Temporal Constraint Networks in Action

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Abstract.
This paper presents an integration of the well known Temporal Constraint Networks representation into a causal approach for reasoning about actions called AL^2 [9]. As a result, temporal constraints are allowed in the conditions of the causal rules that describe the domain behavior. We show the adequacy of the AL^2 understanding of causation (based on the concept of *pertinence*) for naturally introducing the set of time points used in constraints, including not only action occurrences, but also the instants of relevant changes in fluents.

1 Introduction

Temporal Reasoning has historically represented one of the main areas inside Artificial Intelligence from its very beginning, not only as a research field by itself, but also affecting other areas that usually need to deal with temporal domains. Unfortunately, research in temporal reasoning has evolved into independent and, sometimes, diverging trends, which have grown motivated by different goals. Among these branches, perhaps the most differentiated ones are: *nonmonotonic temporal reasoning* – action formalisms –, *temporal modal logic* and *temporal constraints*. Although considerable effort has been done in mixing action formalisms and temporal modal logic [13, 1, 2], there is not much work combining action based approaches with temporal constraints (as an exception, see [12]).

In this paper we introduce an integration of *Temporal Constraint Networks* (TCN) [3] into a nonmonotonic logic for reasoning about actions and change called AL^2 [9]. Our aim is to focus on the representational aspects, trying to bridge the gap between the clearly different aims of both orientations. On the one hand, quantitative temporal constraints are concerned with solving temporal numerical relations among a given set of events, provided that they have actually occurred. Each event "expresses" by itself all the needed knowledge for describing a particular situation. So, the stress is laid on solving, in the most efficient way, the restrictions about *temporal distances* among events. On the other hand, action formalisms are thought for describing the behavior of a dynamic system, paying more attention to the quality of the used representation — usually, a nonmonotonic logical approach. The system is described in terms of the properties that can be identified on it (*fluents*), especially focusing on the way in which these properties *evolve* along time, rather than on temporal distances among events.

We show how the obtained result provides an expressivity enhancement, both from an action formalism point of view and for a temporal constraint representation. As a main contribution, we show how the AL^2 treatment of causality (based on the concept of *pertinence*) naturally allows the introduction of temporal constraints, fixing the set of "events" to be used for measuring temporal distances. In this way, we are able to express constraints, not only among action occurrences, but also among relevant changes for the same or for different fluents.

The paper is organized as follows. In the next section we recall the usual TCN definitions, but under a logical model-based terminology. Section 3 introduces the basic definitions of the nonmonotonic action logic AL^2, which are extended in the next section for dealing with temporal constraints. In the last two sections, we establish comparisons to related work and draw some conclusions and directions for future work.

2 Model based Temporal Constraint Networks

Since we are interested in introducing TCN into a logical formalism, we will first propose a rephrasing of TCN definitions under a logical model-based terminology. We define a logic for TCN, called L_{TC}, describing its syntax and semantics in the following way. The language of L_{TC} consists of a finite set of *time point names*, *TP* $= \{t_1, ...t_n\}$, an infinite set of non-nested binary connectors $\trianglerighteq_d$ (where d is an integer number) and the propositional connectives ($\wedge$, $\vee$ and $\neg$). We will also handle the usual derived operators $\supset$ and $\equiv$ standing for material implication and classical equivalence. We define an *atomic constraint* as any formula of the shape

$$t_1 \trianglerighteq_d t_2 \tag{1}$$

where $t_1, t_2 \in$ *TP* and d is any integer number. Intuitively, this constraint points out that the elapsed time from t_1 to t_2 is, *at most*, d temporal units. We define an L_{TC} *well formed formula* as any propositional combination of atomic constraints. A *theory* is then defined as a set of well formed formulas.

The semantics of L_{TC} relies on assigning an integer value to each time point name. Thus, an *interpretation* M will be a mapping from the set of time points *TP* to the integer numbers M : *TP* $\longrightarrow \mathbf{Z}$. We say that an interpretation M *satisfies* an atomic formula $t_1 \trianglerighteq_d t_2$, written $M \models t_1 \trianglerighteq_d t_2$, iff $M(t_2) - M(t_1) \leq d$. Satisfaction for general propositional formulas is defined in the usual way.

