Efficient and Optimally Secure Key-Length Extension for Block Ciphers via Randomized Cascading

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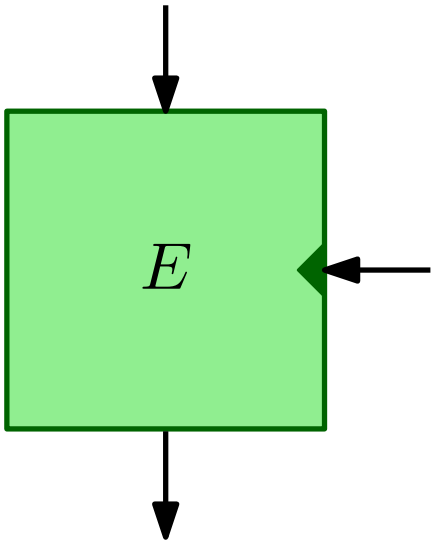
Eurocrypt 2012

Outline

Outline

Block Ciphers



- e.g. DES, IDEA, AES

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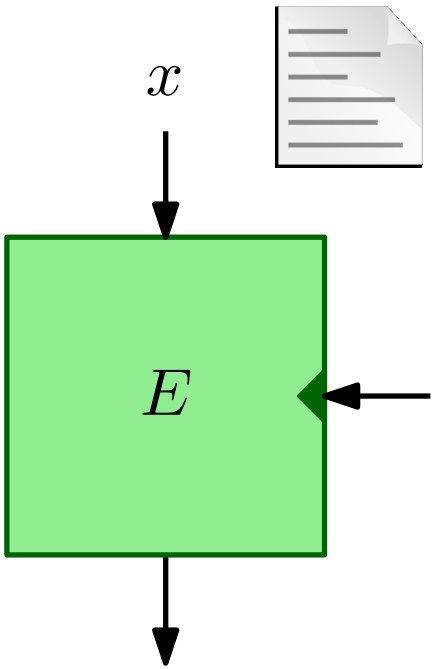

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- $\{0,1\}^n$

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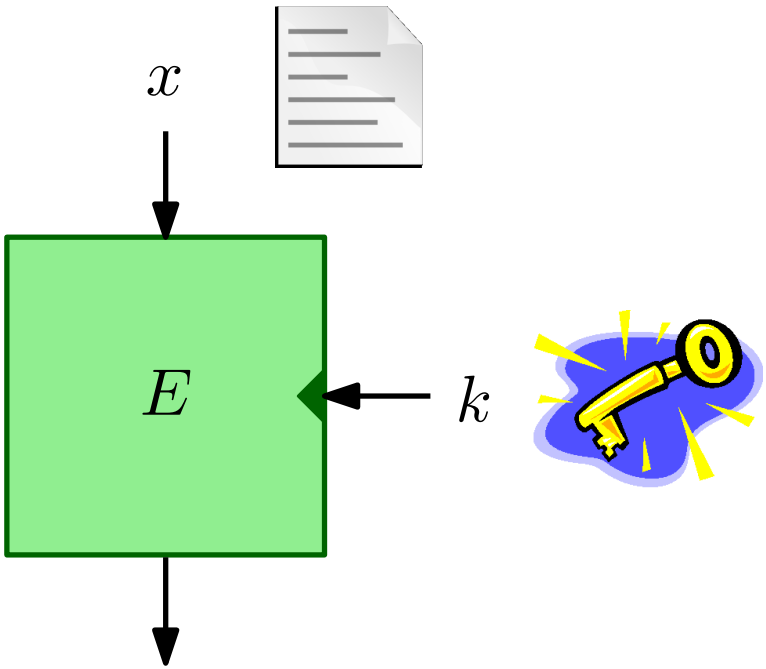

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- $\{0,1\}^{\kappa} \times \{0,1\}^{n}$

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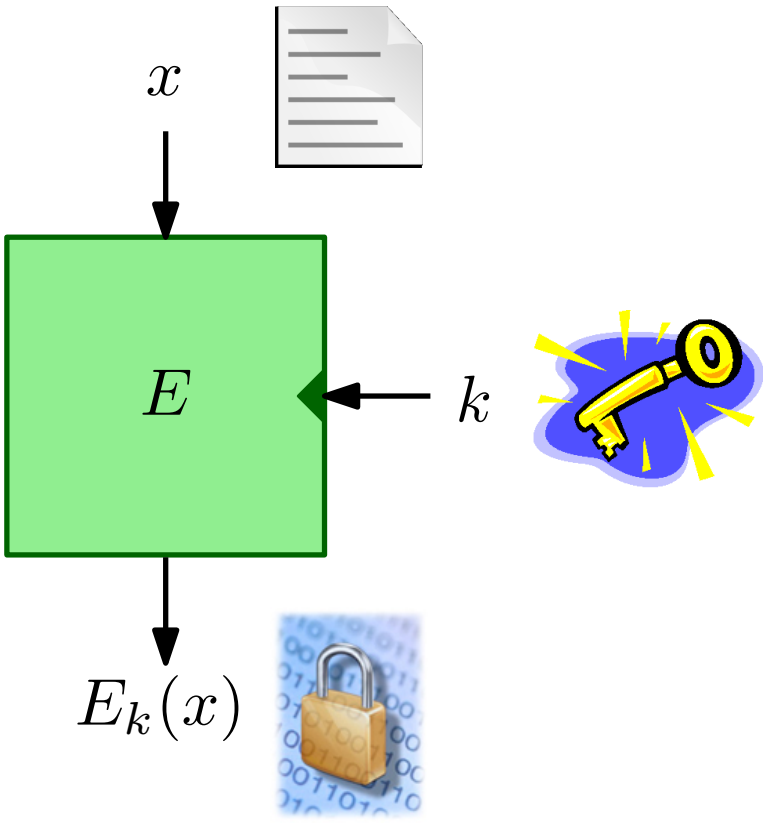

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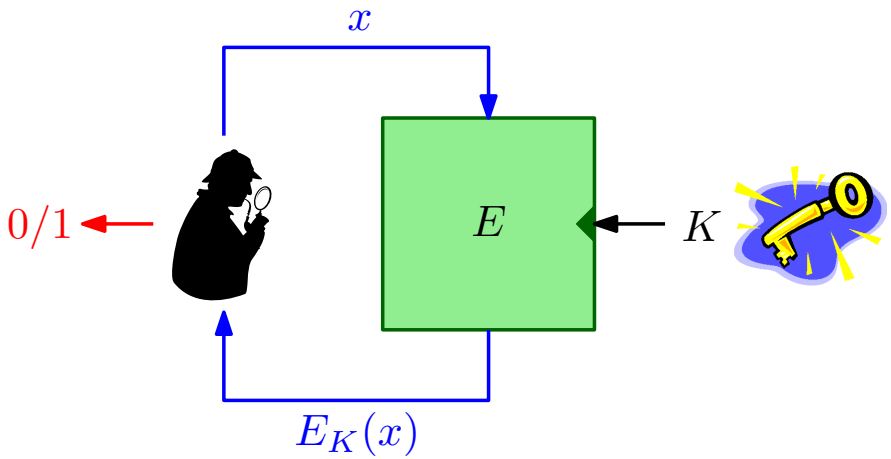
- $E\colon \{0,1\}^\kappa \times \{0,1\}^n \to \{0,1\}^n$

Block Cipher Security: Pseudo-Random Permutations



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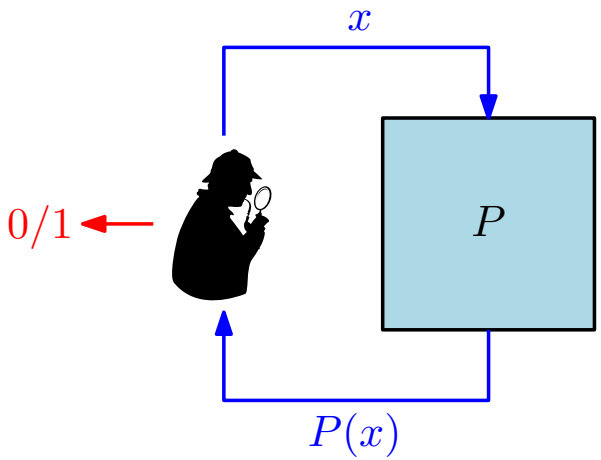

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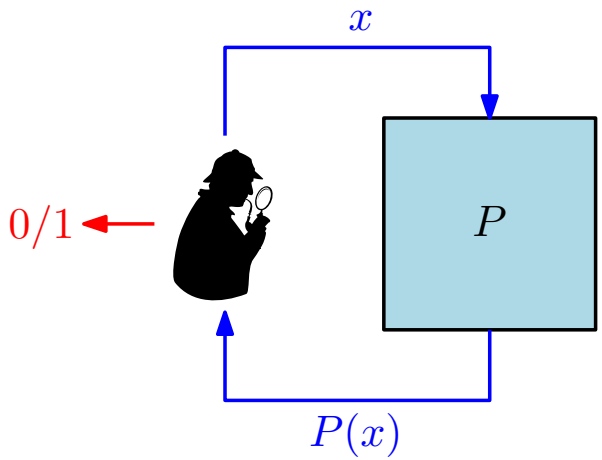

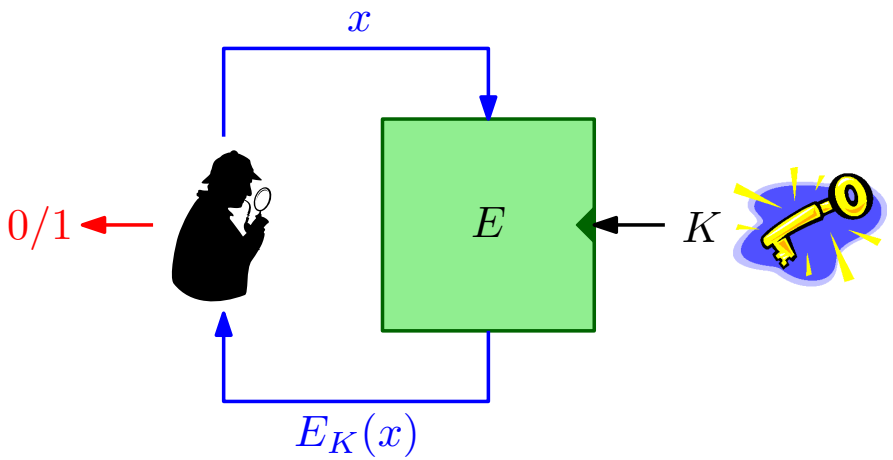

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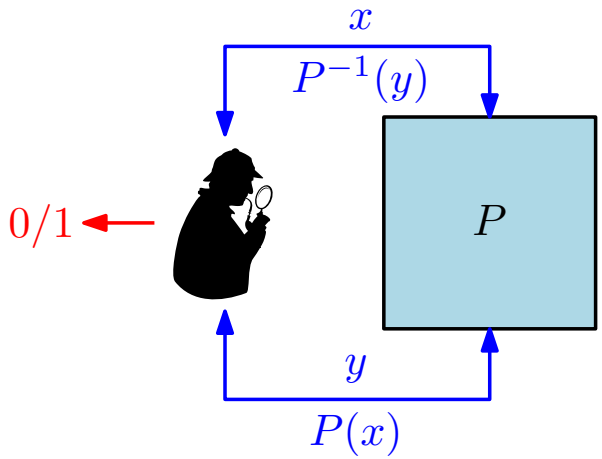

