CPA [GM84]

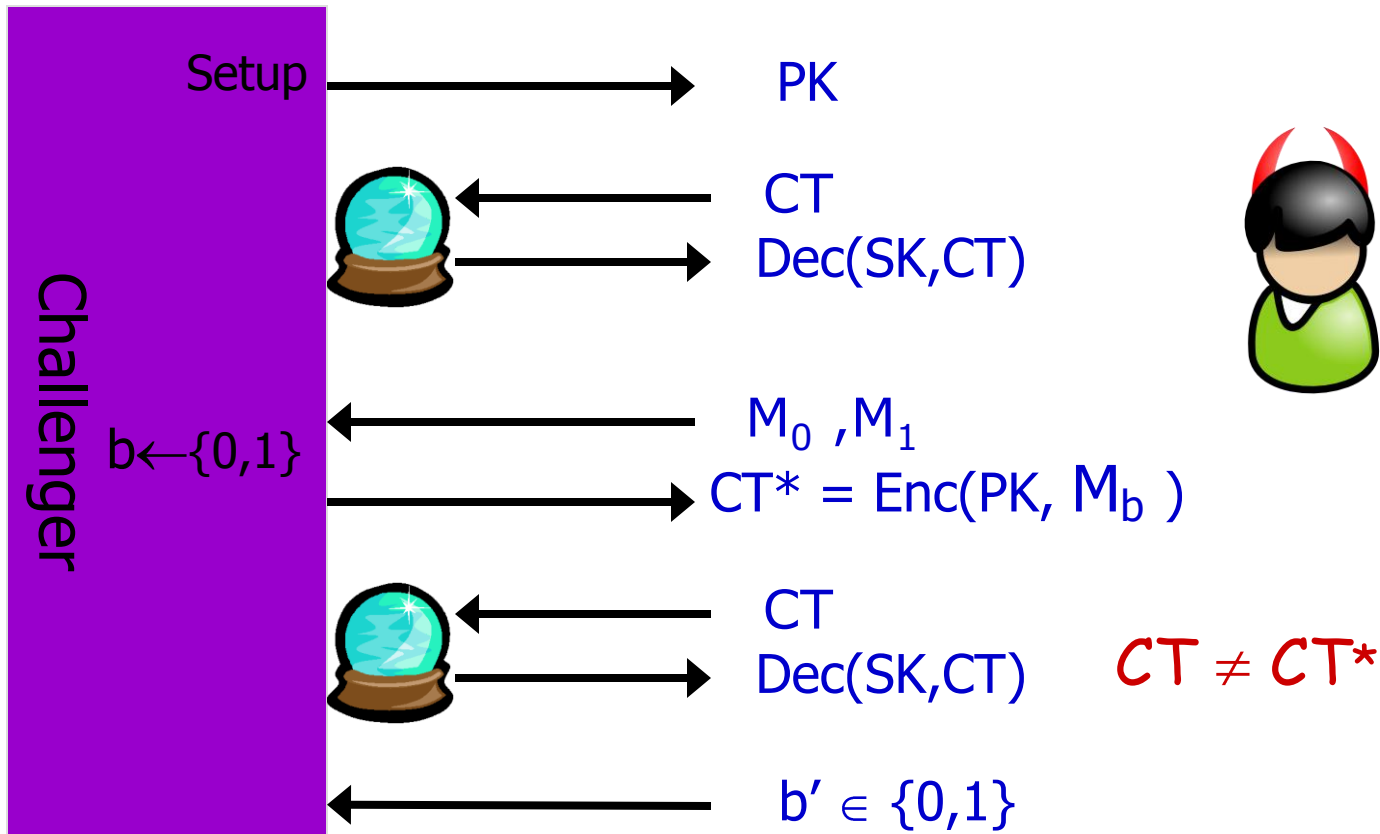
Indistinguishability under Chosen Plaintext Attack



$$\text{Adv}_A = \Pr[b=b'] - 1/2$$

IND-CCA [NY90,DDN91,RS91]

