

# Combining Two Heuristics to Solve a Supply Chain Optimization Problem

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**Abstract.** In this paper, we consider a real-life supply chain optimization problem concerned with supplying a product from multiple warehouses to multiple geographically dispersed retailers. Each retailer faces a deterministic and period-dependent demand over some finite planning horizon. The demand of each retailer is satisfied by the supply from some predetermined warehouse through a fleet of vehicles which are only available within certain time windows at each period. Our goal is to identify a combined inventory and routing schedule such that the system-wide total cost over the planning horizon is minimised. This problem in essence is an amalgamation of two classical NP-hard optimization problems: the Dynamic Lotsizing problem and the Vehicle Routing problem. In this paper, we propose an efficient rolling horizon heuristic that combines two heuristics to solve this problem. Numerical experiment results show that our approach can achieve, on average, within 10% of the lower-bound proposed by Chan, Federgruen and Simchi-Levi (1998) for some specific instances generated from Solomon benchmarks.

**Key Words:** Meta-Heuristics, Planning, Scheduling, Search.

## 1 Introduction

Consider a logistics system consisting of multiple warehouses and multiple geographically dispersed retailers. Each retailer faces a deterministic and period-dependent demand over a finite planning horizon that must be fulfilled by the supply from a predetermined warehouse through a fleet of vehicles. Under supply chain management philosophy, these supply activities should be co-ordinated in order to exploit economies of scale and other benefits arising from supply chain integration. Hence, the goal is to identify a combined inventory and routing strategy such that the system-wide total cost over the planning horizon is minimised. The replenishment for each retailer at each period includes two activities: pickup from the predetermined warehouse and delivery to the retailer. These two activities must be performed within certain time windows, and incur a warehouse-specific pickup service duration and a retailer-specific delivery service duration. There is a fixed number of capacitated vehicles available within certain time window at each period. The maximum available supply that can be provided by each warehouse at the beginning of each period is known. Note that the demands of some retailers at certain period may exceed this supply quantity, hence shortage (backlogging) is possible for some retailers. The cost of a route consists of a fixed component and a component which is proportional with the total travelling distance. The inventory cost at each retailer includes (a) a retailer-specific fixed service cost for each

replenishment, and (b) the holding or shortage cost, which is proportional with the stock level.

This problem, which we call the multi-warehouse multi-retailer distribution problem (MMDP), is essentially an amalgamation of two classical optimization problems: dynamic capacitated lot-sizing problem [Florian *et al.* (1980)] and vehicle routing problem with time-windows (VRPTW) [Solomon (1987)], both of which are NP-hard. MMDP is a generalisation of the classical Inventory Routing Problem (IRP) in logistics management. For a survey on inventory-routing problems, the reader may refer to Federgruen and Simchi-Levi (1995). In Campbell *et al.* (1998), a simplified version of IRP was considered. Even for that problem, the authors, who proposed an integer programming approach, reported that "This model is not very practical for two reasons: the huge number of possible delivery routes, and although to a lesser extent, the length of the planning horizon. To make this integer program computationally tractable, we only consider a small (but good) set of routes and aggregate time periods towards the end of the planning horizon." Hence, even for small instances of IRP, the resulting mathematical program can be huge.

To our knowledge, a logistics problem as complicated as MMDP has not been extensively studied and much less experimented in the literature. In this paper, we propose a very simple yet effective rolling horizon heuristic for solving MMDP. This heuristic can be regarded as a meta-heuristic, in the sense that it incorporates two classical heuristics, and guides them in a novel way that makes it an effective approach for MMDP. These classical heuristics are good candidates because they have proven to work well in industry for solving classical inventory and routing problems. It turns out that by extending these classical heuristics slightly to take all our problem constraints into consideration, and by embedding them within our proposed meta-heuristic, the resulting approach is intuitive, easy to implement, and produces good solutions! Experimental results show that our approach can achieve, on average, within 10% of the lower-bound proposed by Chan, Federgruen and Simchi-Levi (1998) for some IRP instances generated from Solomon benchmarks [Solomon (1987)]. It also solves large-scale MMDP problems that we generated very quickly and effectively.

