Institute for Theoretical Computer Science

Leibniz Universität Hannover

Complexity of Model Checking for Modal Dependence Logic

Johannes Ebbing
joint work with Peter Lohmann

Institut für Theoretische Informatik
Leibniz Universität Hannover

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Overview

Motivation

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- CTL is used for verifying systems e.g. integrated circuits
- MDL seen as first approach of DCTL (= CTL + dependence atom)
- MDL is based on teams, not on single assignments / worlds

Introduction

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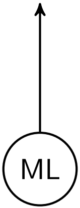

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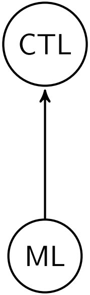

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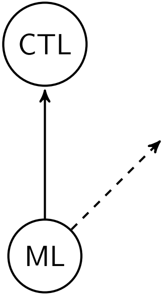

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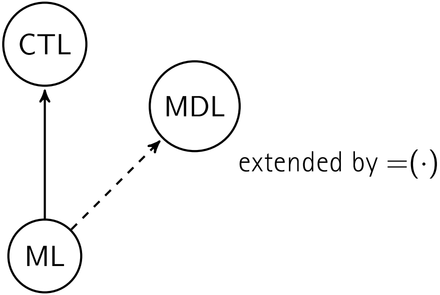

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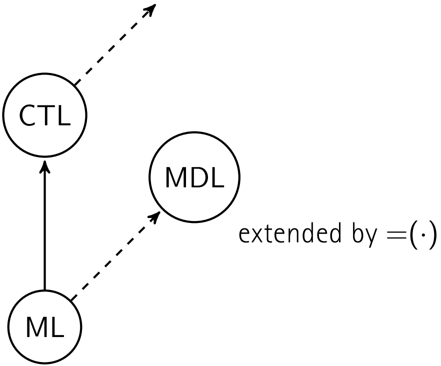

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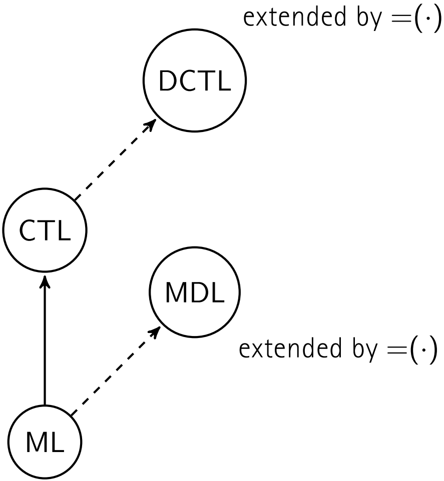

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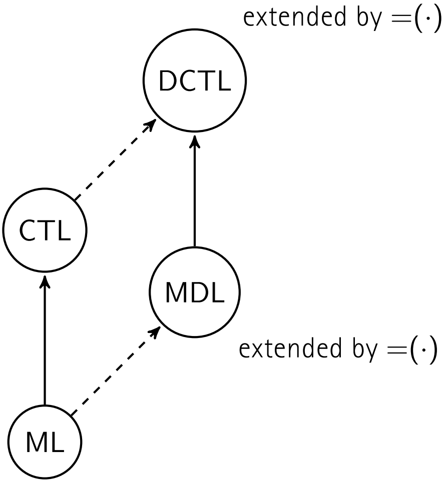

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If $s \in S$, s is called world. We call S the set of worlds. R is called the transition relation between the worlds, and π is called the labeling function.

Modal Dependence Logic

Syntax of Modal Logic

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$$\varphi ::= q \mid \neg q \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \Box \varphi \mid \Diamond \varphi \mid =\!(p_1, \ldots, p_n; q) \mid \neg =\!(p_1, \ldots, p_n; q)$$

Modal Logic

Semantics of Modal Logic

Let $W = (S, R, \pi)$ an AP-Kripke structure, $T \subseteq S$ a set of initial states, and φ an ML formula and $p \in AP$. Inductive over the structure of φ we define

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$W, T \models p$	iff	$p \in \pi(s)$ for all $s \in T$
$W, T \models \neg p$	iff	$p \notin \pi(s)$ for all $s \in T$
$W, T \models \varphi \vee \psi$	iff	there are sets $T_1, T_2 \subseteq T$ with $T_1 \cup T_2 = T$, $W, T_1 \models \varphi$ and $W, T_2 \models \psi$

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$W, T \models \Diamond \varphi$	iff	there is a set $T' \subseteq T$ s. t. $W, T' \models \varphi$ and for all $a \in T$ there is an $a' \in T'$ with $(a, a') \in R$.