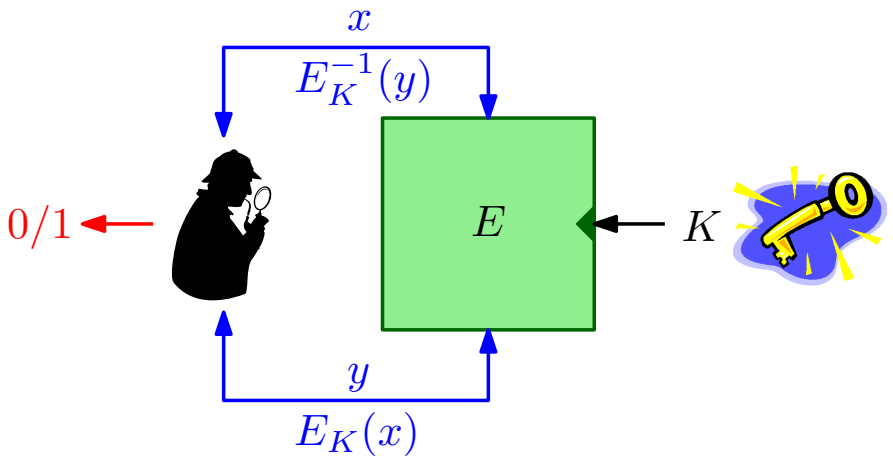

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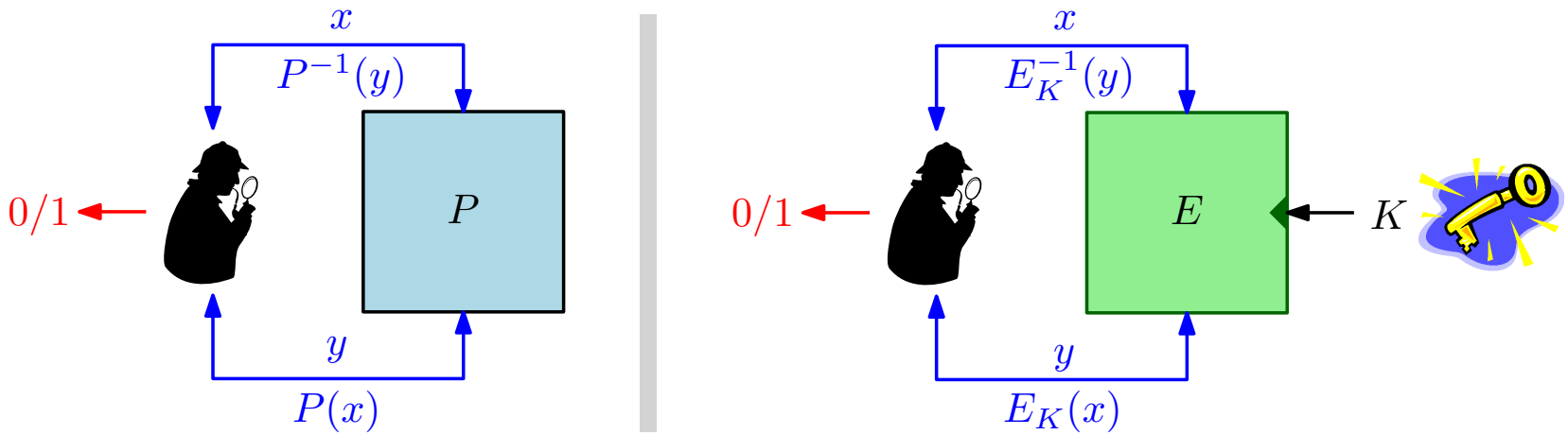

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$$\Delta^D(P, E_K) := |\Pr[D(P) = 1] - \Pr[D(E_K) = 1]|$$

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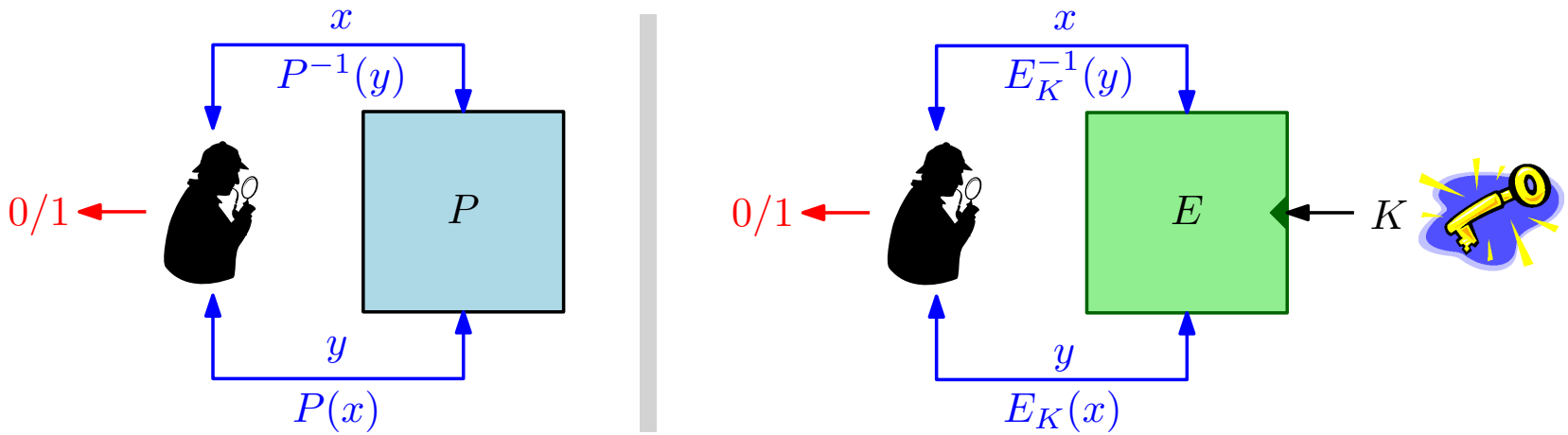

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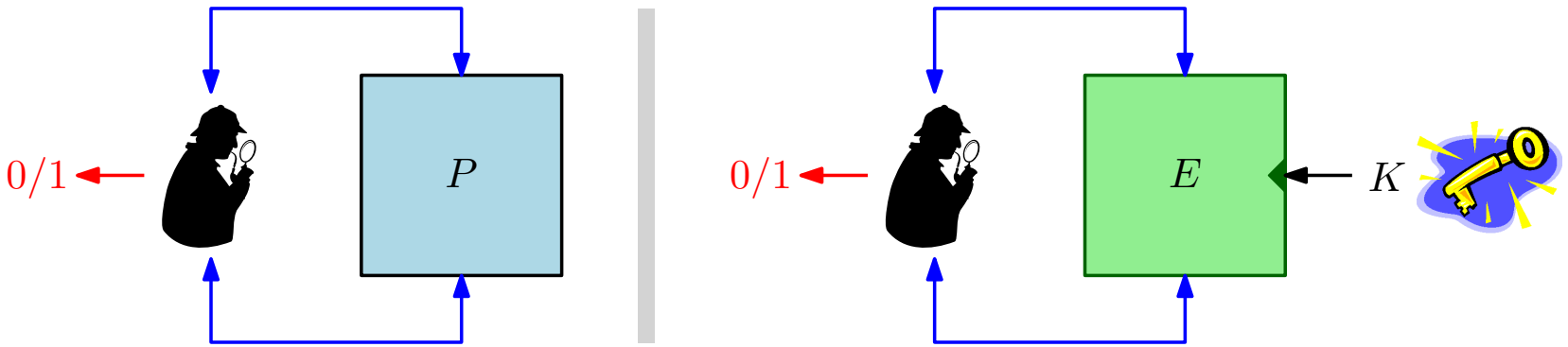

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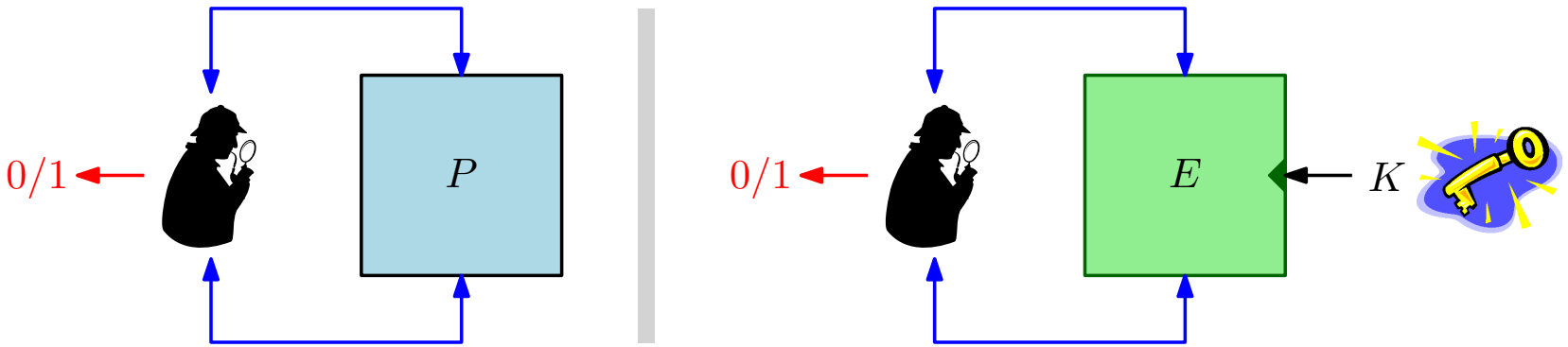

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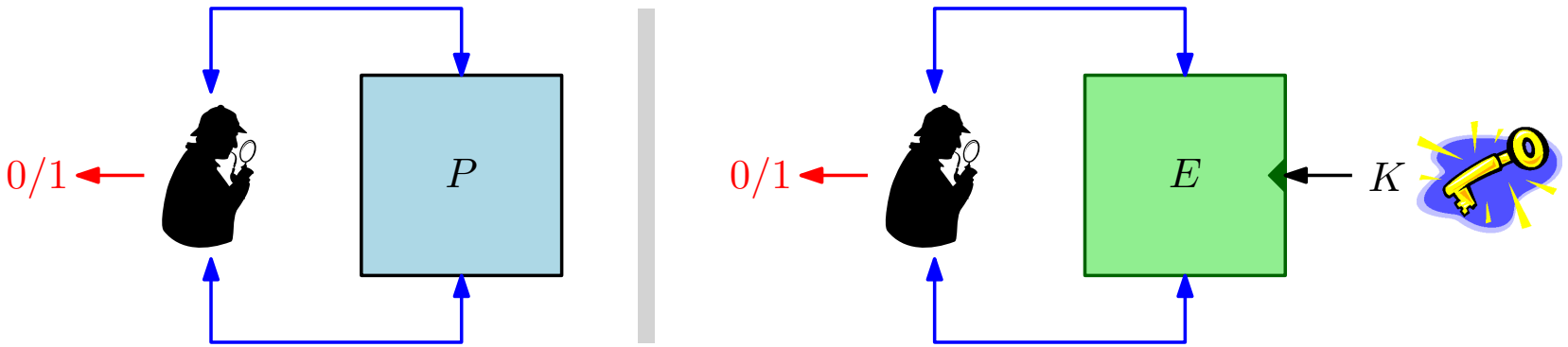

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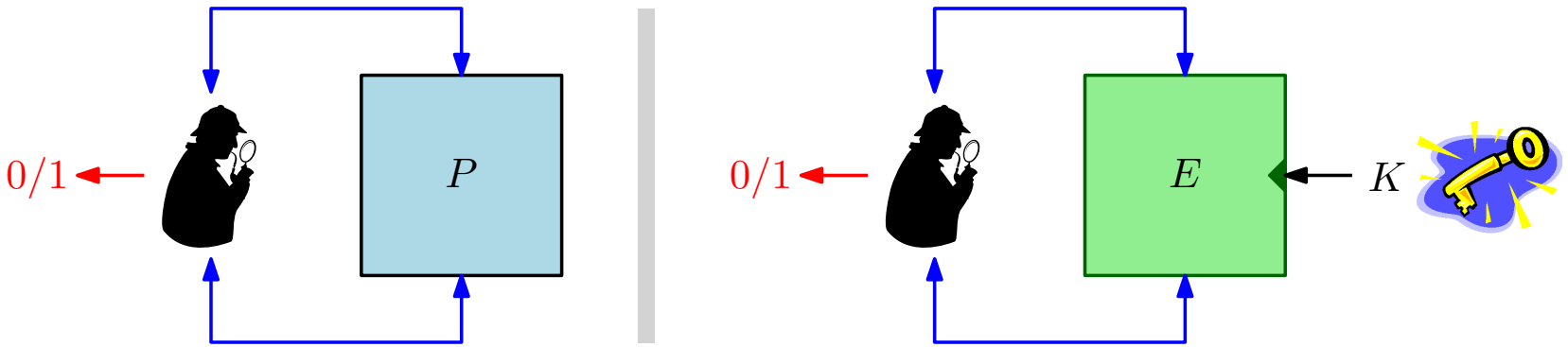

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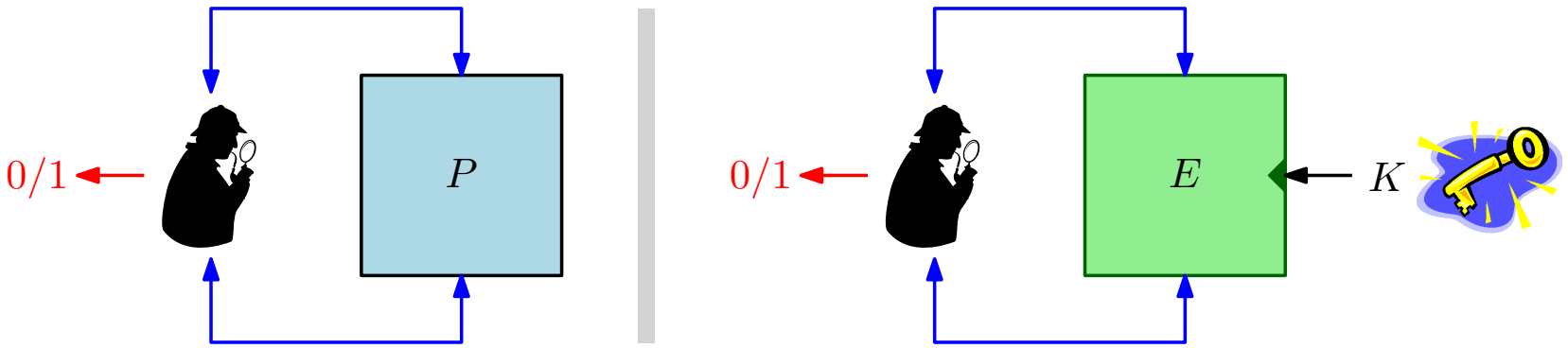