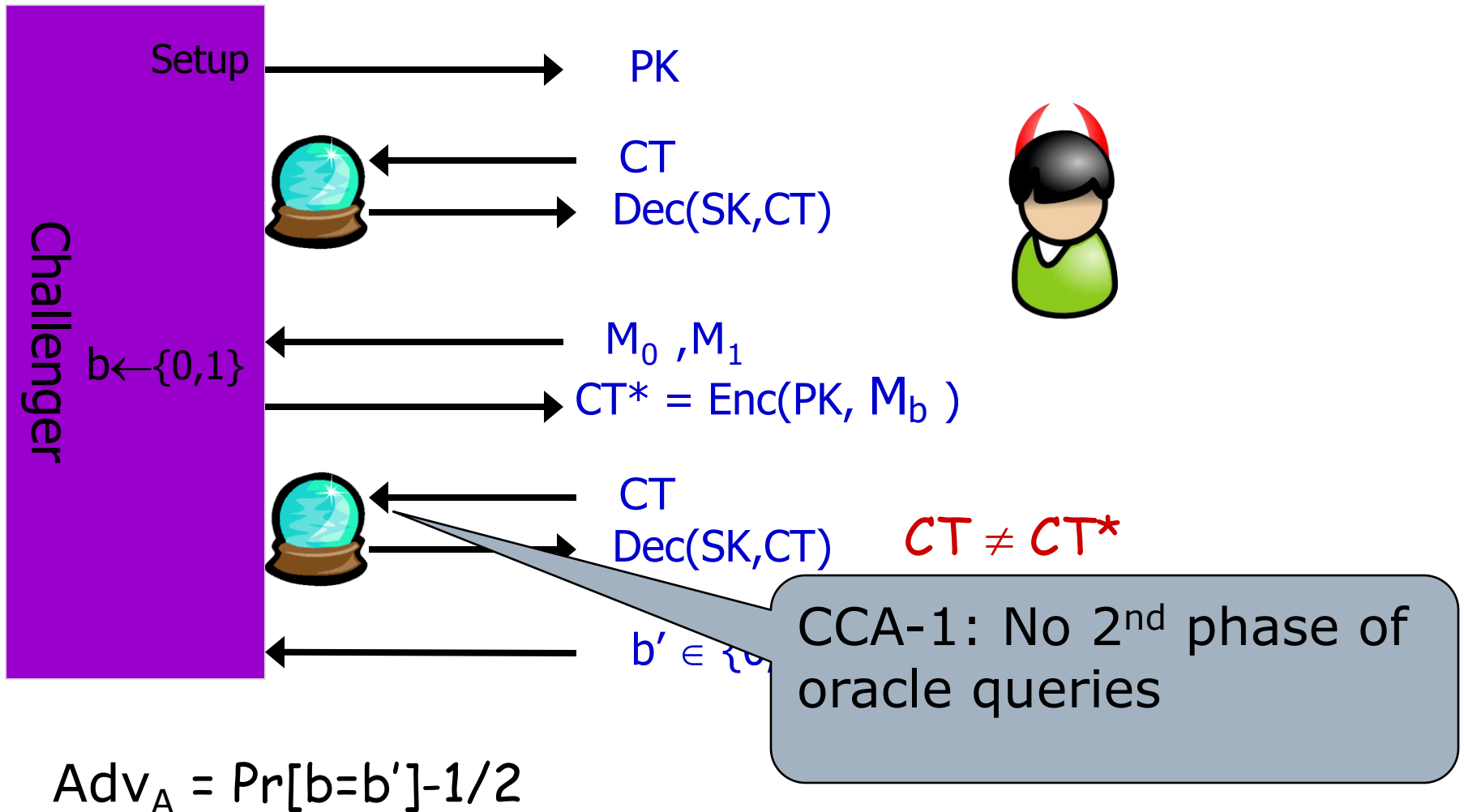
Indistinguishability under Chosen **Ciphertext** Attack



$$Adv_A = \Pr[b=b'] - 1/2$$

IND-CCA [NY90,DDN91,RS91]