## 2 Literature Review and Notations

The one-warehouse multi-retailer distribution problem was first introduced by Anily and Federgruen (1990). The authors restricted their analysis to a class of replenishment strategies in which their regional partitioning scheme is asymptotically optimal. Subsequent works were restricted to other classes of strategies. Gallego and Simchi-Levi (1990) proved that Direct Shipping Strategies are within 6% of optimality under some assumptions. Herer and Roundy (1997)

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as well as Viswanathan and Mathur (1997) showed a good empirical performance for the so-called power-of-two strategies. Under this policy each retailer is replenished at constant intervals which are power of two multiples of a common base planning period. Chan, Federgruen and Simchi-Levi (1998) showed the effectiveness of the class of so-called Fixed-Partitioning policy under similar assumptions stated in Anily and Federgruen (1990). They observed that the solutions generated by their approach are relatively close to the lower bound: the optimality gap with respect to the lower bound is always less than 16% and in most cases no more than 10% for a set of randomly generated problems.

As far as heuristic approaches are concerned, Campbell *et al.* (2001) recently proposed a two-phase solution approach. In the first phase, they determine which customers receive a delivery on each day of the planning period and on the size of the deliveries. In the second phase, they determine the actual delivery routes and schedules for each of the day.

The majority of the literature is related to the deterministic models such as the research works listed above. Dynamic and stochastic models are very difficult to solve in general. In early works, the planning horizon for these models is only one day or a very short horizon, such as the works of Federgruen and Zipkin (1984), Golden, Assad, and Dahl (1984), Chien, Balakrishnan, and Wong (1989), Dror and Ball (1987). The approaches differ mainly in how they decide which customers to include, taking the short-term decisions into account. Minkov (1993) and Kleywegt *et al.* (1997) formulate the inventory routing problem as a Markov decision process.

Most of the research in the literature to date assume that the underlying routing subproblem is a pure vehicle routing problem. But in the real life, the routing subproblem of some multi-warehouse multi-retailer distribution systems is a pick up and delivery problem with time window constraints.

In this paper, we assume that one unit of any product consumes one unit capacity of each vehicle, and the cost per unit distance travelled equal 1. We also assume warehouses have infinite supply capacity (which can be easily relaxed under our model). We denote the multiple retailers as  $r_1, r_2, \dots, r_I$ , and the vehicles as  $v_1, v_2, \dots, v_L$ . The following inputs and their notations are used:

$H$ : finite planning horizon, *i.e.*,  $1 \leq t \leq H$

$d_{it}$ : demand for retailer  $r_i$  in period  $t$

$h_{it}$ : unit holding cost of retailer  $r_i$  in period  $t$

$b_{it}$ : unit shortage cost of retailer  $r_i$  in period  $t$

$K_{it}$ : fixed replenishment service cost of retailer  $r_i$  in period  $t$

$S_i$ : fixed service cost incurred each time vehicle  $v_i$  is used

$U_{jt}$ : maximum available supply at warehouse  $w_j$  in period  $t$

$u_{it}$ : inventory capacity of retailer  $r_i$  in period  $t$

$C_l$ : capacity of vehicle  $v_l$

MMDP seeks to find a combined inventory and routing plan that minimizes the sum of holding costs, shortage costs, service costs, and routing costs subject to demands, inventory capacity and vehicle capacity constraints.

### 3 Proposed Solution

The core issues in MMDP are inventory allocation and vehicle route sequencing. These two activities are interrelated in the following way. In order to determine which retailer should be serviced and the amount to be delivered to each retailer in each period, we need the feasibility and cost associated with routing the vehicles. On the other hand, the routing cost and feasibility can only be determined if we

have decided on the selected retailers and the delivery quantities in each period.

The basic idea behind our rolling horizon heuristic is that we first solve the inventory allocation, followed by the vehicle routing problem. We then in turn use the routing cost information of each vehicle to adjust the delivery quantities for all retailers and resolve the vehicle routing problem. We will repeat this process period by period from the first period until the end of the planning horizon.

Before we present the detailed description of our approach, we first discuss two related problems: the dynamic lot sizing problem with backlogging and capacity constraints, and the vehicle routing problem for pickup and delivery with time window constraint.

#### 3.1 Dynamic Lot Sizing Problem

For any fixed retailer  $r_i$ , we can formulate a dynamic lot sizing problem. For simplicity, we drop the index  $i$ , *i.e.* for any period  $t (1 \leq t \leq H)$ ,  $d_t, h_t, b_t, K_t$  and  $u_t$ , denote the quantities described above respectively.

Lot sizing is essentially the problem of balancing the fixed replenishment costs with the inventory holding costs. The decision to be made is what quantity should be ordered at the beginning of each period. The notations and assumptions used in the description of the dynamic lot sizing problem are summarised as follows. All ordering and demands occur at the beginning of each period and the inventory cost is charged according to the inventory level at the end of each period.