As we can see, an atomic constraint $t_1 \trianglerighteq_d t_2$, with $d > 0$, fixes a maximum value for the elapsed time between t_1 and t_2. When distance is negative, requiring $M \models t_1 \trianglerighteq_{-|d|} t_2$ is equivalent to $M(t_1) - M(t_2) \geq |d|$, that is, $|d|$ becomes a minimum for the converse distance time from t_2 to t_1.

As usual, we say that an interpretation M is a *model* of a theory T iff it satisfies all the formulas in T. In the same way, a theory T is said to *entail* a formula ϕ iff all the models of T are models of ϕ.

An interesting property verified by L_{TC}, is the transitivity of constraints. Thus, it can be easily seen that the following formula:

$$t_1 \trianglerighteq_d t_2 \wedge t_2 \trianglerighteq_e t_3 \supset t_1 \trianglerighteq_{d+e} t_3 \tag{2}$$

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is a tautology, for any $t_1, t_2, t_3 \in$ *TP* and any pair of integer numbers d, e.

In order to relate this logical reformulation of Temporal Constraints to their original terminology [3], we establish some direct correspondences. An L_{TC} theory only consisting of atomic constraints receives the name of *Simple Temporal Problem* (STP), and corresponds to what is called a *distance graph*. In this graph, each constraint $t_1 \trianglerighteq_d t_2$ is represented as an arc from t_1 to t_2 with distance d. A model of a theory T is also known as a *solution* for the corresponding graph.

Sometimes, arcs in distance graphs are labeled by an interval, instead of a single distance. This can also be directly represented as an STP in L_{TC} by defining the derived operator:

$$t_1 \xrightarrow{[a,b]} t_2 \stackrel{\text{def}}{=} t_1 \trianglerighteq_b t_2 \wedge t_2 \trianglerighteq_{-a} t_1$$

A typical task in STPs is to check the *consistency* of a graph, that is, checking the existence of a model. We will show that the well known consistency checking method introduced in [3] is also sound and complete with respect to L_{TC}. In short, the method lies in looking for negative cycles inside the distance graph. When translated into L_{TC} terms, this means looking for a particular kind of subtheory (the negative cycle) inside an STP theory.

Let us introduce first some definitions. Given an STP theory T, we define a *path* between t and t', as a subtheory of T consisting of a sequence of constraints like: $\{t \trianglerighteq_{d_1} t_1,\ t_1 \trianglerighteq_{d_2} t_2, \ldots,\ t_{n-1} \trianglerighteq_{d_n} t'\}$. A *cycle* is a non-empty path from t to t. Given a path p, we define its *distance* as the summation of all the d_i in p.

Theorem 1 *An STP theory T is inconsistent iff it contains a cycle p with negative distance $dist(p) < 0$.*

Proof
"$\Longleftarrow$" (Soundness)

Let p be a negative cycle in T. By iterative application of transitivity to the constraints in p, any model of T should satisfy $t \trianglerighteq_{dist(p)} t$, that is, $M(t) - M(t) \leq dist(p)$, which is not possible, since $dist(p) < 0$.
"$\Longrightarrow$" (Completeness)

For a proof sketch, assume that T does not contain negative cycles. Then it is always possible to construct an interpretation M which is a model of T, what contradicts the initial assumption of inconsistence of T. For constructing such an M it suffices with identifying the first "layer" of time points, TP_0 (those that do not occur in the right side of any constraint) or, if they do not exist, an arbitrary element of each first layer -*strongly connected components* in the distance graph. Then, it is always possible to establish a model mapping which assigns, for any $t \in TP_0$, $M(t) = 0$, and for those $t \notin TP_0$, the minimal distance among the paths from some $t' \in TP_0$ to t. Notice that this minimal distance is computable provided that all the cycles are of nonnegative distance. □

As we can see, STP theories have a strong expressive limitation, since they practically do not allow any kind of disjunctive knowledge. The unique way of expressing uncertainty is by imposing minimum and maximum bounds to temporal differences. In fact, in [3], the STPs are generalized into a more expressive representation: *Temporal Constraint Networks* (TCN). A TCN is a graph where each arc (t_1, t_2) is labeled now by *a set of intervals* $\{I_1, \ldots, I_n\}$, meaning that the elapsed time from t_1 to t_2 must be contained in at least one of the I_j.