Indistinguishability under Chosen **Ciphertext** Attack



The Grand Goal: CCA from CPA

CCA



CPA

Some Prior Methods (Standard Model)

NIZK [BFM88,NY90,DDN91,RS91,S99]

- TPD/RSA, Pairings No:DDH, Lattices

Cramer-Shoup plus [CS98,02,...]

- DDH,DCR, Factoring, IBE [CHK04], No:Lattices

Lossy TDFs [PW08,RS09,...]

- DDH, Lattices

1-bit CCA to n-bit CCA [MS09]

- Straightforward appending won't work!

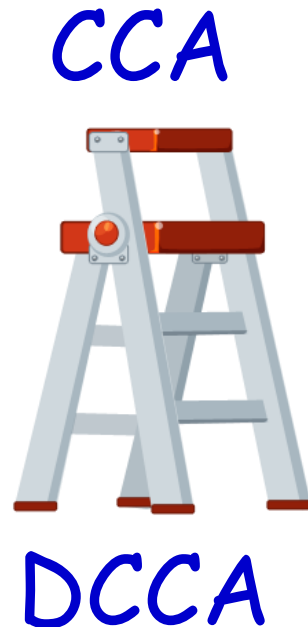


- Neat ideas
- Heavyweight machinery + complex
- We will adapt + generalize some ideas

Our Result

New General Approach for CCA security:

Detectable Chosen Ciphertext Security (DCCA)



DCCA Security: Intuition

CCA secure *if* avoid "dangerous" queries

- 1) Hard to produce bad queries w/o challenge CT
- 2) Can *detect* dangerous queries

Example: Concatenate 1 bit CCA ciphertexts

CT* 

Dangerous Query for CT*: CT = Reorder of CT*

- 1) Hard to produce w/o CT*
- 2) Easy to detect

Detectable Encryption System

$\text{Setup}(1^n) \rightarrow (\text{PK}, \text{SK})$

$\text{Encrypt}(\text{PK}, \text{M}) \rightarrow \text{CT}$

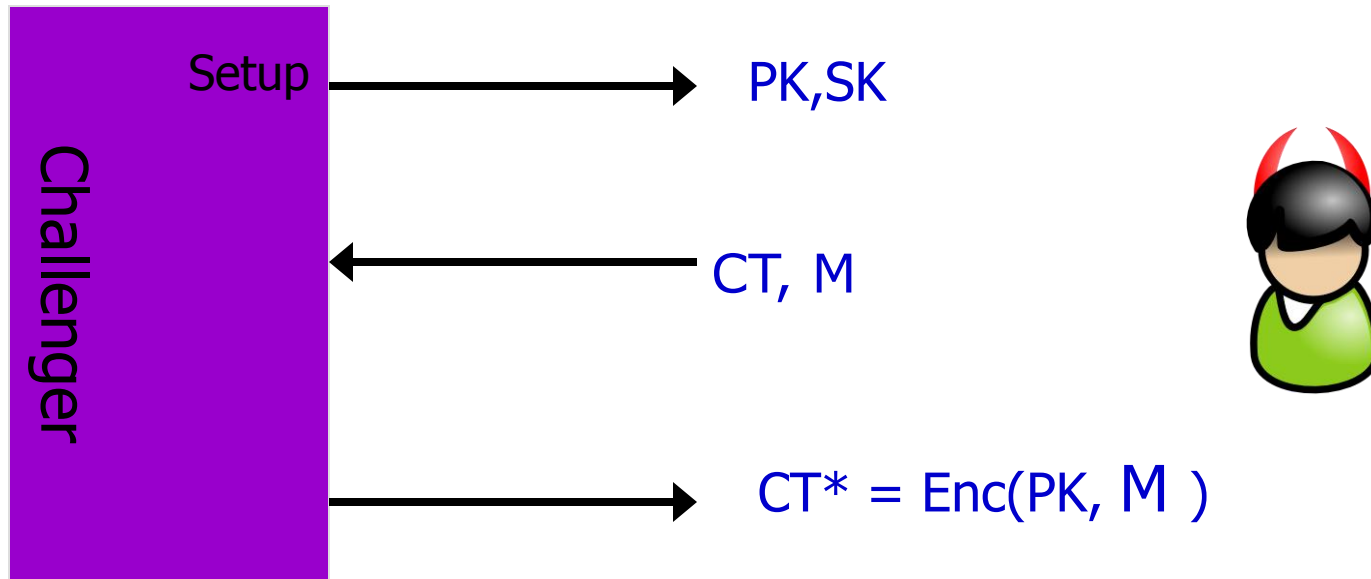
$\text{Decrypt}(\text{SK}, \text{CT}) \rightarrow \text{M}$