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Semantics of the Dependence Atom

Let $W = (S, R, \pi)$ an AP-Kripke structure, $T \subseteq S$ the evaluation team. Let $p_1, \ldots, p_n, q \in AP$. Then $=(p_1, \ldots, p_n; q)$ is called a dependence atom. The truth of the dependence atom on W, T is defined by

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$$W, T \models \;=(p_1, \ldots, p_n; q) \quad \text{iff} \quad \text{for all } s_1, s_2 \in T \text{ it holds that}$$
$$\pi(s_1) \cap \{p_1, \ldots, p_n\} \neq \pi(s_2) \cap \{p_1, \ldots, p_n\}$$
$$\text{or } \pi(s_1) \cap \{q\} = \pi(s_2) \cap \{q\}$$

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$$W, T \models \; \neg =(p_1, \ldots, p_n; q) \quad \text{iff} \quad T = \emptyset$$

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Example: Let $\varphi := \Diamond =\!(p_1, \ldots, p_4; q)$ and $T := \{c_1, c_2, c_3\}$.

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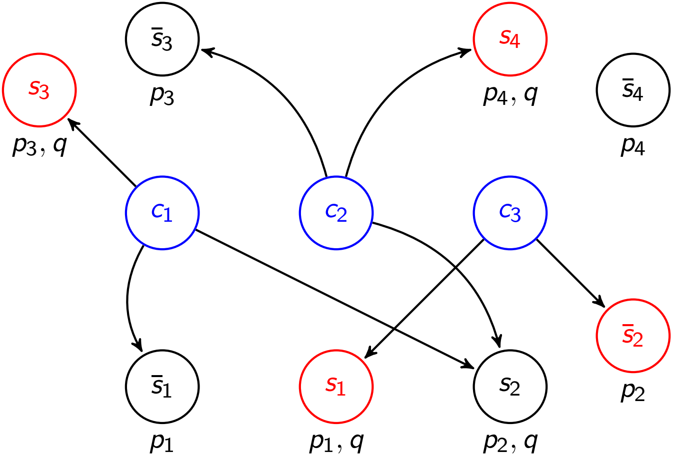

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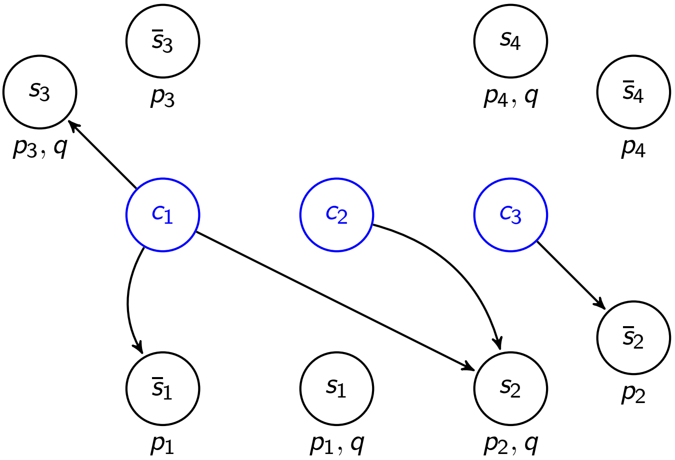

Model Checking

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The Model Checking problem

Let $M \subseteq \{\Box, \Diamond, \wedge, \vee, \neg, =\}$.
Given: A Kripke structure $W = (S, R, \pi)$, a formula $\varphi \in \mathrm{MDL}(M)$ and an evaluation team $T \subseteq S$.
Problem: Check, whether $W, T \models \varphi$.