This same definition can be also done in L_{TC} terms, so that a theory T is said to be a TCN if all its formulas have the shape:

$$\bigvee_{i \in [1,n]} (t_1 \trianglerighteq_{b_i} t_2 \wedge t_2 \trianglerighteq_{-a_i} t_1) \qquad (3)$$

being $t_1, t_2 \in$ *TP* and the a_i, b_i a sequence of $n > 0$ arbitrary pairs of integer numbers. It can be easily seen that each clause like (3) corresponds to an arc (t_1, t_2) labeled with intervals $[a_1, b_1], \ldots, [a_n, b_n]$.

Although TCNs make use of disjunction (as clearly expressed in their L_{TC} representation) they do not fully cover the expressive range of L_{TC}. Think, for instance, in the simple introduction of negation for atomic constraints: $M \models \neg(t_1 \trianglerighteq_d t_2)$, i.e., $M(t_2) - M(t_1) > d$. Negating an atomic constraint allows representing strict inequalities that, otherwise, should have to be introduced by defining new constraint operators (some kind of strict $t_2 \triangleright_d t_1$). But, in fact, we can even go further and represent problems which cannot be encoded into a TCN. Think for instance in the example theory:

$$T = \{(t_1 \xrightarrow{[0,0]} t_2 \wedge t_3 \xrightarrow{[0,0]} t_4) \vee (t_1 \xrightarrow{[0,0]} t_3 \wedge t_2 \xrightarrow{[0,0]} t_4)\}$$

This kind of expressions cannot be translated into a TCN, since the disjunction affects to the whole structure of the network, and not only to the possible combinations of intervals for a fixed set of arcs.

3 AL^2 syntax and semantics

The causal logic AL^2, introduced in [9], is intended for describing narrative-based actions domains. Its syntax is fixed by a parameter n which delimits the length of the narratives, and by two disjoint sets: the *action names*, $\mathcal{A}$, which describes all the actions that can be performed, and the *fluent names*, $\mathcal{F}$, containing all the (boolean) properties that we identify for describing each system situation. We will use the letter q to denote any action or fluent name, and letters f and a to denote fluents and actions respectively.

An *atom* has the shape q_i where $i \in [0, n]$ is a non-negative integer, pointing out a situation along the narrative. By $Atoms(\mathcal{A}, \mathcal{F}, n)$ we denote the set of all possible atoms. We call a *literal* to any atom q_i or its negation $\neg q_i$. General AL^2 formulas may be of two possible types:

i) *non-causal* formulas: defined as combinations of atoms with the classical propositional connectives ($\wedge$, $\vee$,$\neg$). We will also include a new type of atomic construction, $(\texttt{prev}\ q)_i$, which will almost stands for q_{i-1} excepting for its causal behavior (as we will see later).
ii) *causal rules*: with shape $F \Leftarrow \phi$ where F is a literal for some f_i and ϕ is a non-causal formula such that, for any subindex j occurring in ϕ, $j \leq i$.

The restrictions imposed on causal rules intuitively say that actions are never "concluded" as a result of a rule (we will be only interested in concluding fluent literals) and that the condition of the rule must refer to the same or to previous situations, with respect to the situation of the conclusion.

The semantics of AL^2 has as main feature the introduction of an *extra valuation* of formulas, called *pertinence*, parallel to the classical truth valuation. This additional valuation points out whether a formula has been *affected by change*, i.e., if it has been caused to take a new value. Informally speaking, a formula will be pertinent if either it is a direct observation of an action occurrence, or it is directly or indirectly affected by an action (through chaining of causal rules).