$F(\text{PK}, \text{CT}^*, \text{CT}) \rightarrow \{0,1\}$

Outputs '1' if CT is a "dangerous" query for CT^*

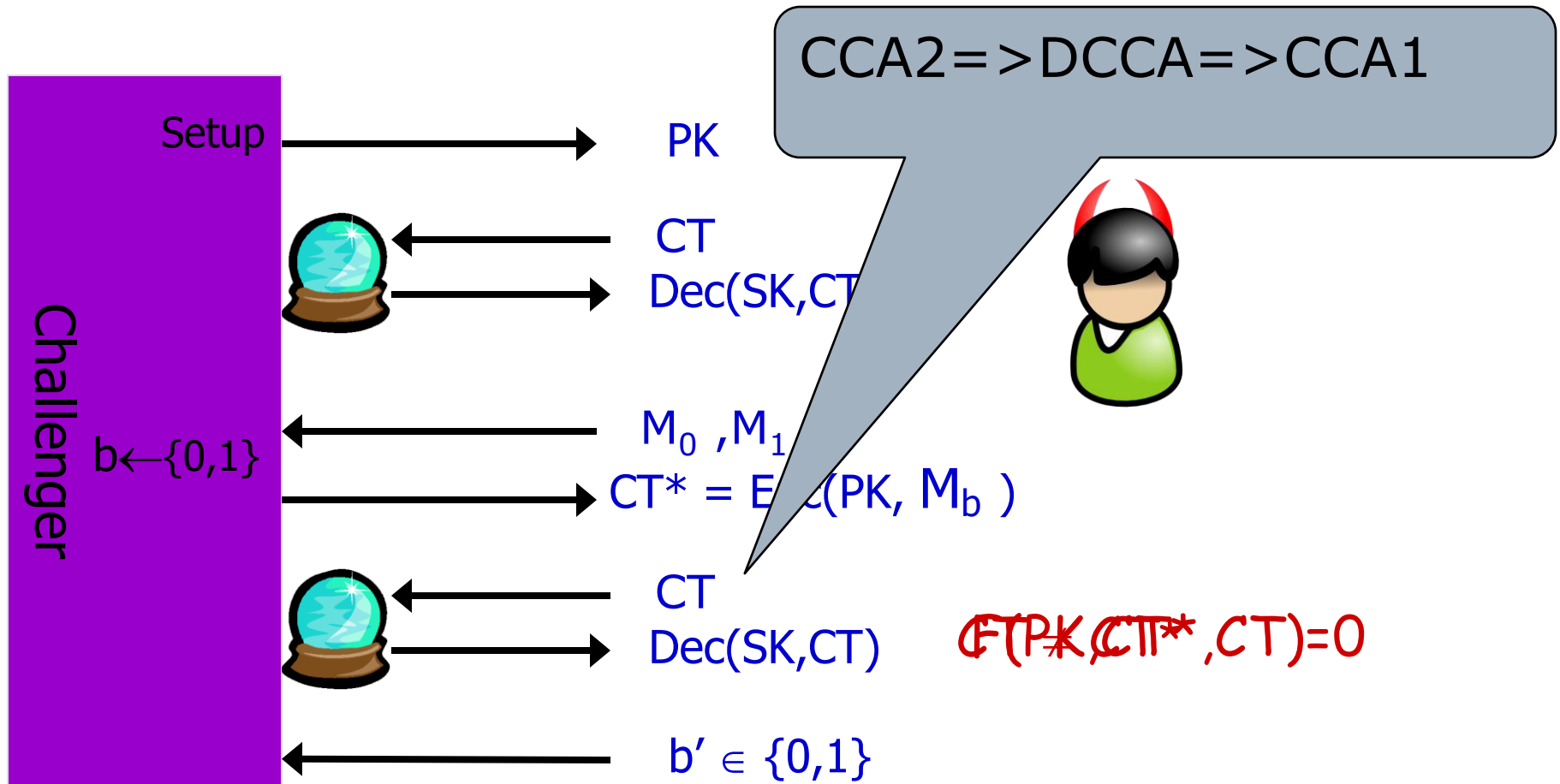
Two Security Properties

Property 1: Hard to Predict (Strong)



$$\text{Adv}_A = \Pr[F(\text{PK}, \text{CT}, \text{CT}^*)=1]$$

Property 2: Indistinguishability



$$\text{Adv}_A = \Pr[b=b'] - 1/2$$

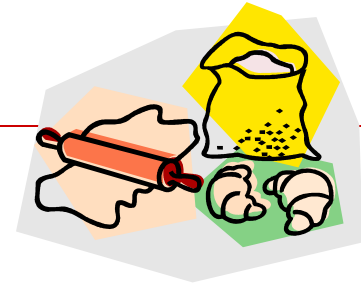
Examples

One bit to many bit CCA

Tag-Based Encryption [MRY04,K06]

Sloppy/Heuristic CCA

The Ingredients



$\text{Msg} \in \{0,1\}^*$ and randomness $\in \{0,1\}^n$
Justified by Pseudo Random Generators

1-Bound

Trivial

Detectable CCA

Our Construction

Setup

Setup(1^n):

- 1) Setup_{1B} (1^n) \rightarrow (PK_A, SK_A)
- 2) Setup_{CPA} (1^n) \rightarrow (PK_B, SK_B)
- 3) Setup_{DCCA} (1^n) \rightarrow (PK_{in}, SK_{in})

$PK = PK_A, PK_B, PK_{in}$

$SK = SK_A, SK_B, SK_{in}$

Encryption

Encrypt(PK,M):

- 1) Choose random $r_a, r_b, r_{in} \in \{0,1\}^n$
- 2) $C_{in} = \text{Enc}_{\text{DCCA}}(\text{PK}_{in}, (M, r_a, r_b); r_{in})$
- 3) $C_A = \text{Enc}_{1B}(\text{PK}_A, C_{in}; r_a), C_B = \text{Enc}_{\text{CPA}}(\text{PK}_B, C_{in}; r_b)$
- 4) $\text{CT} = C_A, C_B$

$$C_A = \boxed{(M, r_a, r_b); r_{in}}; r_a$$

$$C_B = \boxed{(M, r_a, r_b); r_{in}}; r_b$$

Decryption

Decrypt($SK, CT = (C_A, C_B)$):