Let  $q_t$  denote the order quantity in period  $t$ . Let  $initInv_t$  and  $inv_t$  be the inventory level at the beginning and end of period  $t$  respectively. Clearly,  $inv_t$  is equal to  $initInv_t - d_t + q_t$ . For any period  $t (> 1)$ , it is clear that  $initInv_t$  is equal to  $inv_{t-1}$ .

Our heuristic approach to this problem is a modification of the classical *Silver-Meal Heuristic* due to Silver and Meal (1973), taking inventory capacity and backlogging cost into consideration. Without loss of generality, in the following, we are only concerned with the computation of  $q_1$ . Computing other quantities  $q_t$  can be done by calling this heuristic iteratively. Let  $\mu_t$  denote the current average period cost with respect to  $q_1$ , which is the total cost incurred from periods 1 to  $t$  by ordering  $q_1$  units at the beginning of period 1 divided by  $t$ , *i.e.*,  $\mu_t = \frac{1}{t}(K_1\delta(q_1) + \sum_{l=1}^t (h_l inv_l + b_l \gamma_l))$  where:  $\delta(q_1) = 1$  if  $q_1 > 0$  and 0 otherwise; and  $\gamma_l = d_l - initInv_l$  if  $initInv_l < d_l$  and 0 otherwise. We say that  $d_i$  can be covered by  $q_1$  iff  $0 \leq (initInv_1 + q_1) + d_i - \sum_{l=1}^t d_l \leq u_t$  for all  $t (1 \leq t \leq i)$ .

#### Modified Silver-Meal Heuristic:

##### 1. Initialisation.

- (a). Initialize  $q_1 = 0$ .
- (b). Determine the maximum value  $\Delta$  such that  $d_1$  can be covered by  $q_1$ .
- (c). If  $\Delta > 0$ , then compute  $\mu_1$ . Update  $inv_t$  from 1 to  $T$ . Set  $q_1 = q_1 + \Delta$ .
- (d). Else STOP.

##### 2. For period $t = 2$ to $H$ , do the following:

- (a). Determine the maximum value  $\Delta$  such that  $d_t$  can be covered by  $q_1$ .
- (b). If  $\Delta > 0$ , then compute  $\mu_t$ , else STOP.
- (c). If  $\mu_t < \mu_{t-1}$ , then update  $inv_t$  from 1 to  $T$  and set  $q_1 = q_1 + \Delta$ . Otherwise, STOP.

### 3.2 Pickup and Delivery Problem

The routing problem arising in MMDP is a pickup and delivery problem with time windows (PDPTW). The objective is to construct a set of routes for a fleet of vehicles which services a set of customers with known demands and time windows. The objective is to minimize the routing cost, defined by the sum of fixed service costs due to vehicle usage (i.e.  $S_l$ 's) and the travel costs (which is assumed to be total distance travelled). Each vehicle starts and ends at a given location. Each customer possesses an original location (pickup location), a destination location (delivery location), a pickup time window, a delivery time window, a pickup service duration, and a delivery service duration. The pickup (resp. delivery) service of a customer can only begin within the pickup (resp. delivery) time window. Each route must satisfy the pairing constraint, since the corresponding pickup and delivery locations must be serviced by the same vehicle. The quantity of the demand at the pickup location is positive, but the quantity of the demand at the delivery location is negative. For example, if the size of the pickup job is  $+d$ , the size of the delivery job should be  $-d$ . When a vehicle services a pickup job, its load is increased by the job load. When it services a delivery job, its load is reduced by that amount. The load of each vehicle  $l$  at any time cannot exceed the vehicle capacity ( $C_l$ ).

Most of the literature on pickup and delivery problems focus on the dial-a-ride problem (DARP). The size of the job in DARP is 1 at the pickup location and  $-1$  at the delivery location. Hence it can be regarded as a special case of PDPTW. We propose the following two-phase heuristic for solving PDPTW. In phase 1, we apply an insertion heuristic to construct an initial solution, which is improved via a greedy local improvement method in phase 2. Conceivably, our proposed heuristic may be improved using more powerful search strategies such as tabu search. However, in this paper, our purpose is not to propose a new algorithm to solve PDPTW per se, but rather to investigate the effectiveness of combining heuristics to solve a more complex problem.