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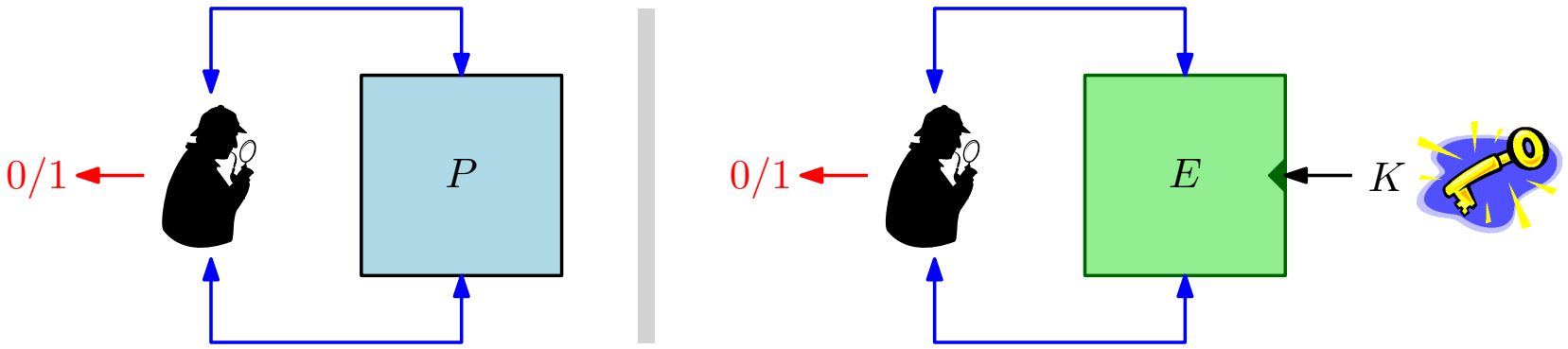

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generic attack!

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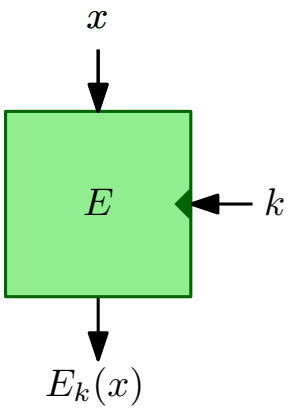

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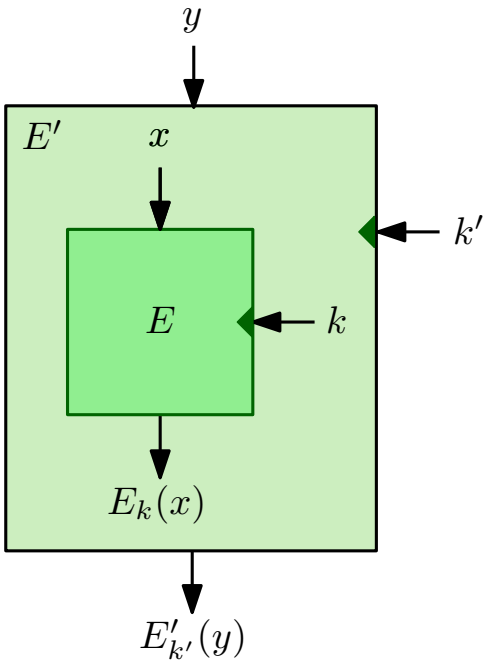

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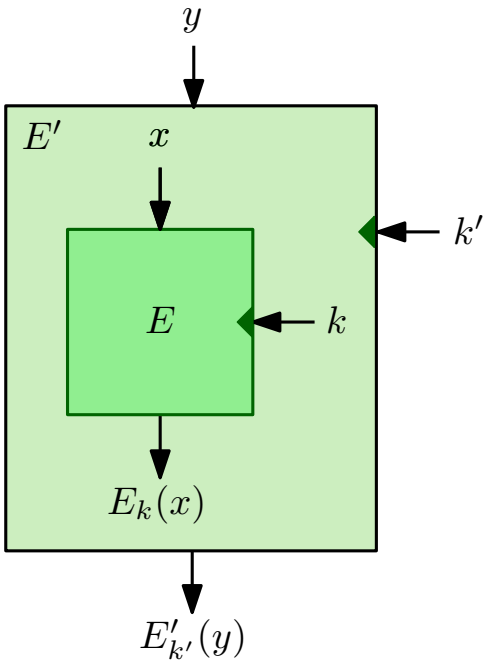

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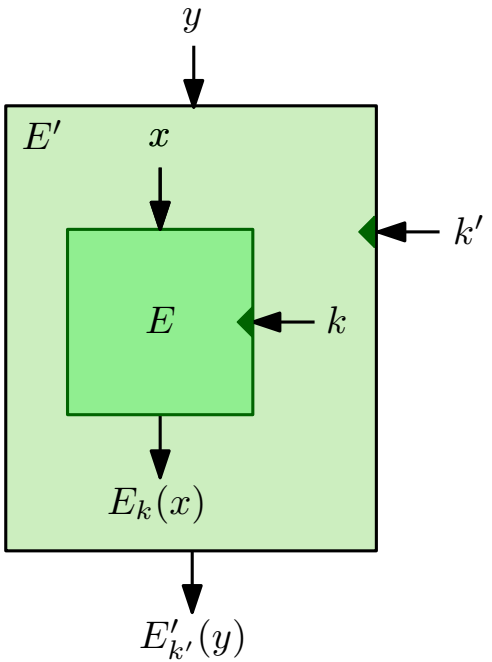

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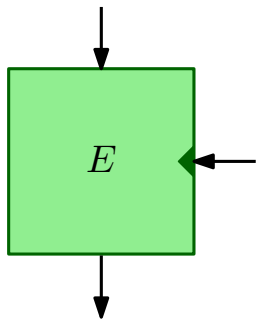
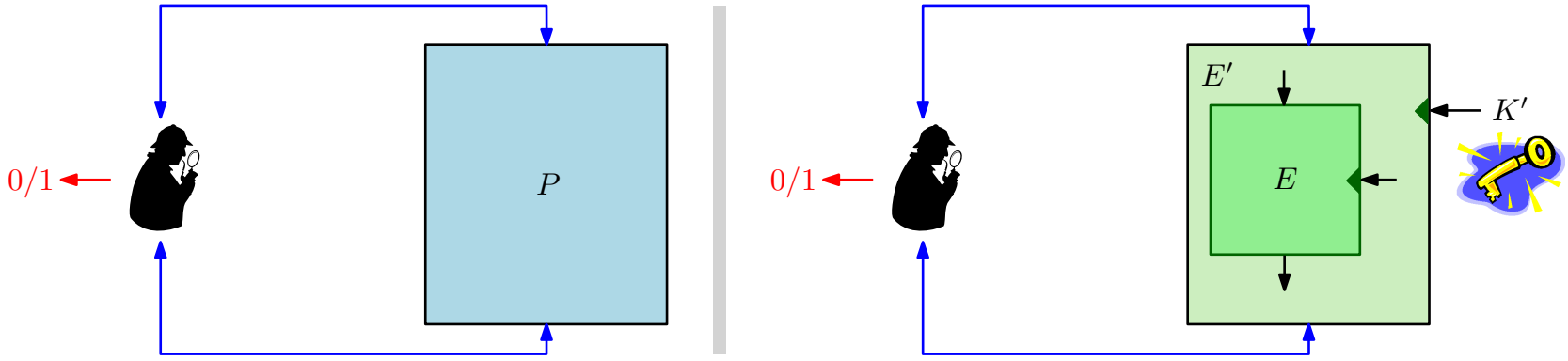
Generic Security: Ideal Block Cipher Model

- $\forall k$: independent uniformly random permutation



Key-Length Extension in ICM



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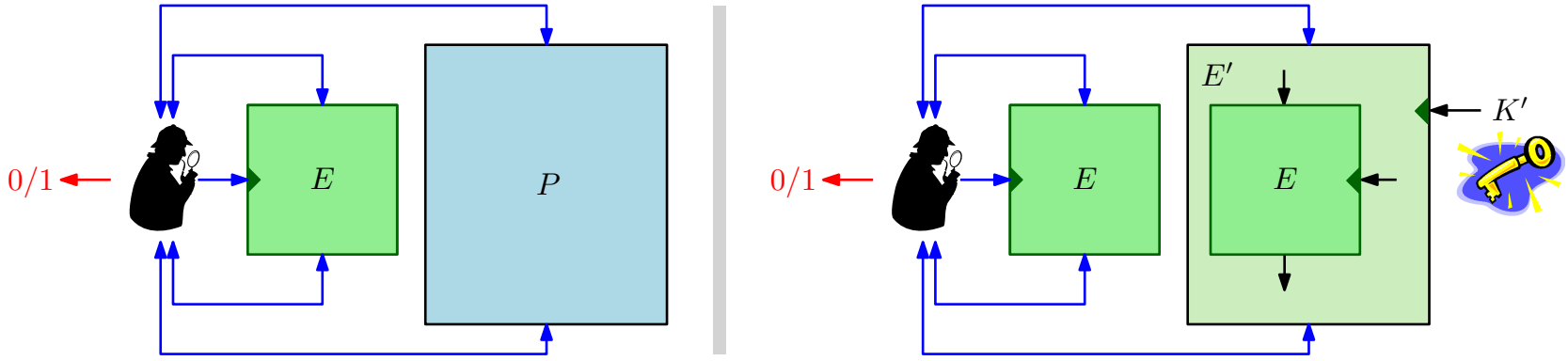

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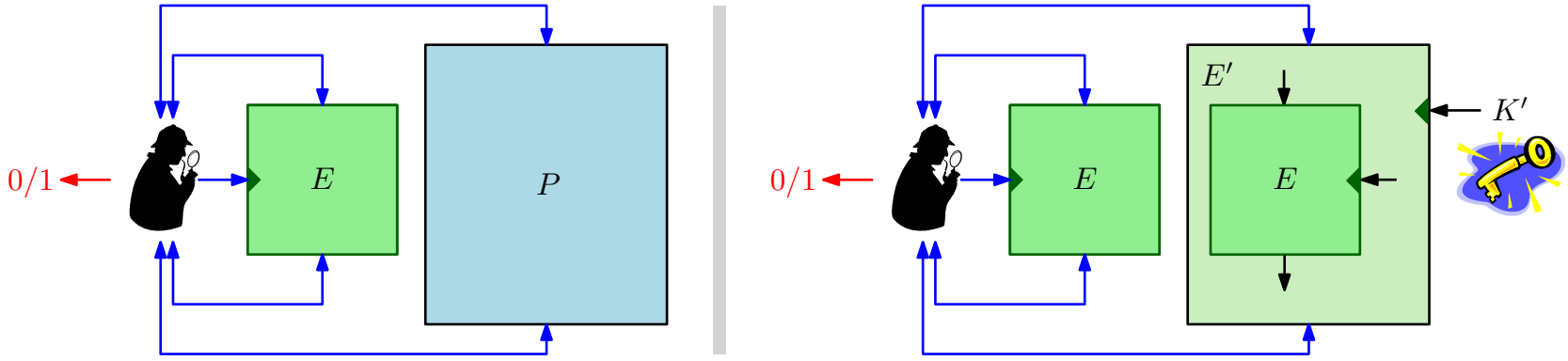