- 1) $C_{in}' = Dec(SK_A, C_A)$
- 2) $(M', r_a', r_b') = Dec(SK_{in}, C_{in}')$
- 3) $C_A' = Enc_{1B}(C_{in}'; r_a')$, $C_B' = Enc_{CPA}(C_{in}; r_b')$
- 4) If $C_A \neq C_A'$ OR $C_B \neq C_B'$ **reject**; else M'

$$C_A = \boxed{(M, r_a, r_b); r_{in}; r_a}$$

$$C_B = \boxed{(M, r_a, r_b); r_{in}; r_b}$$

Idea: Recover (M, r_a, r_b) then re-encrypt

A Few Comments

$$C_A = \boxed{(M, r_a, r_b); r_{in}; r_a}$$

$$C_B = \boxed{(M, r_a, r_b); r_{in}; r_b}$$

Features: Naor-Yung 2-key & Myers-shelat nesting

Embedded Randomness vs. NIZK

Proof w/ embedding randomness:

Good: Decrypt from either side

Problem: Embedding challenge



What is the trouble?

$$C_A^* = \boxed{C_{in}^* = (M, r_a, r_b); r_{in}; r_a} \quad C_B^* = \boxed{C_{in}^* = (M, r_a, r_b); r_{in}; r_b}$$

Challenge CT = C_A^* , C_B^* encryptions of C_{in}^*

Problem Query: Get C_{in}' s.t. $F(PK_{DCCA}, C_{in}^*, C_{in}') = 1$

Bad Event: Query $C = C_A, C_B$ s.t.

(1) $C_A \neq C_A^*$

(2) $\text{Dec}(SK_A, C_A) = C_{in}'$ where $F(PK_{DCCA}, C_{in}^*, C_{in}') = 1$

Nested Indist. Game

If prove under this game we are done!

Attacker gets

Challenge Inner encryption of msg + randomness or all 0's



z=1

$$C_A^* = C_{in}^* = (M, r_a, r_b); r_{in}; r_a$$

$$C_B^* = C_{in}^* = (M, r_a, r_b); r_{in}; r_b$$

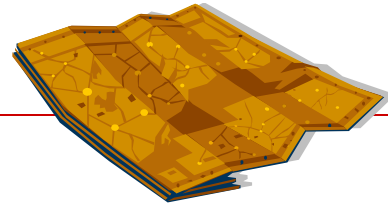


z=0 No embedded randomness

$$C_A^* = C_{in}^* = (00...00); r_{in}; r_a$$

$$C_B^* = C_{in}^* = (00...00); r_{in}; r_b$$

Proof Overview



Eliminate bad event \Rightarrow Security follows from DCCA

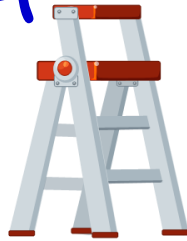
- (1) Eliminate with $z=0$ (no embedded randomness)
- (2) Indirectly infer $z=1$ case from (1)
- (3) Finish off

Summary

- New abstraction: Detectable CCA security
- Build CCA from it
- Cover 1 to many bit enc. , tag-based, & more
- Embedded randomness --- blessing & problems
- Indirect inference on bad event

Our Picture (not necessarily to scale)

CCA



DCCA

CCA-1

CPA

Thank you

Bad Event Analysis (no embedded randomness)

Show probabilities are close

Nested

IND-CPA

$(00\dots 00); r_{in}; r_a$

$(00\dots 00); r_{in}; r_b$

Right-Erased

$(00\dots 00); r_{in}; r_a$

$1111\dots 111; r_b$

Switch -Decrypt

Bounded CCA

Full-Erased

$1111\dots 111; r_a$

$1111\dots 111; r_b$

=negl(n) unpredictability

No Bad Event for embedded randomness

Suppose it did happen => We break DCCA indist.

- 1) Run Indist Game on A (while playing DCCA)
- 2) Submit $\text{Msg}_1 = (M, r_a, r_b)$, $\text{Msg}_0 = (00\dots00)$
- 3) Get back either $(M, r_a, r_b); r_{in}$ or $(00\dots00); r_{in}$
- 4) Create challenge CT (know SK_A, SK_B)
- 5) Use DCCA oracle to answer non-dangerous queries

What if get dangerous query? **Stuck!**

But then we know it must be Msg_1 => breaks DCCA!