Define the *insertion cost* of a pickup job to be inserted at a specified location on a vehicle as follows. Fix the pickup job at that specified location and insert the corresponding delivery job after the pickup job at the position that minimizes the resulting routing cost while preserving feasibility. If found, the insertion cost is the resulting cost minus the old routing cost; otherwise, it is set to infinity.

#### Insertion Heuristic for PDPTW

1. *Initialisation: the unassigned pool consists of the set of all unassigned pickup jobs. The route of each vehicle is empty. Let the insertion cost of the best insertion move be infinity.*
2. *If the unassigned pool is empty, then STOP.*
3. *For each unassigned pickup job and each insertion position on each vehicle, compute the insertion cost. If it is smaller than the insertion cost of the best insertion move, update the best insertion move.*
4. *If the insertion cost of the best insertion move is not infinity, we insert the corresponding pickup and delivery job pair, delete this pickup job from the unassigned pool, and let the insertion cost of the best insertion move be infinity. Otherwise, STOP.*
5. *Goto Step 2.*

By using the above Insertion Heuristic, we obtain an initial solution. We apply the following Greedy Improvement Heuristic for PDPTW to improve the quality of the initial solution.

There are two basic move operations in our Greedy Improvement Heuristic: the relocate operation and the exchange operation.

The relocate operation involves a pickup and delivery job pair  $(PJ_1, DJ_1)$  on vehicle  $v_1$ , the current insertion position  $l_1$  of  $PJ_1$ , and the one possible insertion position  $l_2$  of  $PJ_1$  on vehicle  $v_2$  ( $v_2 \neq v_1$ ). Notice that *both* pickup and delivery jobs are removed from  $v_1$  and inserted into  $v_2$ . The cost of a relocate operation is the increase in the routing cost due to the operation. If the new routes are not feasible with respect to the vehicle capacity and time window constraints, the cost is set to infinity.

The exchange operation involves two pickup and delivery job pairs  $(PJ_1, DJ_1)$  on vehicle  $v_1$  and  $(PJ_2, DJ_2)$  on  $v_2$  ( $v_2 \neq v_1$ ), and the insertion positions  $l_1$  of  $PJ_1$  and  $l_2$  of  $PJ_2$ . In this operation, we remove  $PJ_1$  from  $v_1$  and  $PJ_2$  from vehicle  $v_2$  and then insert  $PJ_1$  to the position  $l_2$  on  $v_2$  and  $PJ_2$  to the position  $l_1$  on  $v_1$ . The cost of an exchange operation is the increase in the routing cost due to the operation. If the new routes are not feasible with respect to the vehicle capacity and time window constraints, the cost is set to infinity.

#### Greedy Improvement Heuristic for PDPTW

1. *Input the initial solution generated by the Insertion Heuristic.*
2. *Find the operation with the minimal cost among all possible relocate and exchange operations.*
3. *If this minimal cost is positive, then STOP. Otherwise, execute the corresponding operation and goto Step 2.*

### 3.3 Rolling Horizon Heuristic

Based on the two heuristics described above for solving the dynamic lot sizing problem and the pick up and delivery problem with time windows respectively, we propose the following meta-heuristic for solving MMDP. These two heuristics are treated as black boxes and called in the following Rolling Horizon Heuristic.

After the determination of the delivery quantity to all retailers by calling the Modified Silver-Meal Heuristic at the beginning of certain period  $t$ , we need to check whether the maximum available supply at each warehouse (i.e.  $U_{jt}$ ) is sufficient to satisfy the delivery quantity to all related retailers. If it cannot be satisfied, we first sort all related retailers according to their respective distances to the warehouse. We then delete the delivery jobs that are furthest from the warehouse until the maximum available supply can satisfy the total delivery quantity from this warehouse in period  $t$ .

We then construct the routes for period  $t$  by calling the Insertion Heuristic and Greedy Improvement Heuristic. As there are fixed number of vehicles available at each period, some pickup and delivery job pairs may not be serviced. All these unserved jobs are deleted. For all served jobs, in order to balance the routing cost and the inventory cost, we readjust the delivery quantity for all related retailers at the beginning of certain period  $t$  by using the routing cost information, computed as follows.

For each vehicle  $v$  used, let the Hamiltonian cost covering the pickup locations and delivery locations on the corresponding route be  $L_v$ . We regard  $\lambda L_v$  as the additional fixed service cost for all retailers  $R(v)$  serviced by vehicle  $v$  in period  $t$ , where the  $\lambda$  is a small positive parameter ( $0 \leq \lambda \leq 1.5$ ). The delivery quantity for each retailer in  $R(v)$  will cover the demands for some periods from period  $t$  onward. We randomly select one retailer from  $R(v)$  with the minimal value of the number of covered periods by the delivery quantity to this retailer and adjust the delivery quantity to this retailer by cov-

ering one more period, repeat this process for  $R(v)$  whenever the total increased holding cost is less than  $\lambda L_v$ .