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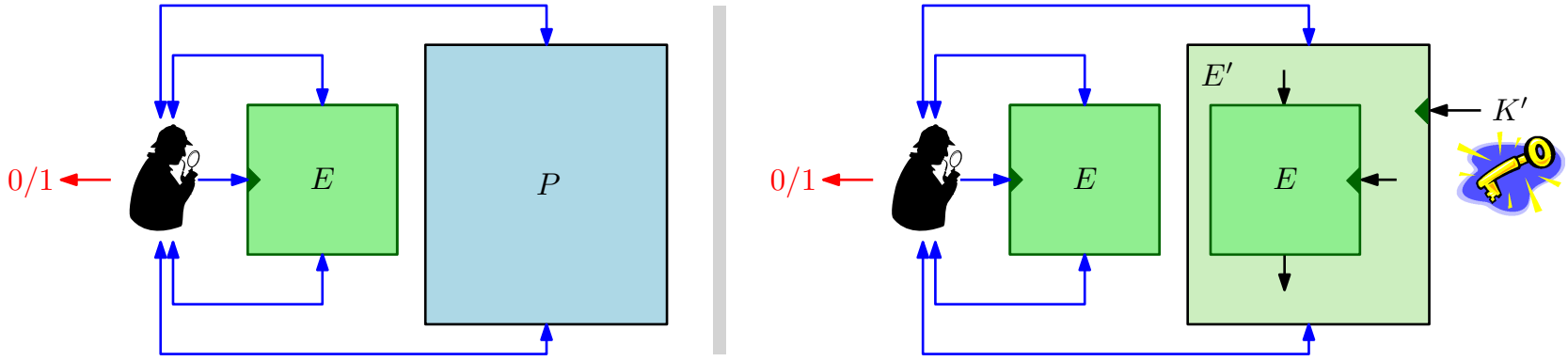

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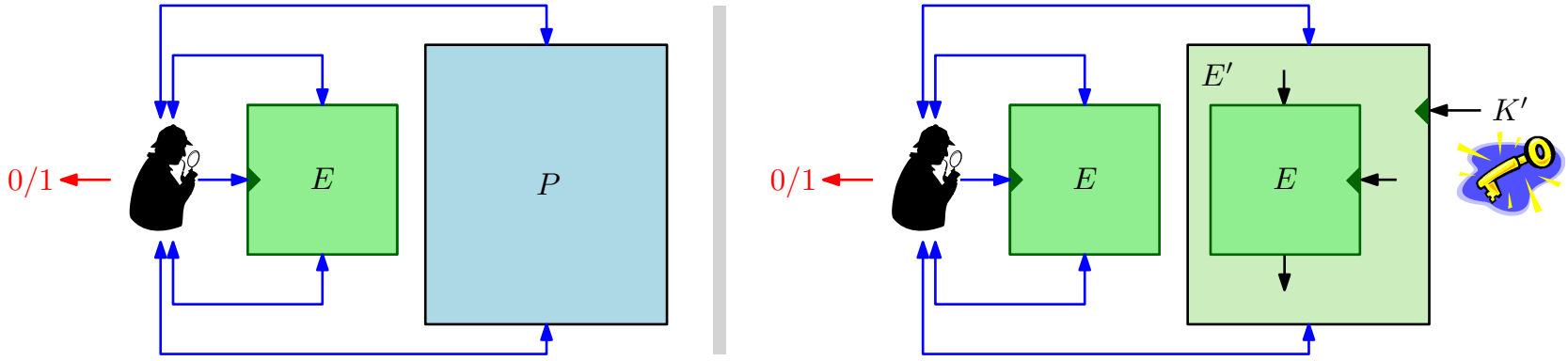

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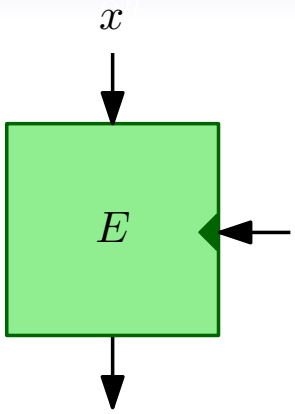

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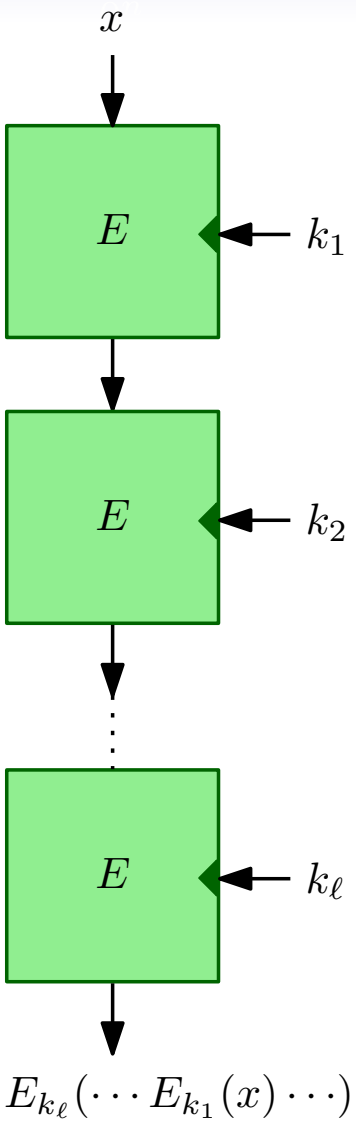

- Re-encrypt with independent keys

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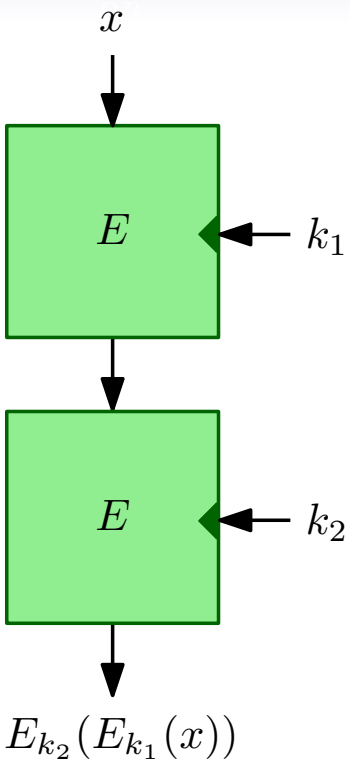

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  - Meet-in-the-middle attack
    (2^{κ} evaluations of E and E^{-1})

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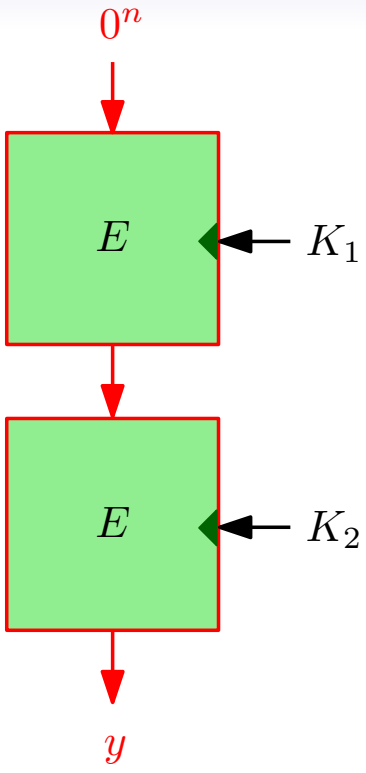

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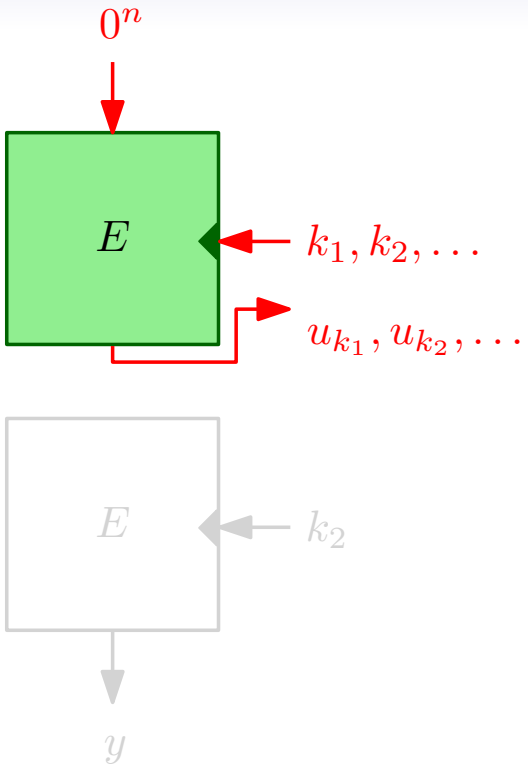

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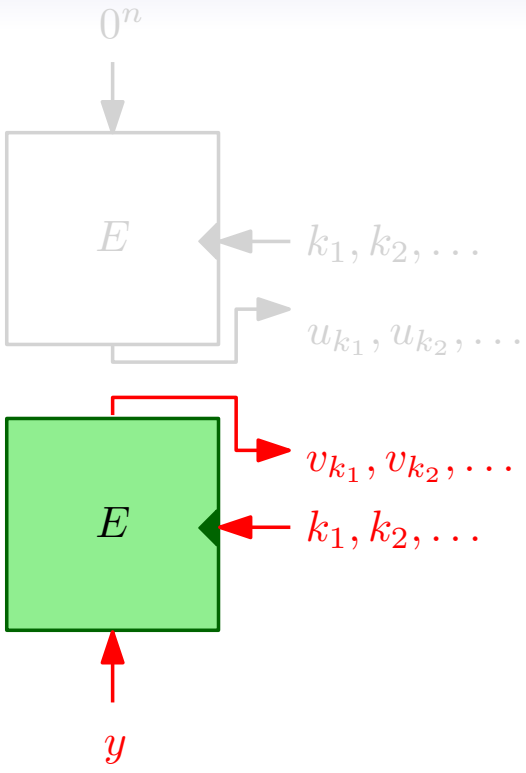

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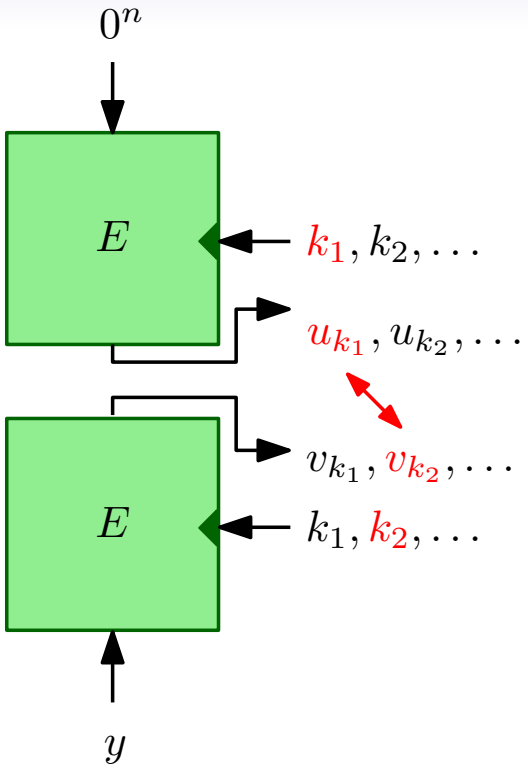

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    4. if $u_{k_i} = v_{k_j}$: verify (k_i, k_j)

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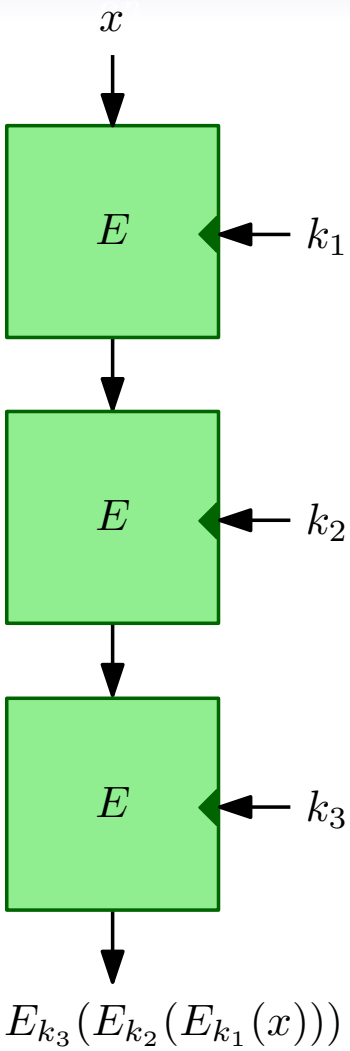

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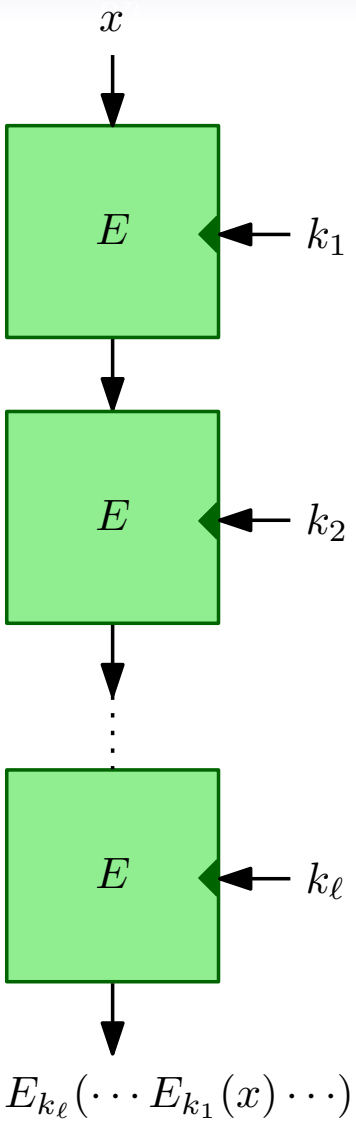