Formally, an interpretation in AL^2 will assign a pair of values to each formula. Apart from the usual truth value (true or false, $\{\mathtt{t},\mathtt{f}\}$), a formula will also be assigned a *pertinence* value: "pertinent" or "non-pertinent", denoted respectively as $\mathtt{p}$ and $\mathtt{n}$. An AL^2 *interpretation* I is a mapping $I : Atoms(\mathcal{A},\mathcal{F},n) \longrightarrow \{\mathtt{t},\mathtt{f}\} \times \{\mathtt{p},\mathtt{n}\}$ assigning truth and pertinence to each atom. In this way, we can consider a four-valued logic with the combinations $\{\mathtt{tp},\mathtt{tn},\mathtt{fp},\mathtt{fn}\}$. An interpretation I may be alternatively described by a pair $\langle S,P\rangle$ of sets of atoms for representing truth and pertinence respectively, so that, given an atom q_i: $q_i \in S$ iff $I(q_i) \in \{\mathtt{tp},\mathtt{tn}\}$; whereas $q_i \in P$ iff $I(q_i) \in \{\mathtt{tp},\mathtt{fp}\}$.

There are, however, two particular restrictions depending on the type of the atom q_i. First, when the atom is placed in the initial situation, $i = 0$, it will always be non-pertinent – there does not exist any $q_0 \in P$. This restriction will avoid application of rules at the initial state. Second, when the atom is an action, a_i, it will be assigned to be pertinent iff it is assigned to be true, that is, $a_i \in P$ iff $a_i \in S$. This restriction intuitively points out that an action is pertinent iff it has occurred and, together with the first restriction, that no action may occur at the initial situation.

Interpretations are extended in order to assign a truth and a pertinence value to each possible formula, following the rules in Figure 1.

$$I((\mathtt{prev}\ q)_i) = \begin{cases} \mathtt{tn} & \text{if } I(q_{i-1}) \in \{\mathtt{tp},\mathtt{tn}\} \\ \mathtt{fn} & \text{if } I(q_{i-1}) \in \{\mathtt{fp},\mathtt{fn}\} \end{cases}$$

$$I(\neg\phi) = \begin{cases} \mathtt{t}\alpha & \text{if } I(\phi) = \mathtt{f}\alpha \\ \mathtt{f}\alpha & \text{if } I(\phi) = \mathtt{t}\alpha \end{cases}$$

$$I(\psi \Leftarrow \phi) = \begin{cases} \mathtt{tn} & \text{if } I(\phi) \neq \mathtt{tp} \\ \mathtt{fp} & \text{if } I(\phi) = \mathtt{tp} \text{ and } I(\psi) \neq \mathtt{tp} \\ \mathtt{tp} & \text{otherwise} \end{cases}$$

$I(\phi \wedge \psi)$	fn	fp	tn	tp
fn	fn	fp	fn	fp
fp	fp	fp	fp	fp
tn	fn	fp	tn	tp
tp	fp	fp	tp	tp

$I(\phi \vee \psi)$	fn	fp	tn	tp
fn	fn	fp	tn	tp
fp	fp	fp	tp	tp
tn	tn	tp	tn	tp
tp	tp	tp	tp	tp

Figure 1. Interpretation of a formula.

Atomic expressions of shape $(\mathtt{prev}\ q)_i$ are used just to check the previous truth value of some symbol q without regarding its pertinence. For this reason, a $\mathtt{prev}$ expression is never assigned to be pertinent. Truth for propositional formulas is valuated as usual, maintaining pertinence valuation completely independent. A propositional formula is pertinent iff at least one of the atomic expressions occurring in the formula is pertinent. Finally, causal formulas are the only way of relating truth and pertinence. A causal formula will be true when: if its condition is true and pertinent, then its consequent is also true and pertinent.

Typically, an AL^2 domain description will consist of two sets of formulas $\langle B,O\rangle$: the set B called *background knowledge*, which contains causal rules, and the set O of *observations*, containing noncausal formulas. Since causal rules should be satisfied for any situation in the narrative, we will use abbreviations like $f_i \Leftarrow a_i \wedge g_{i-1}$ to stand for the set of rules instantiating $i = 1 \dots n$.

An interpretation I is said to be a *model* of a theory T iff it assigns to each formula $\phi \in T$ any value from $\{\mathtt{tp},\mathtt{tn}\}$. The set of models of T induces a *monotonic* entailment relation that is not expressive enough for representing the default rule of *inertia*: "under no evidence on the contrary, everything remains unchanged." The absence of an inertia default rule directly leads to the well-known *frame* problem [8, 14], unnaturally forcing us to represent what things *do not change*. To solve this problem, AL^2 introduces a models selection process, which also implements the causal behavior of the conditional expressions $F \Leftarrow \phi$ (solving, as explained in [9], the so-called *ramification* problem [16, 6, 7, 4, 15]). This models selection has two steps: first, we minimize pertinence among models (*selected* models) and, second, we require all the models to satisfy an *a posteriori* condition related to inertia (*causal* models).