In the following Rolling Horizon Heuristic, we will generate 30 solutions for each instance by varying the value of  $\lambda$  from 0 to 1.5 incremented by  $L = 0.05$  for each iteration.

**Rolling Horizon Heuristic:**

1. *Initialisation: Let  $\lambda = 0.0$  and  $L = 0.05$ . Let the planning horizon be  $H$  periods.*
2. *While  $\lambda \leq 1.5$  do the following:*
  - 2.1 *For period  $t = 1$  to  $H$ , do:*
    - (a) *Determine the delivery quantity to each retailer in period  $t$  using the Modified Silver-Meal Heuristic.*
    - (b) *Construct vehicle routes to service these retailers using the Insertion Heuristic and Greedy Improvement Heuristic.*
    - (c) *Adjust the delivery quantity to each retailer in period  $t$  by using the routing cost information and  $\lambda$ .*
    - (d) *Update the inventory and initial inventory for all retailers from period  $t$  to period  $H$ .*
  - 2.2 *Set  $\lambda = \lambda + L$ .*
3. *Choose the best solution among the solutions generated in Step 2.*

**4 Numerical Experiments**

In this section, we report numerical experiments conducted to verify our proposed method. For all experiments described in this section, we let the number of warehouses and retailers to be 1 and 100 respectively. The planning horizon  $H = 5$ . We assume that the length of each period is 1. Hence there are 5 periods.

**4.1 MMDP Problems**

We generated MMDP test instances based on the generalization of Solomon benchmark VRPTW problems, described as follows. The locations, time windows, demands and service durations of retailers are exactly the same as the customer data in the Solomon test cases. We set the holding cost and backlogging cost for all retailers at all periods to be 1 and 10, respectively. We set the inventory capacity of all retailers at all periods are to infinity. Likewise, for the single warehouse, the location and the time windows for all periods are the same as the location and the time window of the depot. The maximum available supply at each period is set to infinity. Hence the pickup location for any retailer is the location of the single warehouse. The vehicle information at each period is the same as the vehicle information in the Solomon test cases.

For each instance, we obtained 30 solutions by varying the value of  $\lambda(0 \leq \lambda \leq 1.5)$ . Due to space constraints, we will show sample experimental runs for C101, R101 and RC101. Figures 1 to 3 show the objective value (i.e. total cost) obtained by varying the values of  $\lambda$ . As shown in the figures, we observe that for random and semi-random instances (R101 and RC101), the value of  $\lambda$  affects the quality of the solution, although this cannot be generally said of the C instances. We also observed similar behavior patterns in other R, C and RC generalized Solomon instances. Hence, there is no particular value of  $\lambda$  that will work well for all instances, whether for R or RC test cases. For C test cases, experiments reveal that setting  $\lambda = 0$  almost always produces the best solution.