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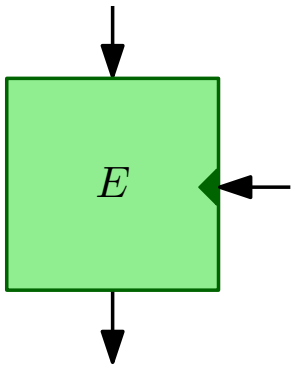
- Longer Cascades
  - Security improves for $\kappa < n$ in ICM (Gaži and Maurer, AC'09)

Approach II: Key Whitening



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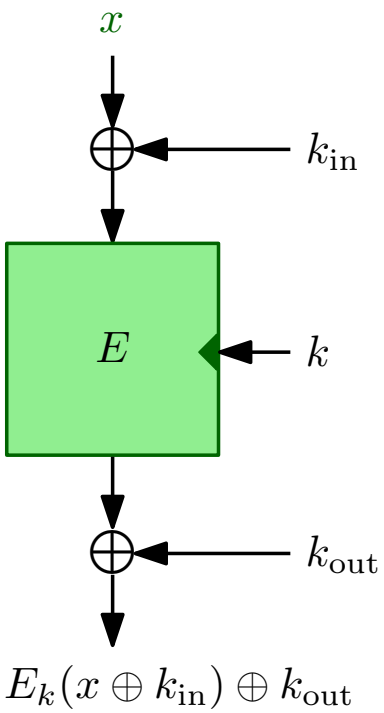

DESX [Rivest]

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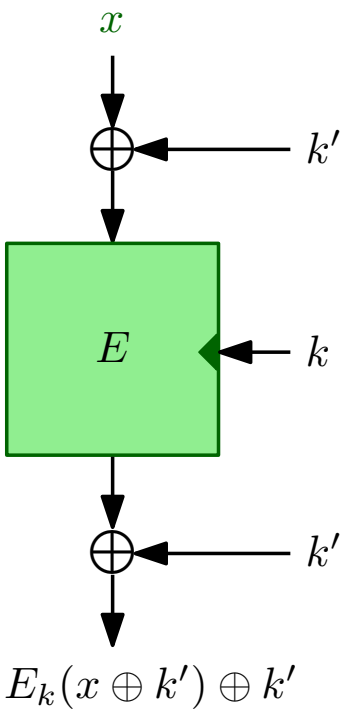

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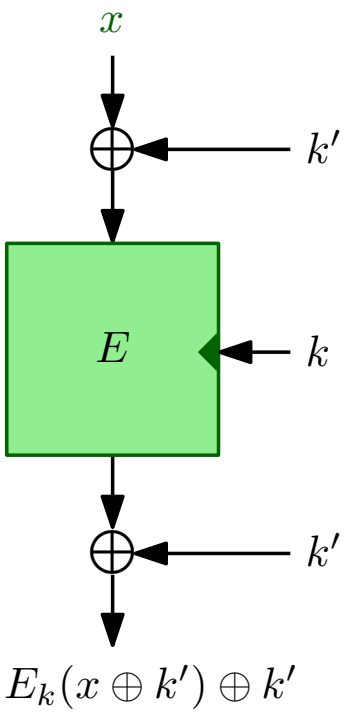

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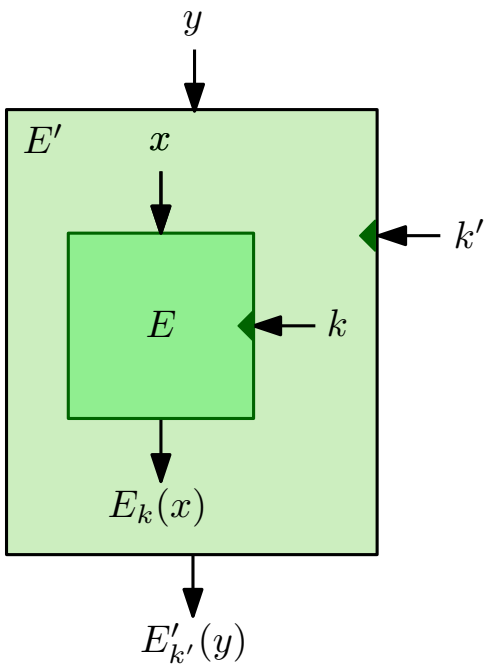
- Also secure if $k_{\text{in}} = k_{\text{out}}$

Can we do better?



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- security beyond $2^{\max\{\kappa,n\}}$ requires 3 BC queries

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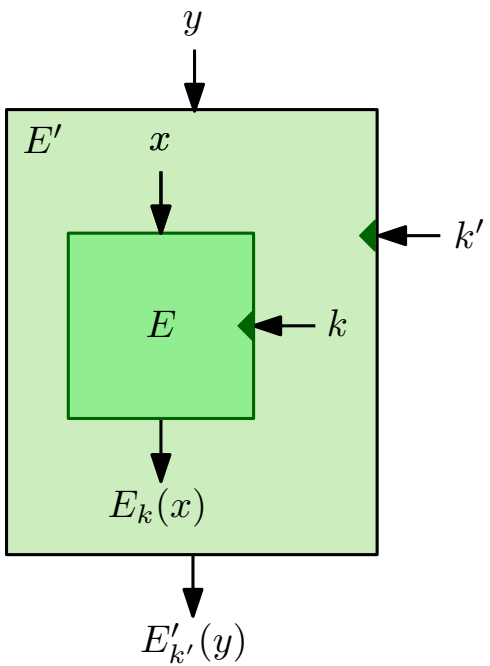

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What can be achieved with at most 2 queries to E?

Outline

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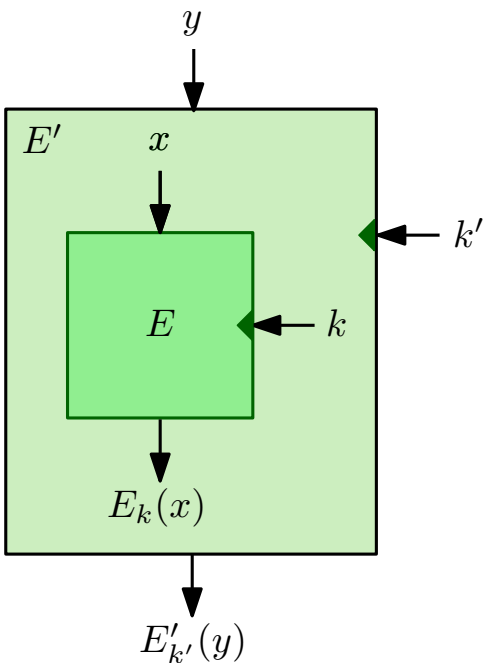
One Query Is Not Enough

Any one-query construction can achieve at most $2^{\max\{\kappa,n\}}$ security!



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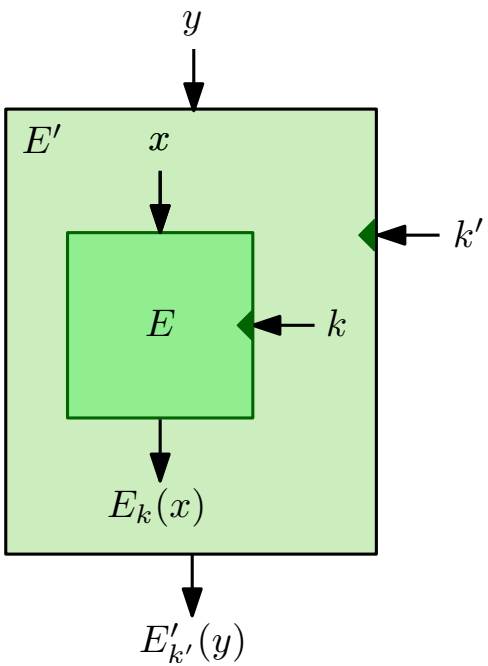

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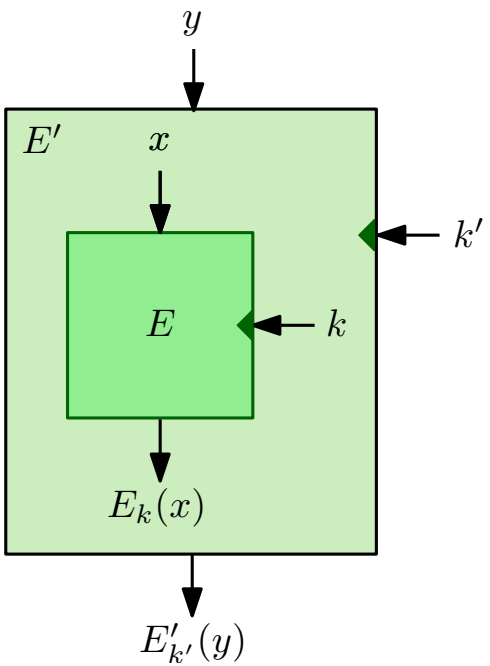

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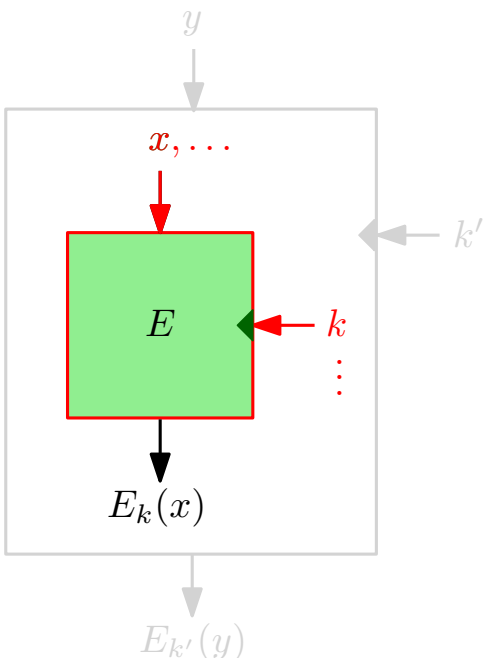

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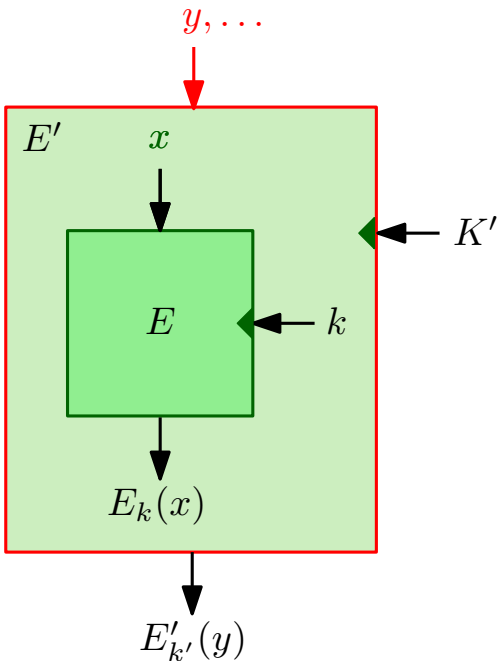

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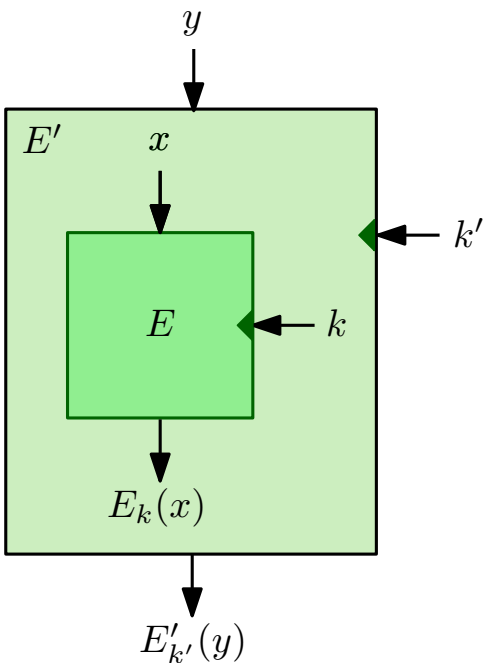