Definition 1 ($\leq\mathtt{p}$) Given two AL^2 interpretations $I = \langle S,P\rangle$ and $I' = \langle S',P'\rangle$, we say that I is lower than I', $I \leq_{\mathtt{p}} I'$, iff $S = S'$ and $P \subseteq P'$. □

Definition 2 (AL^2 selected model) A model $I = \langle S,P\rangle$ of a theory T is said to be a *selected model* iff it is a minimal model, with respect to the $\leq_{\mathtt{p}}$ ordering relation. □

Definition 3 (Causal model) A selected model $I = \langle S,P\rangle$ of a theory T is said to be *causal* iff satisfies the condition: for any $i > 0$, and any f_i, if $f_i \notin P$ then ($f_i \in S$ iff $f_{i-1} \in S$). □

Intuitively, selected models impose the condition that things are nonpertinent by default, whereas causal models add to this requirement that any fluent nonpertinent should maintain its previous truth value. The entailment relation induced by causal models, $\models_c$, is now nonmonotonic. Let us see an example:

Example 1 (Yale Shooting scenario) We have a gun for killing a turkey. To this aim, we may perform an action $load$ that makes the gun become $loaded$, and an action $shoot$ that kills the turkey if the gun was loaded. Initially, the gun is loaded, the turkey is alive and we perform action $shoot$. □

Let $\mathcal{A} = \{load, shoot\}$, $\mathcal{F} = \{loaded, alive\}$ and $n = 1$. The domain description for the example could be simply formulated in AL^2 by the theory T containing the causal rules:

$$loaded_i \Leftarrow load_i \tag{4}$$

$$\neg alive_i \Leftarrow shoot_i \wedge (\mathtt{prev}\ loaded)_i \tag{5}$$

and the observations: $\{alive_0, loaded_0, shoot_1, \neg load_1\}$. It can be easily checked that the unique causal model of T is $\langle S,P\rangle$ with $S = \{alive_0, loaded_0, loaded_1, shoot_1\}$ and $P = \{shoot_1, alive_1\}$. Notice that our background knowledge does not specify what happens to $loaded$ when we $shoot$, and so, inertia takes charge of keeping $loaded$ true, as in the initial situation. Let us see what happens if we add a rule stating that the shot unloads the gun:

$$\neg loaded_i \Leftarrow shoot_i \wedge (\mathtt{prev}\ loaded)_i \tag{6}$$

Now, the extended theory has a unique causal model (in fact, it is the unique AL^2 model) in which the gun is finally unloaded. This example shows the nonmonotonic property of the $\models_c$ entailment, as we had that $T \models_c loaded_1$, whereas, by adding the new rule, $T \cup (6) \not\models_c loaded_1$ (in fact, $T \cup (6) \models_c \neg loaded_1$).

4 Adding temporal constraints to AL^2

In this section, we extend AL^2 into a new logic called AL^2_{TC} which incorporates temporal constraints. The syntax will include new atomic expressions (the already seen atomic constraints) to be freely combined in any noncausal formula. The intended utility of the temporal constraints will be to impose or test conditions on the elapsed *number of situations* that have occurred between two given events [3]. However, this immediately gives rise to a question: what is considered to be an *event* in an action-based formalism? or, in other words, which will be the set of time points, *TP* for the atomic constraints?

A first natural answer to this question is thinking on action occurrences as events, and on the situations in which they occur as time points. However, in AL^2 it is possible to go further, and consider also as an event the moment in which any atom, action or fluent, becomes pertinent. This results in a richer expressivity, allowing temporal constraints to relate *instants that depend on fluents*.

Let us introduce the formal definitions. Given a set of actions $\mathcal{A}$, a set of fluents $\mathcal{F}$ and the fixed length n, we define the set of *time point names*, $TP(A,F,n)$ as all those $t(q)_i$, for any $q \in \mathcal{A} \cup \mathcal{F}$ and $i \in [0,n]$. Notice that time point names have also a subindex, that is, they depend on the situation in which they are placed. The intuitive meaning of $t(q)_i$ is that it represents, at situation i, the number of the latest situation, $j \leq i$, in which q became pertinent. In other words, the last time in which q was affected by change.

