

Public Key Compression and Modulus Switching for Fully Homomorphic Encryption over the Integers

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Fully homomorphic encryption

- Multiplicatively homomorphic: RSA.

$$\begin{aligned} c_1 &= m_1^e \bmod N \\ c_2 &= m_2^e \bmod N \end{aligned} \Rightarrow c_1 \cdot c_2 = (m_1 \cdot m_2)^e \bmod N$$

- Additively homomorphic: Paillier

$$\begin{aligned} c_1 &= g^{m_1} \bmod N^2 \\ c_2 &= g^{m_2} \bmod N^2 \end{aligned} \Rightarrow c_1 \cdot c_2 = g^{m_1+m_2} \bmod N^2$$

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 - Open problem until Gentry's breakthrough in 2009.

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Fully Homomorphic Encryption Schemes

- 1. Breakthrough scheme of Gentry [G09], based on ideal lattices. Some optimizations by [SV10].
 - Implementation [GH11]: PK size: 2.3 GB, recrypt: 30 min.
- 2. van Dijk, Gentry, Halevi and Vaikuntanathan's scheme over the integers [DGHV10].
 - Implementation [CMNT11]: PK size: 1 GB, recrypt: 15 min.
- 3. RLWE schemes [BV11a,BV11b].
 - FHE without bootstrapping [BGV11]
 - Batch FHE (next talk !)
 - Implementation with homomorphic evaluation of AES [GHS12]
- This talk: smaller PK for DGHV (10 MB) and improved attack against DGHV.

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The DGHV Scheme

- Ciphertext for $m \in \{0, 1\}$:

$$c = q \cdot p + 2r + m$$

where p is the secret-key, q and r are randoms.

- Decryption:

$$(c \bmod p) \bmod 2 = m$$

- Parameters:



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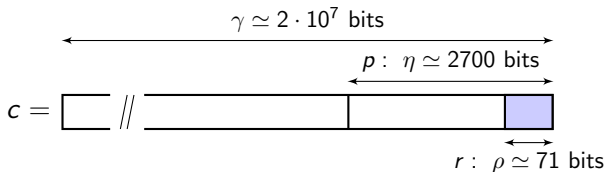
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Homomorphic Properties of DGHV

- Addition:

$$\begin{aligned}c_1 &= q_1 \cdot p + 2r_1 + m_1 \\c_2 &= q_2 \cdot p + 2r_2 + m_2\end{aligned} \Rightarrow c_1 + c_2 = q' \cdot p + 2r' + m_1 + m_2$$

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with

$$r'' = 2r_1r_2 + r_1m_2 + r_2m_1$$

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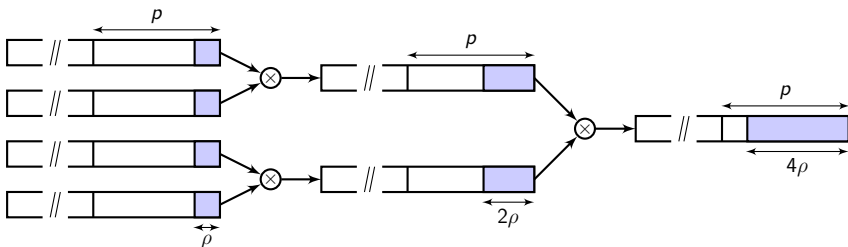
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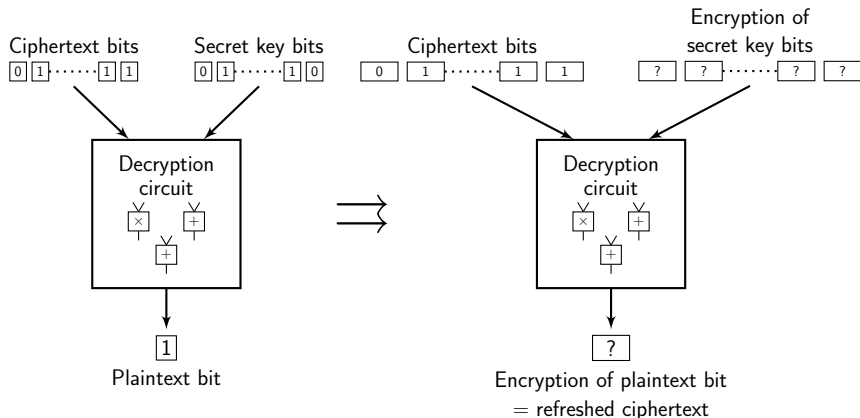
Somewhat homomorphic scheme

- The number of multiplications is limited.
 - Noise grows with the number of multiplications.
 - Noise must remain $< p$ for correct decryption.



Fully Homomorphic Encryption

- Gentry's breakthrough idea: refresh the ciphertext by evaluating the decryption circuit homomorphically: bootstrapping.



Public-key Encryption with DGHV

- Ciphertext

$$c = q \cdot p + 2r + m$$

- Public-key: a set of τ encryptions of 0's.

$$x_i = q_i \cdot p + 2r_i$$

- Public-key encryption:

$$c = m + 2r + \sum_{i=1}^{\tau} \varepsilon_i \cdot x_i$$

for random $\varepsilon_i \in \{0, 1\}$.