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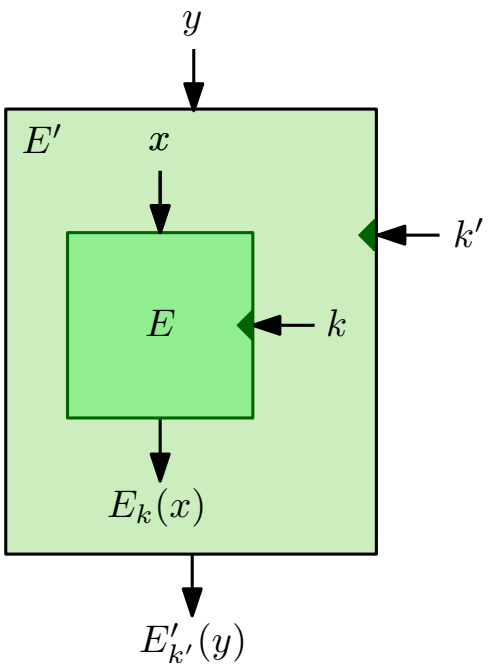

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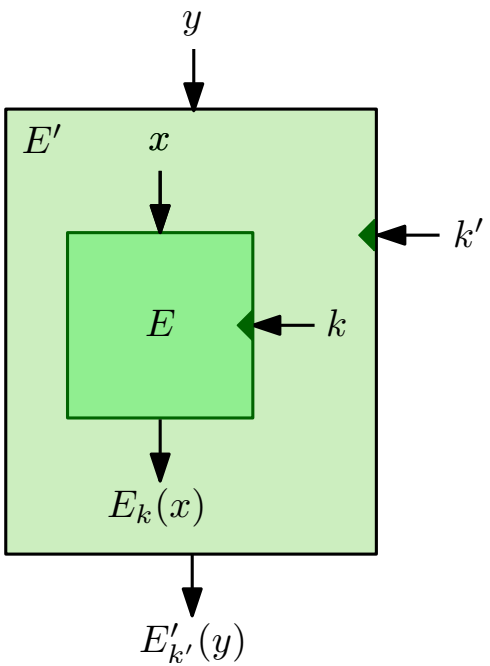

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- Non-injective queries do no better

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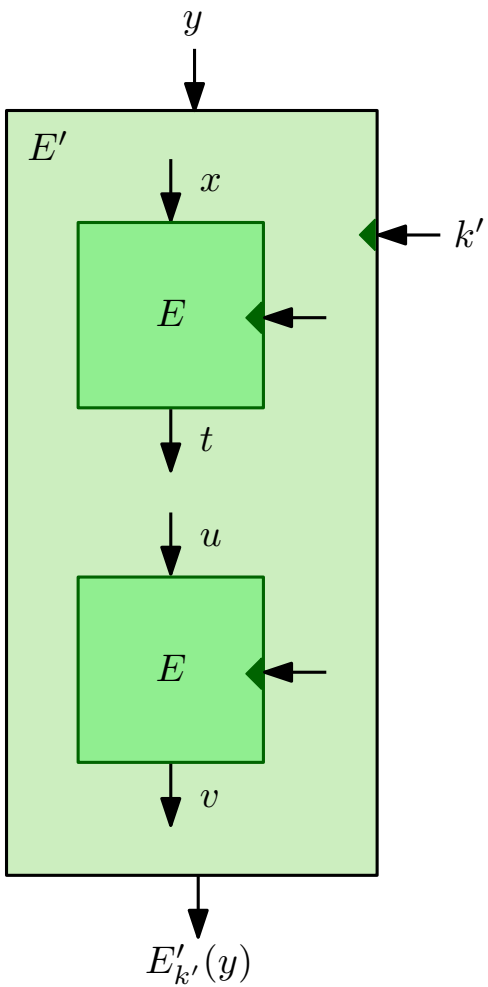

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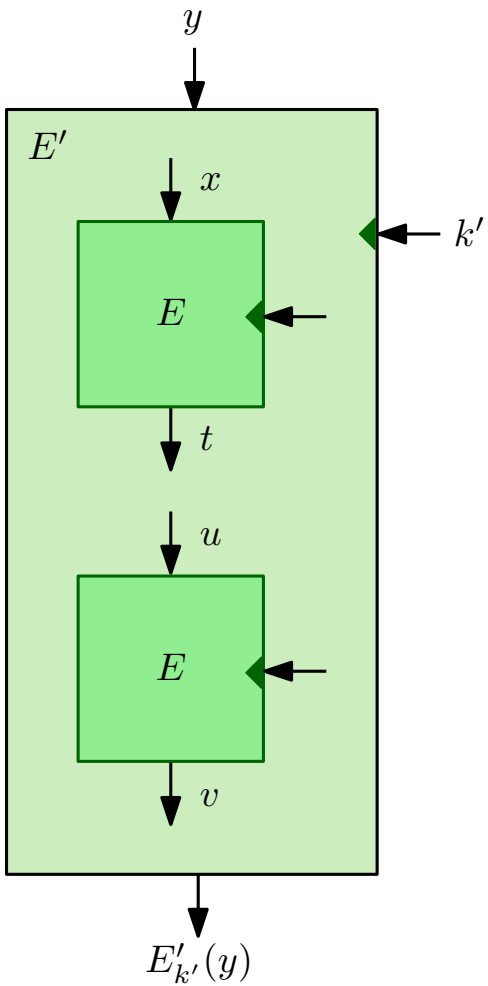

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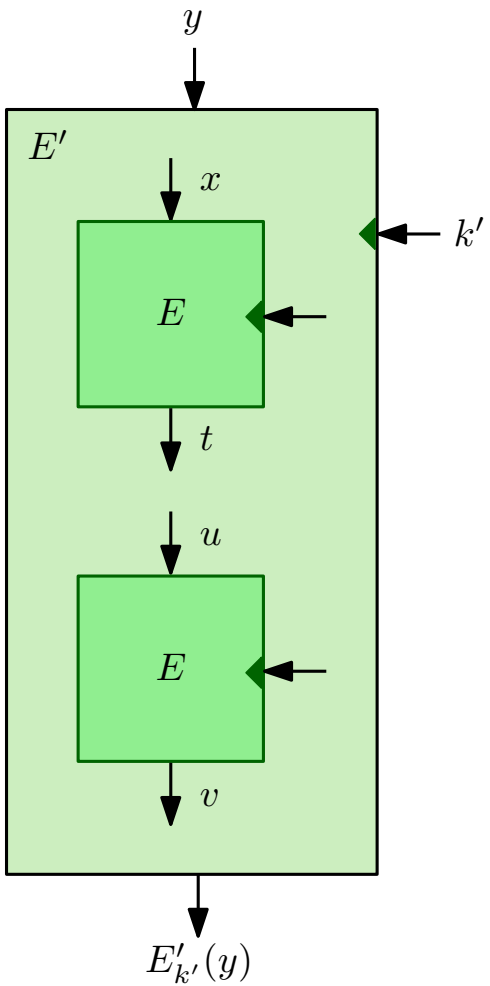

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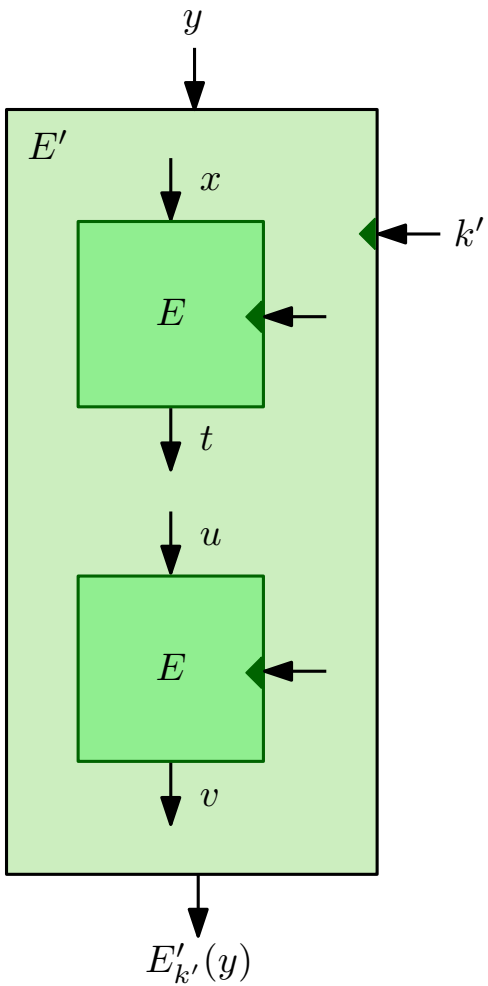

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There is room for security increase, we achieve it!

Outline

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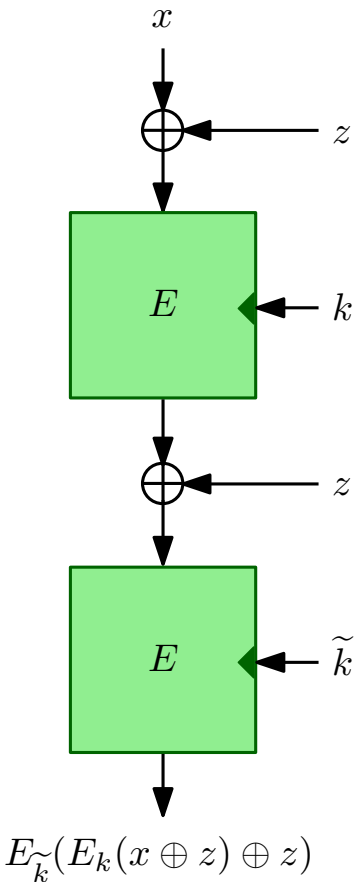
The Double XOR-Cascade



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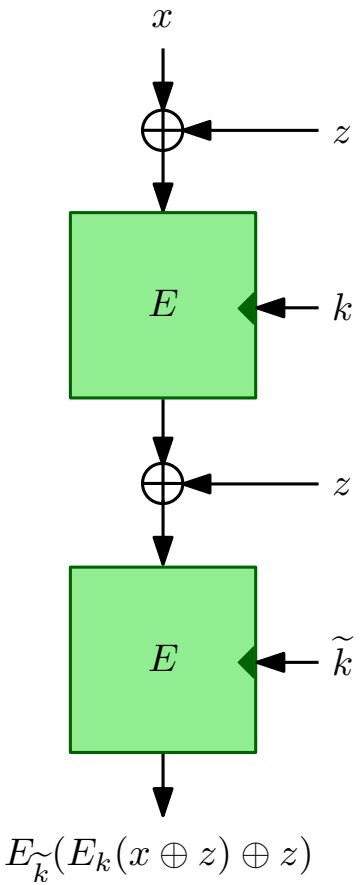

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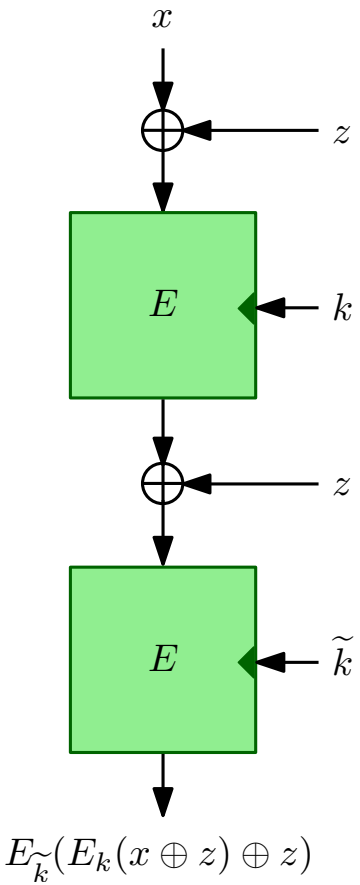

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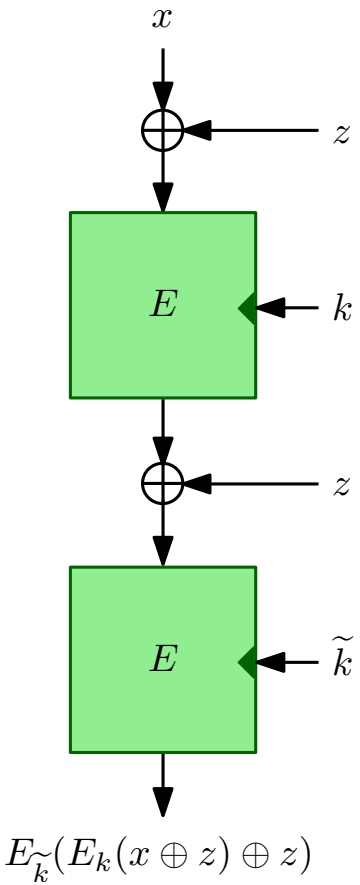