The syntax of AL^2_{TC} is the same one of AL^2 but allowing a new type of atomic expression called *atomic constraint* with shape:

$$t(q)_i \trianglerighteq_d t(q')_i \qquad (7)$$

Notice that the set of events we are handling introduces a new feature not considered in traditional temporal constraint problems: an event may have not occurred. As we will see, we consider that the satisfaction of any temporal constraint like (7) implies that both q and q' have actually occurred in the past. For this reason, we will call *strong* to these kind of constraints and, for the expressivity sake, we will define the dual *weak* constraints:

$$t(q)_i \succeq_d t(q')_i \qquad (8)$$

that will allow q or q' not to occur.

The semantics of AL^2_{TC} is defined in the following way. An AL^2_{TC} interpretation I is defined as a triple $I = \langle S, P, M\rangle$, where S and P have the same meaning as in AL^2 and M is a mapping from *TP*(A,F,n) to the nonnegative integer numbers $[0,n]$, satisfying $M(t(q)_i) \leq i$, for any q_i. An assignment $M(t(q)_i) = 0$ will point out that q_i has never become pertinent. For satisfaction of formulas in AL^2_{TC}, we just add the following rules:

$$I(t(q)_i \trianglerighteq_d t(q')_i) = \begin{cases} \mathtt{tn} & \text{if } M(t(q)_i) \neq 0 \text{ and} \\ & M(t(q')_i) \neq 0 \text{ and} \\ & M(t(q')_i) - M(t(q)_i) \leq d \\ \mathtt{fn} & \text{otherwise} \end{cases}$$

$$I(t(q)_i \succeq_d t(q')_i) = \begin{cases} \mathtt{tn} & \text{if } M(t(q)_i) = 0 \text{ or} \\ & M(t(q')_i) = 0 \text{ or} \\ & M(t(q')_i) - M(t(q)_i) \leq d \\ \mathtt{fn} & \text{otherwise} \end{cases}$$

to the already introduced in Figure 1. As we can see, temporal constraints are never assigned a pertinence value (they behave in a similar way to the `prev` expression). Besides, strong constraints require both compared instants to be different from 0, whereas weak constraints will be directly true when any of those instants happen to be 0.

We directly recall the definition of *model* in AL^2, using now this extended definition of interpretation. As for *selected* models, we only need to redefine the ordering relation in the following way:

Definition 4 ($\leq$p) Given two AL^2_{TC} interpretations $I = \langle S, P, M\rangle$ and $I' = \langle S', P', M'\rangle$, we say that I is lower than I', written $I \leq_{\mathsf{p}} I'$, iff $S = S'$, $M = M'$ and $P \subseteq P'$. □

That is, in order to minimize pertinence, we fix not only the truth S, but also the time points assignment M.

Definition 5 (Causal model) A selected model $I = \langle S, P, M\rangle$ of a theory T is said to be *causal* iff it satisfies the conditions:

1. For any $f_i \notin P$, $f_i \in S$ iff $f_{i-1} \in S$
2. For any $M(t(q)_i) = 0$, there is no $q_j \in P$, $j \leq i$
3. For any $M(t(q)_i) \neq 0$,
   $M(t(q)_i) = max\{j \leq i$ such that $q_j \in P\}$

□

The two new conditions fix the meaning of time points with respect to the pertinences occurring in the selected model. For time points $t(q)_i$ assigned to 0, we require that q has never been pertinent up to situation i. For time points different from 0, the value must coincide with the last recent situation j before i in which q was pertinent.

Notice that, until this step of models selection, we had not imposed any relation between the assigned number to a time point $t(q)_i$ and the information about q_i contained in the model. This means that, for AL^2_{TC} models and selected models, the assigned numbers to time points are completely arbitrary with respect to pertinence of fluents and actions.

We will illustrate how temporal constraints can be mixed with an usual action theory by an elaboration of example 1.