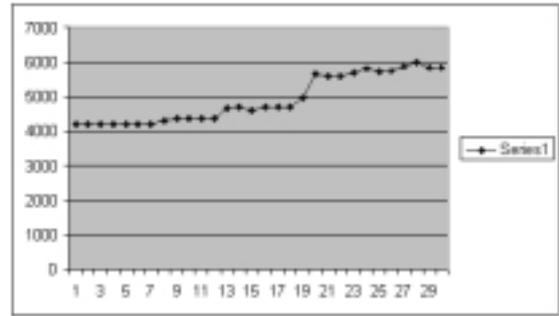


Figure 1. Results of C101 instance

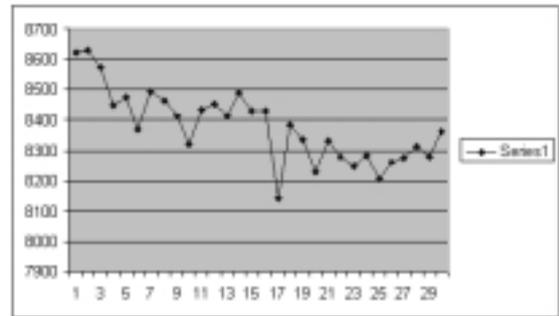


Figure 2. Results of R101 instance

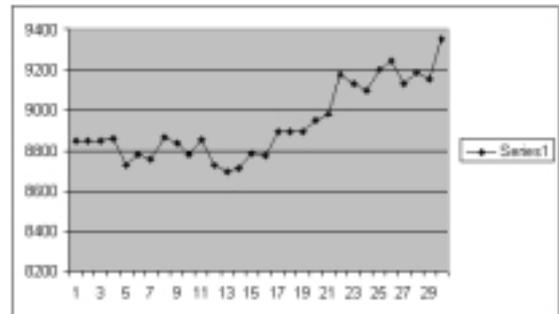


Figure 3. Results of RC101 instance

## 4.2 Problems with Known Lower Bounds

In this subsection, we consider a special case of the MMDP problem which was studied in Chan, Federgruen and Simchi-Levi (1998) (abbrev. CFS). Our purpose is to compare the objective values obtained by our approach against the lower bound proposed in CFS. In order for the lower bound proposed in CFS to be still valid, we need to make the following modifications to the problem instances.

1. In CFS, each retailer faces a constant demand rate, which is a special case of MMDP achieved by setting all period-dependent demands for each retailer to be equal.
2. Since demands are constant over time, there is no difference between the infinite horizon considered in CFS and the fixed planning horizon considered in this paper.
3. In CFS, backorders are not allowed, which can be emulated in MMDP by setting the unit shortage costs to be a large constant.
4. In accordance with CFS, we set the holding cost per unit time  $h = 6$  for all retailers and periods, the routing cost per unit distance to be 1, and the fixed service cost for all vehicles to be equal.

All other parameter values are the same as the experimental setup described in the above sub-section.

Notice that since the holding cost is much larger than the routing cost, the parameter  $\lambda$  has no effect on improving the quality of the solutions. Hence, for the experiments in this section, we conveniently use  $\lambda = 0$ .

The lower bound  $B^*$  obtained in CFS is given as follows. Let  $C$  be the identical vehicle capacity and the  $S$  be the fixed service cost of each used vehicle. Let  $D_i$  be the demand rate for retailer  $r_i$  and  $l_i$  be the distance between the retailer  $r_i$  ( $1 \leq i \leq 100$ ) and the warehouse. Then,  $B^* = H \sum_{i=1}^{100} [\frac{D_i(2l_i+S)}{C} + \frac{hD_i}{2}]$ . For the planning period  $H = 5$ , the values of the lower bound ( $B^*$ ) and the corresponding values of our solutions ( $Z_H$ ) are compared in Table 2.

**Table 2.** Our Cost  $Z_H$  over the Lower Bound  $B^*$

	$Z_H$	$B^*$	$\frac{Z_H}{B^*}$
C101	32529.7	29901.1	1.08791
C201	30352.4	27946.1	1.08611
R101	26579.6	23715.5	1.12077
R201	25465.5	22239.1	1.14508
RC101	31480.6	28820.3	1.09231
RC201	29514.2	26452.1	1.11576

From Table 2, we observe that the quality of the solutions generated by our approach is very promising. It is on average within 10% of the lower bound proposed by CFS.

## 5 Conclusion

In this paper, we presented a realistic model for managing a complex supply chain optimization problem involving inventory and route scheduling over multiple warehouses, multiple retailers and time-dependent demands. The underlying routing problem we considered is the pickup and delivery problem with time window constraints, which is more realistic yet much more computationally challenging. We proposed an efficient meta-heuristic based on combining known heuristics. The key feature of this heuristic is that in order to balance the routing and inventory costs, we use the Hamiltonian cost related to each vehicle to adjust the delivery quantities for the retailers. Experimentally, we observed that this adjustment process can generate

a better solution for some test cases. In order to verify the quality of the generated solutions, we performed further numerical experiments for cases where there are known lower bounds. The solutions generated by our approach have objective values which are within 10% on average of the lower bounds proposed by Chan, Federgruen, and Simchi-Levi (1998).

In this paper, we used the same parameter  $\lambda$  for all routes in all periods during each iteration. What we believe could be promising in obtaining better solutions is to use different parameters  $\lambda$  for different routes in each iteration. Moreover, by using the routing cost information, perhaps a different process (compared to ours) can be used to adjust the delivery quantity for all retailers at a certain period. These ideas require more analysis and extensive numerical experimentation.

The main purpose for using the Solomon test cases to generate instances of MMDP is so that these instances can be generated by others for the purpose of comparison with our results. These instances can foreseeably evolve into benchmark problem instances for MMDP.

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