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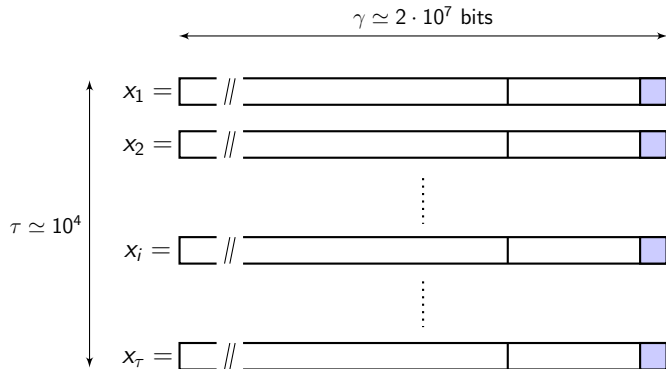
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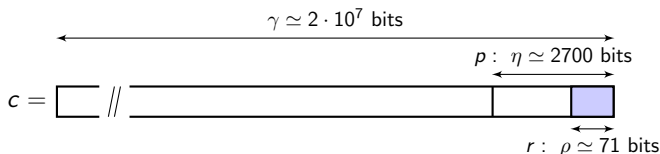
Public Key Size



- Public-key size: $\tau \cdot \gamma = 2 \cdot 10^{11}$ bits = 25 GB !
 - In [CMNT11], with quadratic encryption, PK size of 1 GB.

New: DGHV Ciphertext Compression

- Ciphertext: $c = q \cdot p + 2r + m$



- Compute a pseudo-random $\chi = f(\text{seed})$ of γ bits.

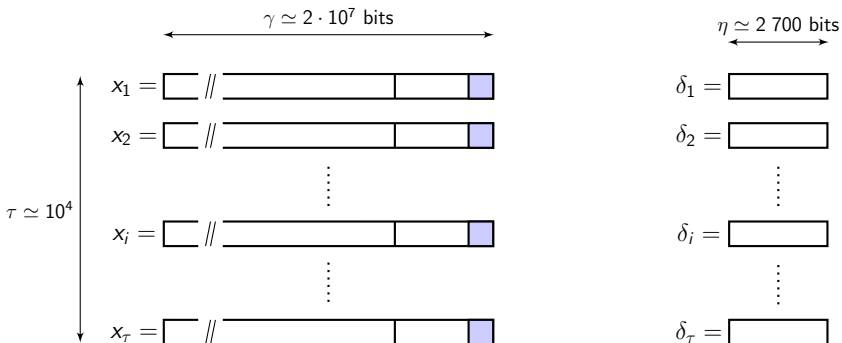
$$\chi = [] || [] [] []$$

$$\delta = \chi - 2r - m \bmod p$$

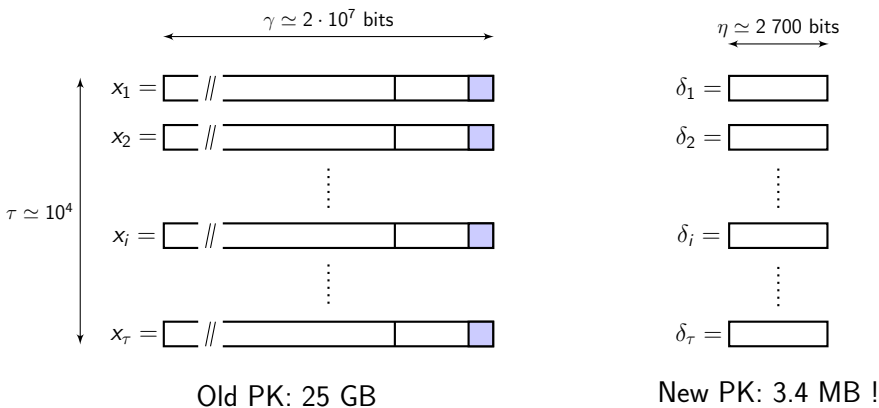
$$c = \chi - \delta [] || [] [] []$$

- Only store *seed* and the small correction δ .
- Storage:** $\simeq 2700$ bits instead of $2 \cdot 10^7$ bits !

Compressed Public Key



Compressed Public Key



Security of Compressed PK

- Original DGHV scheme is semantically secure, under the approximate-gcd assumption.
 - Approximate-gcd problem: given a set of $x_i = q_i \cdot p + r_i$, recover p .
- Compressed public key
 - *seed* is part of the public-key, to recover the x_i 's, so we cannot argue that $f(\text{seed})$ is pseudo-random.
 - Security in the random oracle model only, still based on approximate-gcd.