Finishing it off



$z=1$

$$C_A^* = C_{in}^* = (M, r_a, r_b); r_{in}; r_a \quad C_B^* = C_{in}^* = (M, r_a, r_b); r_{in}; r_b$$



$z=0$

No embedded randomness

$$C_A^* = C_{in}^* = (00\dots00); r_{in}; r_a \quad C_B^* = C_{in}^* = (00\dots00); r_{in}; r_b$$

N.I. easy to prove from DCCA if no bad events

CCA security follows immediately

Could CCA-1 work?

Idea: Replace DCCA component w/ CCA-1

Problem 1: Proof needs to detect

Problem 2: Counterexample (w/natural CCA-1 scheme)

Ex. 1: n-bit DCCA from 1 bit CCA

Idea: Use basic concatenation

$\text{Enc}(\text{PK}, m) \rightarrow C_1 = \text{Enc}(\text{PK}, m_1), \dots, C_n = \text{Enc}(\text{PK}, m_n)$



$F(\text{PK}, CT^*, CT): \exists (i, j) \text{ s.t. } CT_i^* = CT_j$

Ex. 2: Tag-Based Encryption [MRY04,K06]

Tag-Based Encryption:

- (1) Each ciphertext associated with a tag
- (2) Is CCA secure as long as Tag_{CT^*} not queried

$F(\text{PK}, CT^*, CT): \text{Tag}_{CT^*} = \text{Tag}_{CT}$

Examples: CHK04-lite, Kiltz06, PW08 (CCA-1 version), DDN91 (w/o signature)

Ex. 3: Heuristic/Sloppy CCA

Idea: DCCA easier to meet than CCA

(1) Heuristic approach

(2) Sloppy: E.g. "Slack" bit in group representation

CT: 

Apply transformation in case messed up

Could CCA-1 work?

Idea: Replace DCCA component w/ CCA-1

Problem 1: Proof needs to detect

Problem 2: Can create an oracle that breaks it



(CT^*): Decrypts CT^* , encrypts M in another CT'

Q1: The oracle is strong! Is there middle ground?

Q2: Structure for CCA-1? Proof idea?

Prior Methods (Standard Model)

NIZK [BFM88,NY90,DDN91,RS91,S99]

- NIZK proves well formness
- NIZKs are rare: TPD/RSA, Pairings No:DDH, Lattices

Cramer-Shoup plus [CS98,02,...]

- Efficient systems from number theory
- DDH,DCR, Factoring, IBE [CHK04], No:Lattices

Prior Methods (Standard Model)

Lossy TDFs [PW08,RS09,...]

- Randomness recovery \Rightarrow use to verify CT
- Change PK in proof
- DDH, Lattices

1-bit to many bit CCA[MS09]

- General techniques
- Partial randomness recovery

BE-Nested vs. BE-Right-Erase



Standard IND-CPA reduction

- Know SK_A, SK_{in} , not SK_B
- Observe BE using SK_A

Switch Decrypt

Switch from using SK_A to SK_B to decrypt

- These are equivalent from Attacker's view
- Best of both worlds: Challenge CT not embed randomness, but queries must!

BE-Right-Erased vs. BE-Full-Erased

Full-Erased

1111...111 ; r_a

1111...111 ; r_b

$C_{in}^* = (00...00); r_{in}$ is gone!

Unpredictability: $\Pr[\text{Bad event in Full Erase}] = \text{negl}(n)$

BE-Right-Erased vs. BE-Full-Erased



1-Bounded CCA reduction

- Know SK_B , SK_{in} , not SK_A
- **Problem:** Cannot observe bad event using SK_B
- **Solution:** "Peek" at 1 A query using 1-Bounded $1/Q$ chance of seeing it