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$$\text{MDL-MC}(M) := \left\{ \langle W, T, \varphi \rangle \;\middle|\; \begin{array}{l} W = (S, R, \pi) \text{ an } AP\text{-Kripke structure}, \\ T \subseteq S, \varphi \in \mathrm{MDL}(M) \text{ and } W, T \models \varphi \end{array} \right\}.$$

Complexity Results on MDL-MC

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Theorem

Model checking for modal dependence logic is NP-complete.

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Theorem

Model checking for modal dependence logic is NP-complete.

We have to show:

- MDL-MC is in NP
- MDL-MC is NP-hard

The MDL Model Checking problem is in NP

Algorithm

```
bool check(W = (S, R, π), φ, T)
  case φ
  when φ = p
    foreach s_i ∈ T
      if not p ∈ π(s_i) then
        return false;
    return true;
  when φ = ¬p
    foreach s_i ∈ T
      if p ∈ π(s_i) then
        return false;
    return true;
```

The MDL Model Checking problem is in NP

Algorithm

```
when φ = ψ ∨ θ
  guess two sets A, B ⊆ T;
  if not A ∪ B = T then
    return false;
  return (check (W, ψ, A) and check (W, θ, B));
when φ = ψ ∧ θ
  return (check (W, ψ, T) and check (W, θ, T));
```

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Algorithm

when $\varphi = \Diamond\psi$
  guess set of states T' ;
  foreach $s \in T$
    if there is no $s' \in T'$ with $(s, s') \in R$ then
      return false;
  return check (W, ψ, T');
when $\varphi = \Box\psi$
  $T' := \emptyset$;
  foreach $s' \in S$
    foreach $s \in T$
      if $(s, s') \in R$ then
        $T' \leftarrow T' \cup \{s'\}$
  return check (W, ψ, T');

The MDL Model Checking problem is in NP

Algorithm

```
when φ = =(p_1, ..., p_n; q)
  foreach (s, s') ∈ T × T
    if π(s) ∩ {p_1, ..., p_n} = π(s') ∩ {p_1, ..., p_n} then
      if not π(s) ∩ {q} = π(s') ∩ {q}
        return false;
  return true;

when φ = ¬=(p_1, ..., p_n; q)
  return T = ∅
```

The MDL Model Checking problem is NP-hard

Proposition

MDL-MC is NP-hard.

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We reduce 3SAT onto MDL-MC, i.e. for every 3SAT formula ϕ, we find an instance of MDL-MC which is at least as hard as ϕ.

Complexity Results

Operators					Complexity
$\vee$	$\wedge$	$\Box$	$\Diamond$	$=(\cdot)$	
$+$	$+$	$*$	$*$	$+$	NP-complete
$*$	$*$	$*$	$+$	$+$	NP-complete
$+$	$*$	$+$	$*$	$+$	NP-complete
$-$	$*$	$*$	$-$	$*$	in P
$+$	$-$	$-$	$-$	$+$	in NP

Complexity Results with bounded $=(\cdot)$

Operators					Complexity
$\vee$	$\wedge$	$\Box$	$\Diamond$	$=(\cdot)$	
$+$	$+$	$*$	$*$	$+$	NP-complete
$+$	$*$	$*$	$+$	$+$	NP-complete
$*$	$+$	$*$	$+$	$+$	NP-complete
$+$	$*$	$+$	$*$	$+$	NP-complete
$-$	$-$	$*$	$*$	$*$	in P
$-$	$*$	$*$	$-$	$*$	in P
$*$	$-$	$-$	$-$	$*$	in P

We limit the arity of the dependence atom, i.e. all dependence atoms are of the form $=(p_1, \ldots, p_j; q)$ where $j < k$ with a fixed $k \in \mathbb{N}$.

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$+$	$*$	$+$	$*$	$+$	NP-complete
$-$	$-$	$*$	$*$	$*$	in P
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$*$	$-$	$-$	$-$	$*$	in P

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Thank you!