Example 2 (Spoiling Yale Shooting Scenario) Assume now that a shot only kills the turkey if we performed a load at most two situations before (otherwise, the gunpowder in the bullet spoils). Initially the turkey is alive and the gun unloaded. We load the gun and after two situations without performing any action, we shoot. □

The domain description T_{spoil} for this scenario consists of the set B of causal rules:

$$\begin{array}{lcl} loaded_i & \Leftarrow & load_i \\ \neg alive_i & \Leftarrow & shoot \wedge (\mathtt{prev}\ loaded)_i \wedge (t(load)_i \trianglerighteq_2 t(shoot)_i) \\ \neg loaded_i & \Leftarrow & shoot \wedge (\mathtt{prev}\ loaded)_i \end{array}$$

and the set O of observations:

$$\{alive_0, \neg loaded_0,\ \ \neg shoot_1, load_1,\ \ \neg shoot_2, \neg load_2, \neg shoot_3, \neg load_3,\ \ shoot_4, \neg load_4\}$$

After applying the model selection process, it can be checked that the unique causal model of T_{spoil} is $I = \langle S, P, M\rangle$ where:

$$\begin{array}{ll} S = & \{alive_0, load_1, loaded_1, alive_1, loaded_2, alive_2, \\ & loaded_3, alive_3, shoot_4, alive_4\} \\ P = & \{load_1, loaded_1, shoot_4, loaded_4\} \end{array}$$

$M \backslash i$	0	1	2	3	4
$t(alive)_i$	0	0	0	0	0
$t(loaded)_i$	0	1	1	1	4
$t(shoot)_i$	0	0	0	0	4
$t(load)_i$	0	1	1	1	1

[3] In fact, a direct extension for real time labels could easily be proposed.

In a similar way, if we load the gun and we shoot two situations later, then, the only AL^2_{TC} model corresponds to:

$S' = \{alive_0, loaded_1, alive_1, loaded_2, alive_2\}$
$P' = \{load_1, loaded_1, shoot_3, loaded_3, alive_3\}$

$M' \backslash i$	0	1	2	3
$t(alive)_i$	0	0	0	3
$t(loaded)_i$	0	1	1	3
$t(shoot)_i$	0	0	0	3
$t(load)_i$	0	1	1	1

5 Related work

Few bibliography can be found relating temporal constraints with action-based formalisms. The most relevant existing approach is perhaps the language HOT introduced in [12], which introduces temporal constraints into the *Event Calculus* [5]. Language HOT handles the Event Calculus usual predicates $Holds(\phi, E_1, E_2)$ (asserting that fact ϕ permanently holds between events E_1, E_2), and $Occurs(E)$ (asserting that event E has occurred). Besides, a new function $Time(E)$ is used for associating to each event a real number that represents the instant in which E occurred. Apart from propositional formulas for $Holds$ and $Occurs$, HOT allows defining both qualitative and quantitative constraints, being the shape of the latter:

$$Time(E) - Time(E') \in I_1 \cup I_2 \cup \ldots \cup I_k \tag{9}$$

where I_i are intervals over real numbers. As we can see, points $Time(E)$ bear a strong resemblance to $t(q)_i$ (excepting that the former refers to real time instead of a situation). However, the $t(q)_i$ points are more expressive, since they also allow relating instants about fluents, using their pertinence for that purpose. Another common problem that also arises in HOT is the possibility of non-occurrence of events. In fact, HOT constraints directly correspond to what we have called *strong* temporal constraints in AL^2_{TC} (they always require the occurrence of the related events).

The major advantage of AL^2_{TC} with respect to HOT is that the latter does not provide an appropriated solution to the frame and ramification problems[4], and so, our approach means, in this sense, a significative advance.

6 Conclusions

We have presented a logical approach, AL^2_{TC}, which combines interesting representational features from a causal formalism for reasoning about actions, AL^2, and from Temporal Constraint Networks representation. From the AL^2 point of view, the new features provide a first natural step towards the introduction of continuity in the time basis. Our approach differs, in this sense, from the usual treatment given to continuity in action based approaches [11, 10], which handles fluents with a real domain that *evolve continuously*, while the number of situations or the distances among them are kept constant. From the temporal constraints point of view, the features provided by AL^2 constitute a significative increase in representational power. We have avoided two strong restrictions that are implicit assumptions in TCN problems: (1) events are always known to have occurred and cannot be repeated and (2) each event contains all the information about the system state (we cannot express things in terms of system properties). Besides, this is done simultaneously providing the solution to the frame and ramification problems, already present in AL^2.