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PK Generation

$$\begin{array}{l} \chi_i = H(seed, i) \\ \delta_i = [\chi_i]_p + \lambda_i \cdot p - r_i \\ \downarrow \\ x_i = \chi_i - \delta_i \end{array}$$

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Simulation in ROM

$$\begin{array}{l} \uparrow \\ H(seed, i) \leftarrow x_i + \delta_i \\ \delta_i \leftarrow \{0, 1\}^{\eta+\lambda} \\ x_i = q_i \cdot p + r_i \end{array}$$

PK size and timings

Instance	λ	ρ	η	γ	pk size	Recrypt
Toy	42	27	1026	$150 \cdot 10^3$	77 KB	0.41 s
Small	52	41	1558	$830 \cdot 10^3$	437 KB	4.5 s
Medium	62	56	2128	$4.2 \cdot 10^6$	2.2 MB	51 s
Large	72	71	2698	$19 \cdot 10^6$	10.3 MB	11 min

- Updated parameters to take into account the Chen-Nguyen attack.
- PK size: **10.3 MB** instead of 1 GB in [CMNT11].

Hardness assumption for semantic security

- Original DGHV scheme: secure under the **General Approximate Common Divisor** (GACD) assumption.
 - Given polynomially many $x_i = p \cdot q_i + r_i$, find p .
- Efficient DGHV variant: secure under the **Partial Approximate Common Divisor** (PACD) assumption.
 - Given $x_0 = p \cdot q_0$ and polynomially many $x_i = p \cdot q_i + r_i$, find p .
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Solving PACD

- Given $x_0 = p \cdot q_0$ and polynomially many $x_i = p \cdot q_i + r_i$, find p .
- Brute force attack: 2^ρ GCD computations.
 - with $x_0 = q_0 \cdot p$ and $x_1 = q_1 \cdot p + r_1$ and $0 \leq r_1 < 2^\rho$.
- Variant suggested by Phong Nguyen, still in $\mathcal{O}(2^\rho)$:

$$p = \gcd \left(x_0, \prod_{i=0}^{2^\rho-1} (x_1 - i) \bmod x_0 \right)$$

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- Given polynomially many $x_i = p \cdot q_i + r_i$, find p .
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 - From $x_1 = p \cdot q_1 + r_1$ and $x_2 = p \cdot q_2 + r_2$
- Using Chen-Nguyen attack: $\tilde{O}(2^{3\rho/2})$ time.
 - Guess r_1 and apply Chen-Nguyen on r_2
 - $\mathcal{O}(2^\rho) \cdot \tilde{O}(2^{\rho/2}) = \tilde{O}(2^{3\rho/2})$ time and $\tilde{O}(2^{\rho/2})$ memory.
- New attack: $\tilde{O}(2^\rho)$ time and memory.

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New Attack against GACD

- Given polynomially many $x_i = p \cdot q_i + r_i$, find p .
- Variant of the previous equation with $x_1 = p \cdot q_1 + r_1$ and $x_2 = p \cdot q_2 + r_2$

$$p \mid \gcd \left(\prod_{i=0}^{2^p-1} (x_1 - i), \prod_{i=0}^{2^p-1} (x_2 - i) \right)$$

- Product over \mathbb{Z} can be computed in $\tilde{O}(2^p)$ time using a product tree.
- $\tilde{O}(2^p)$ time and memory.
- Problem: many parasitic factors.
 - Can be eliminated by taking the gcd with more products,
 - and by dividing by $B!$ for $B \simeq 2^p$.

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Source Code in SAGE

```
def attackGACD(rho=12,gam=1000,eta=100):  
    p=random_prime(2^eta)  
    s=rho  
  
    B=floor(2^(1.*rho*(s+1)/(s-1)))  
    fa=factorial(B)  
  
    for j in range(1,s):  
        x=p*ZZ.random_element(2^(gam-eta))+ \  
            ZZ.random_element(2^rho)  
        z=prod([x-i for i in range(2^rho)])  
        if j==1: g=z; continue  
        g=gcd(g,z)  
        g=prime_to_m_part(g,fa)  
        if g.nbits()==p.nbits(): break
```

GACD Attack Running Time

Instance	ρ	γ	time	time [CN12]
Micro	12	10^4	40 s	
Toy (Section 8)	13	$61 \cdot 10^3$	13 min	
Toy' ([CN12])	17	$1.6 \cdot 10^5$	17 h	3495 h (est.)

- Chen-Nguyen attack: $\mathcal{O}(2^{3\rho/2})$ time and $\mathcal{O}(2^{\rho/2})$ memory.
- Our attack: $\mathcal{O}(2^\rho)$ time and memory
- Time-memory tradeoffs are possible.

Conclusion

- Smaller public key size for the DGHV fully homomorphic encryption scheme.
 - 10 MB instead of 1 GB
- Better attack against approximate-gcd without $x_0 = q_0 \cdot p$
 - $\tilde{O}(2^\rho)$ complexity instead of $\tilde{O}(2^{3\rho/2})$
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 - Generalization of [CMNT11] quadratic encryption technique to higher degrees.
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- Smaller public key size for the DGHV fully homomorphic encryption scheme.
 - 10 MB instead of 1 GB
- Better attack against approximate-gcd without $x_0 = q_0 \cdot p$
 - $\tilde{O}(2^\rho)$ complexity instead of $\tilde{O}(2^{3\rho/2})$
- In the proceedings:
 - Generalization of [CMNT11] quadratic encryption technique to higher degrees.
 - DGHV without bootstrapping: analogous to RLWE without bootstrapping [BGV11].