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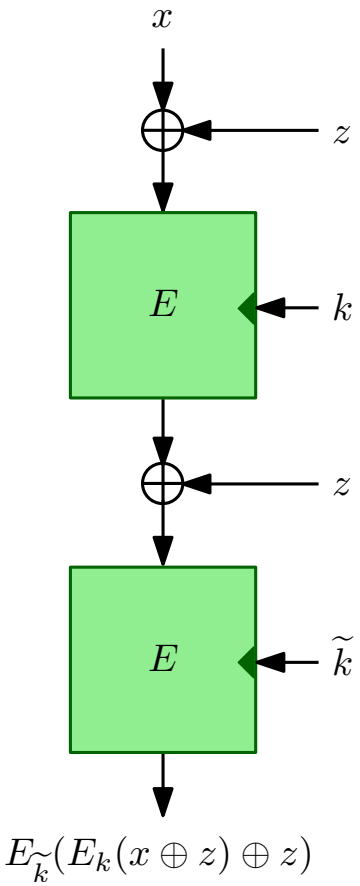

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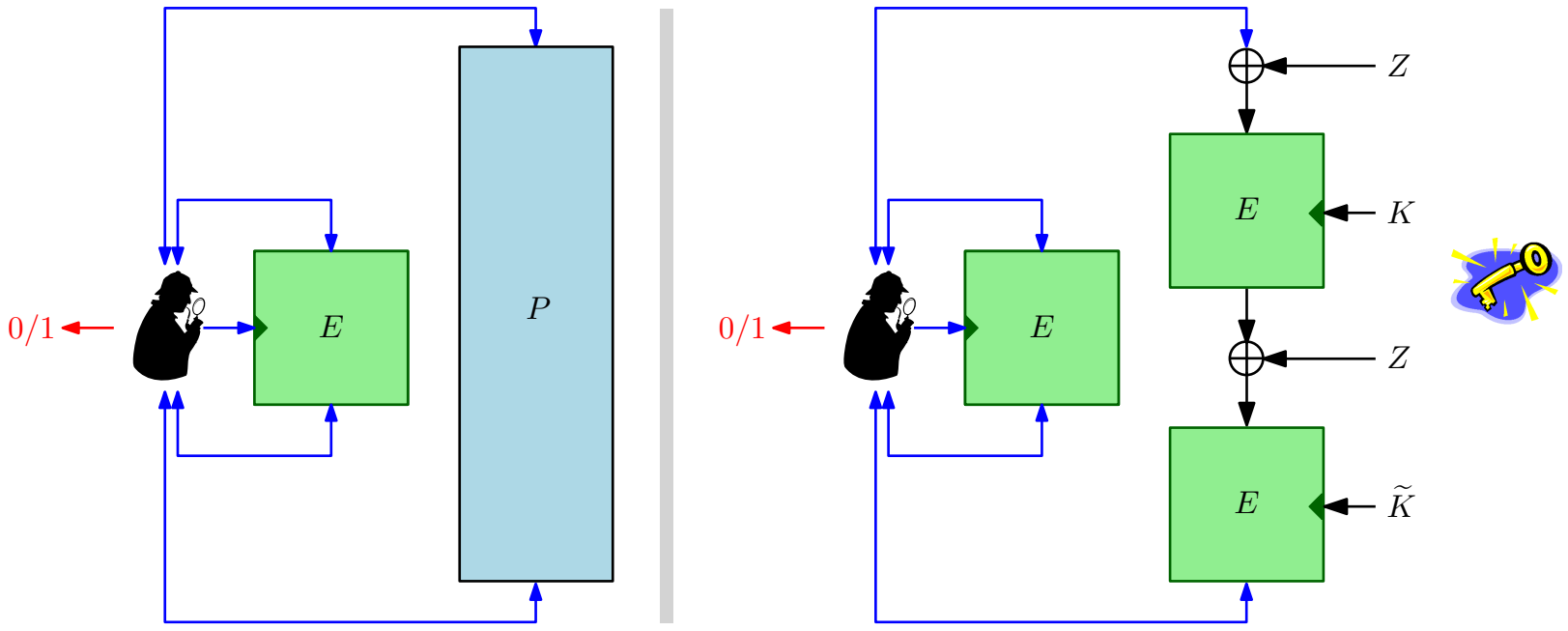
Main Result

Double XOR-Cascade is secure up to $2^{\kappa+n/2}$ queries.

A Glimpse at the Proof



The Initial Setting

$$\Delta^D\left((E,P),(E,2XOR_{K,Z}[E])\right)$$

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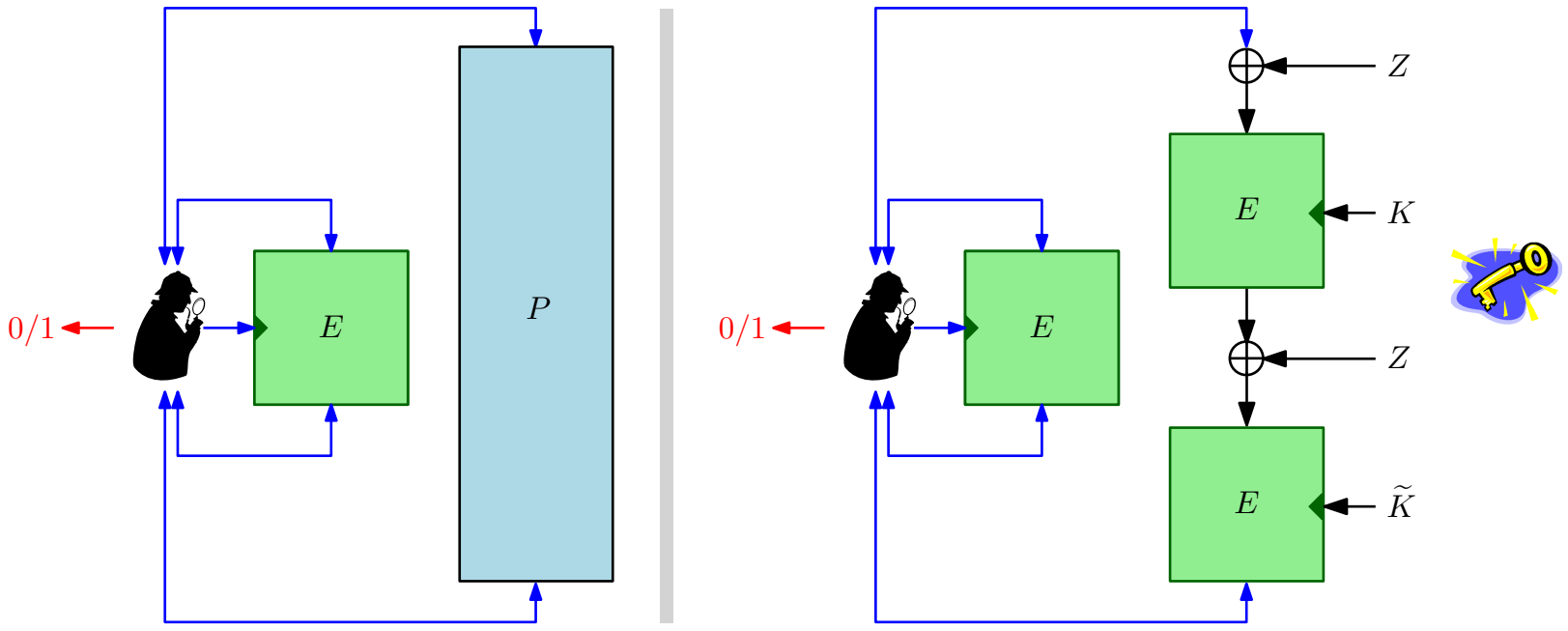

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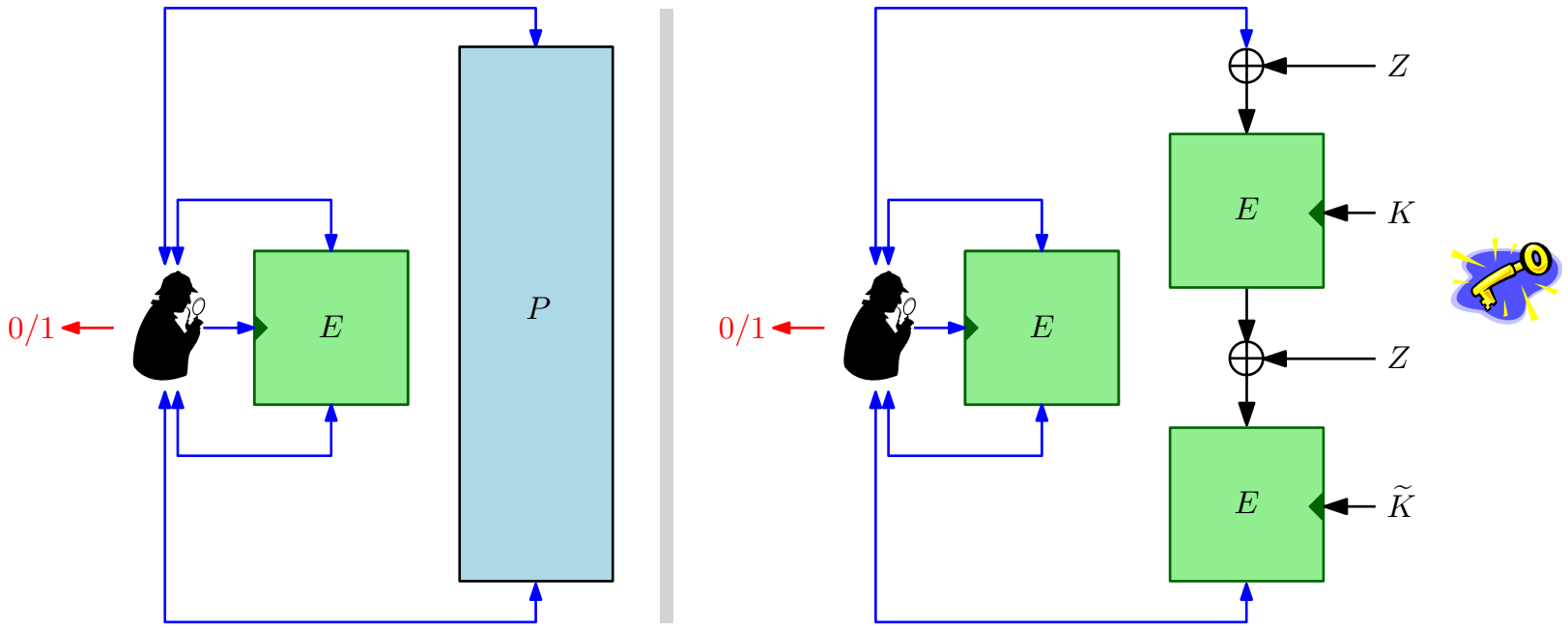

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Main Steps

- Reduce to a simpler combinatorial problem
- Show it is hard

A Glimpse at the Proof (2)

The New Problem: Distinguishing Permutations

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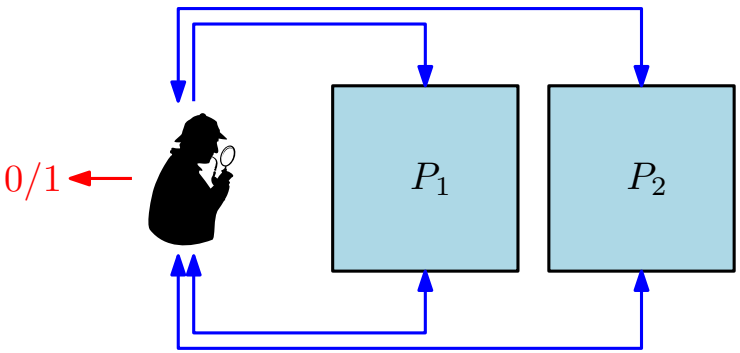

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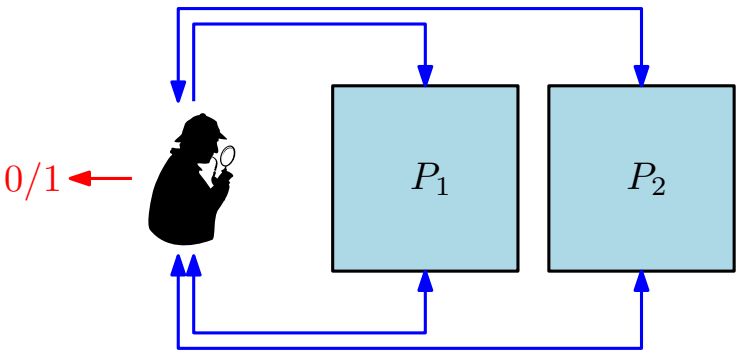