As future work, we will study how the existing TCN techniques can be used for improving inference in particular kinds of AL^2_{TC} theories. For instance, if we restrict the use of constraints for limiting the instants in which actions can be performed this directly corresponds to a STP, a well-known kind of problems for which there exist much work on efficient algorithms.

[4] Notice that, for instance, the study on how to solve ramification problem for the Event Calculus is still a topic of current research [15].

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REFERENCES

[1] Matteo Baldoni, Laura Giordano, Alberto Martelli, and Viviana Patti, 'An abductive proof procedure for reasoning about actions in modal logic programming', in *Proc. of Non-Monotonic Extensions on Logic Programming (NMELP'96)*, (1996).

[2] Marcos A. Castilho, Olivier Gasquet, and Andreas Herzig, 'Formalizing action and change in modal logic I: the frame problem', *Journal of Logic and Computation*, (to appear).

[3] R. Dechter, I. Meiri, and J. Pearl, 'Temporal constraint networks', *Artificial Intelligence Journal*, **49**, 61–95, (1991).

[4] J. Gustafsson and P. Doherty, 'Embracing occlusion in specifying the indirect effects of actions', in *Proc. of the 5th Intl. Conference on Principles of Knowledge Representation and Reasoning*, (1996).

[5] R. Kowalski and M. Sergot, 'A logic-based calculus of events', *New Generation Computing*, **4**, 67–95, (1986).

[6] Fangzhen Lin, 'Embracing causality in specifying the indirect effects of actions', in *Proc. of the Intl. Joint Conference on Artificial Intelligence (IJCAI)*, ed., C. S. Mellish, Montreal, Canada, (August 1995). Morgan Kaufmann.

[7] Norman McCain and Hudson Turner, 'A causal theory of ramifications and qualifications', in *Proc. of the Intl. Joint Conference on Artificial Intelligence (IJCAI)*, ed., C. S. Mellish, pp. 1978–1984, Montreal, Canada, (August 1995). Morgan Kaufmann.

[8] J. McCarthy and P. Hayes, 'Some philosophical problems from the standpoint of artificial intelligence', *Machine Intelligence Journal*, **4**, 463–512, (1969).

[9] R.P. Otero and P. Cabalar, 'Pertinence and causality', in *Proc. of the 3rd Workshop on Nonmonotonic Reasoning, Action, and Change (NRAC),IJCAI'99*, Stockholm, (August 1999).

[10] E. Sandewall, 'Combining logic and differential equations for describing real world systems', in *Proc. of the First Intl. Conference on Principles of Knowledge Representation and Reasoning*, eds., H. Levesque R. Brachman and R. Reiter, pp. 412–420. Morgan Kaufmann, (1989).

[11] E. Sandewall, 'Filter preferential entailment for the logic of action in almost continuous worlds', in *Proc. of the Intl. Joint Conference on Artificial Intelligence (IJCAI)*, ed., C. S. Mellish, pp. 894–899. Morgan Kaufmann, (1989).

[12] E. Schwalb, K. Kask, and R. Dechter, 'Temporal reasoning with constraints on fluents and events', in *Proc. of the 12th National Conference of Artificial Intelligence*, pp. 1067–1072, (1994).

[13] C. B. Schwind, 'A logic based framework for action theories', in *The Tbilisi Symposium on Logic, Language and Computation*, eds., J. Ginzburg, Z. Khasidashvili, C. Vogel, J.-J. Lévy, and E. Vallduví, 275–291, (1997).

[14] M. Shanahan, *Solving the Frame Problem*, The MIT Press, 1997.

[15] M. Shanahan, 'The ramification problem in the event calculus', in *Proc. of the Intl. Joint Conference on Artificial Intelligence (IJCAI)*, ed., C. S. Mellish, pp. 140–146. Morgan Kaufmann, (1999).

[16] Michael Thielscher, 'Ramification and causality', *Artificial Intelligence Journal*, (1997). (To appear. A preliminary version is available as Technical Report TR-96-003, ICSI, Berkeley, CA).