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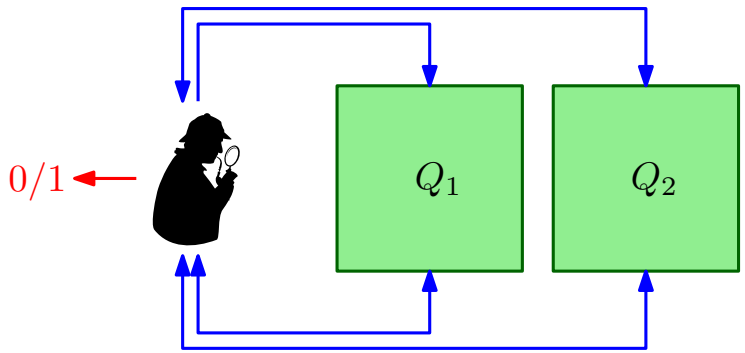

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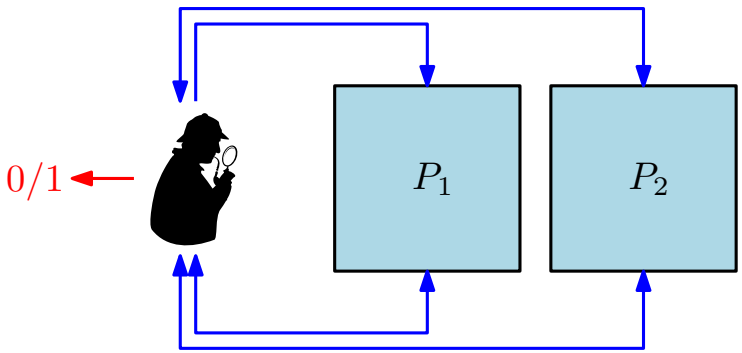

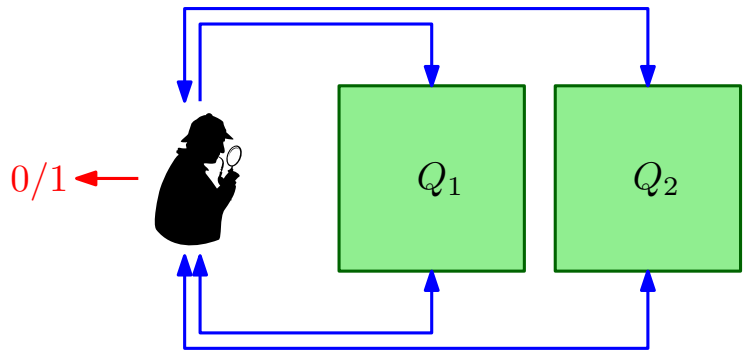

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Hard for $< 2^{n/2}$ queries!

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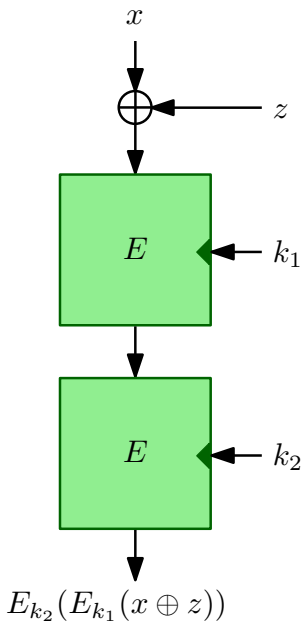

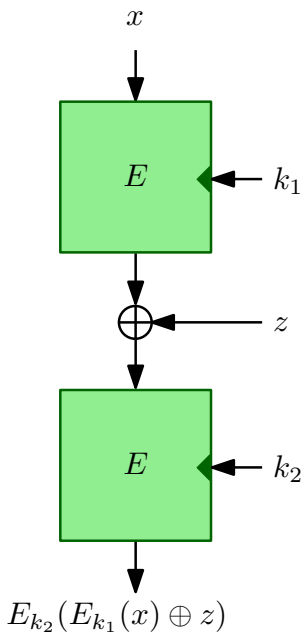

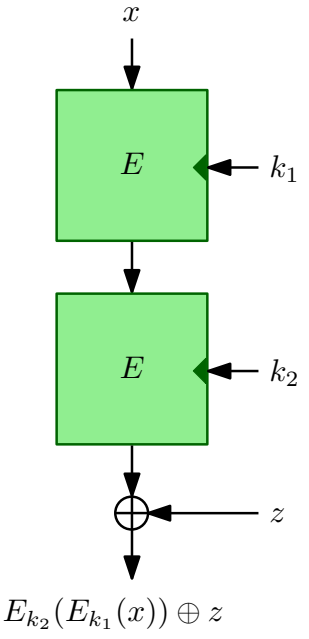

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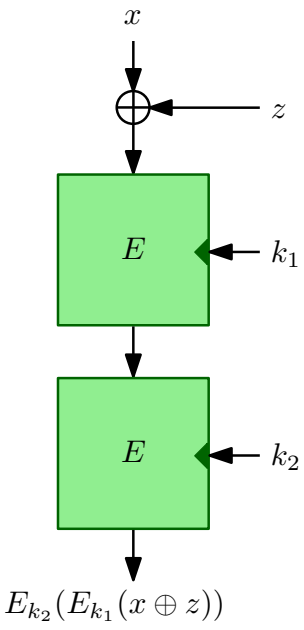

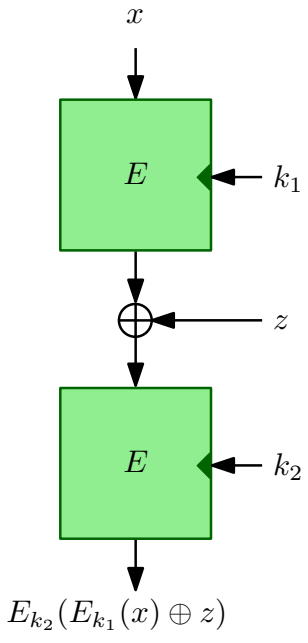

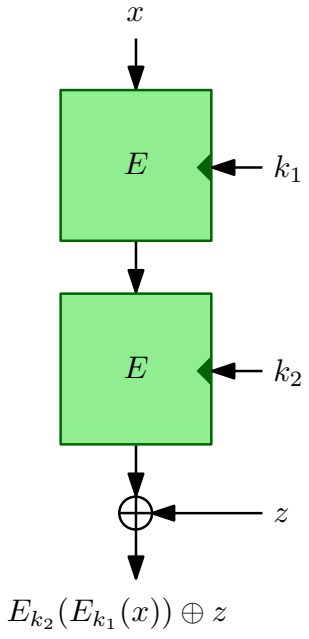

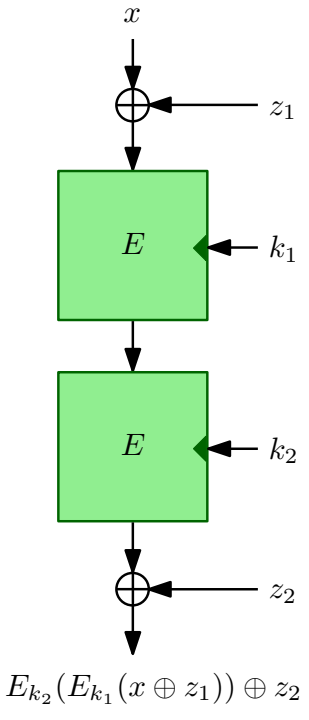

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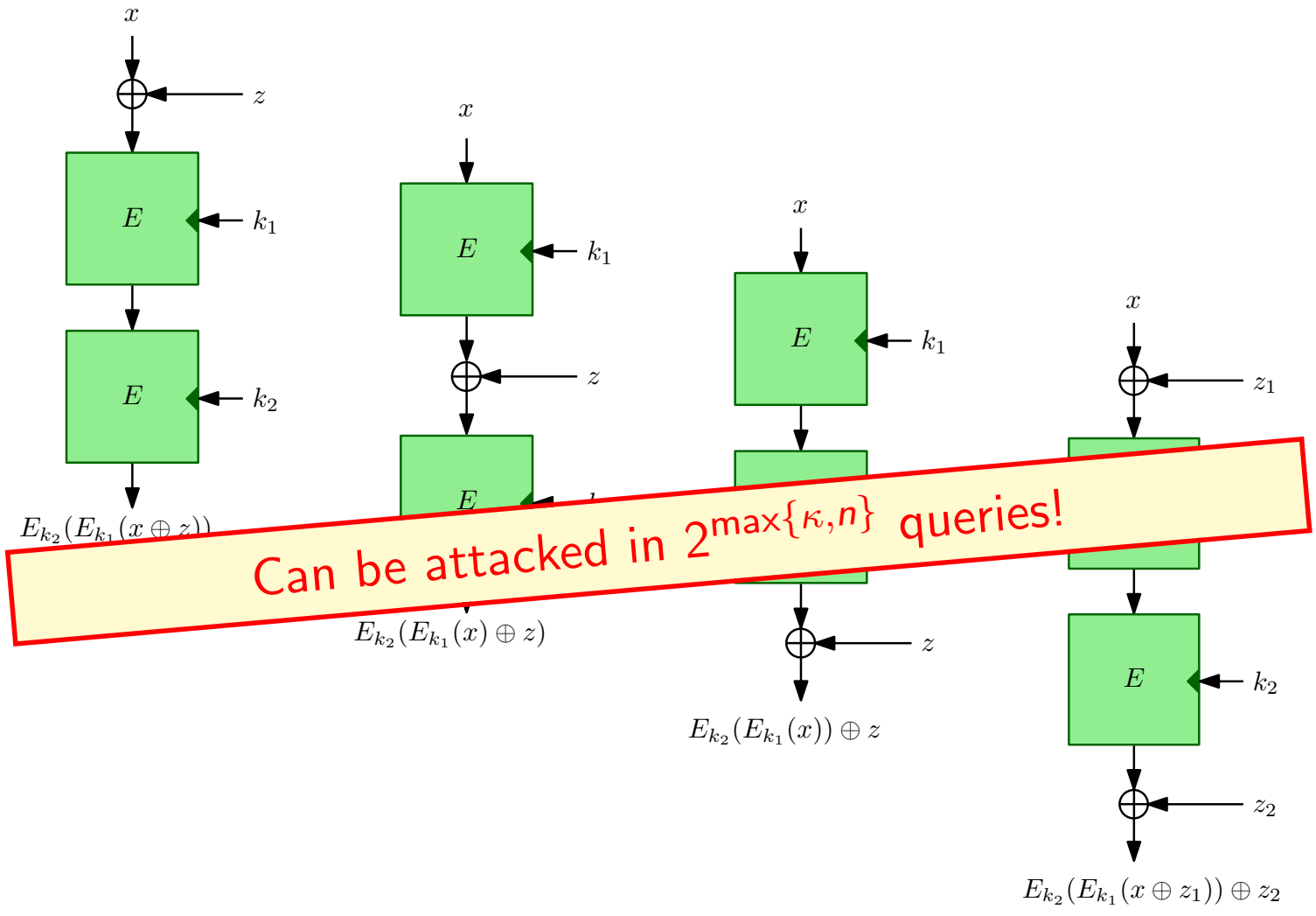

Summary

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    (2 BC queries per invocation)
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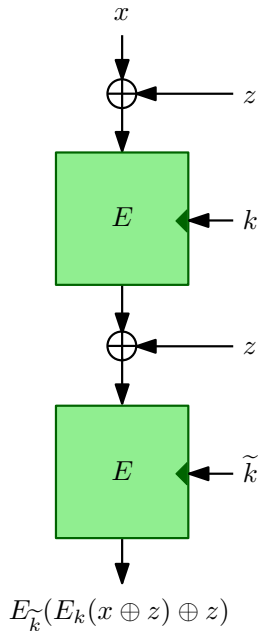

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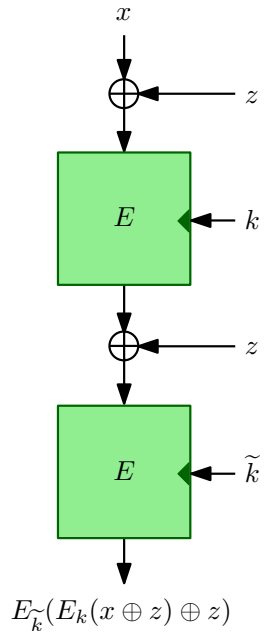

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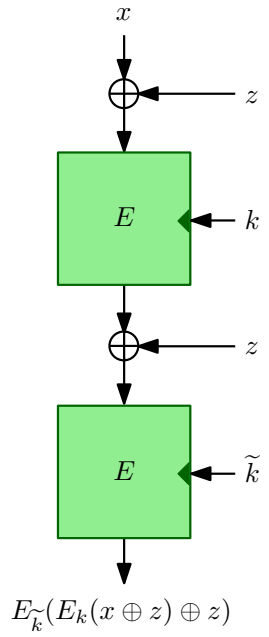

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