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Optimal Security Proofs for Full-Domain Hash, revisited

Cambridge, EUROCRYPT 2012

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procedure KeyGen $p, q \in_R \mathbb{P}$, $N = pq$ $e \in_R \mathbb{Z}_{\varphi(N)}$ Pick $H : \{0,1\}^* \to \mathbb{Z}_N$ return $(pk = (N, e, H), sk = (p, q))$	**procedure Sign**(sk, m) return $\sigma = H(m)^{\frac{1}{e}} \mod N$ **procedure Verify**(pk, m, σ) if $\sigma^e \mod N = H(m)$ then return 1 else return 0

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- RSA-FDH signatures are unique.

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- Can RSA-FDH be tightly secure?
- Exactly 10 years ago at EUROCRYPT 2002 in Amsterdam, Coron answered this by showing that a loss of a factor of q_s is optimal

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  - Proof does not hold for small e
- We show a tight proof for small e
  - Proof is to Φ-Hiding , which is stronger than RSA
- We then show some generalizations and extensions.

Flaw in Coron's Proof

Theorem 1 (Coron)

If there is a reduction $\mathcal{R}$ from RSA-FDH to inverting RSA, with security loss less than q_s, then we can efficiently invert RSA.

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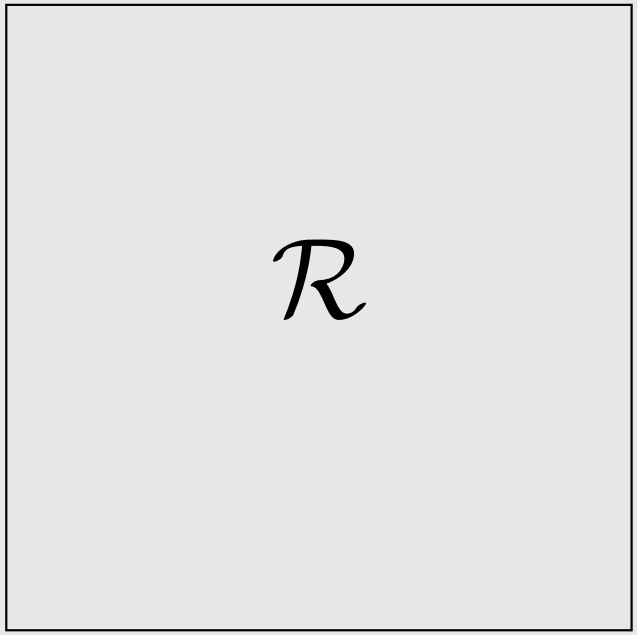

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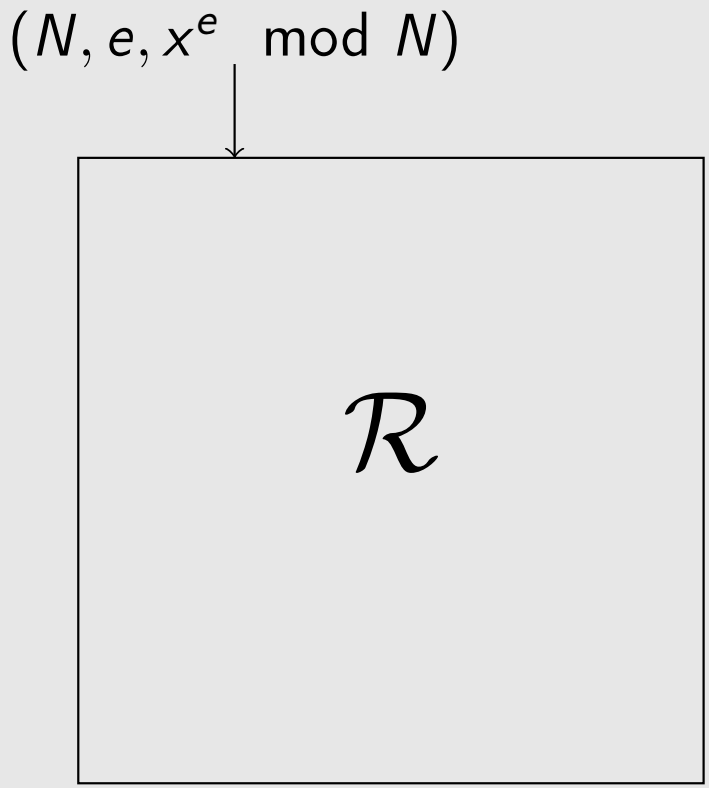

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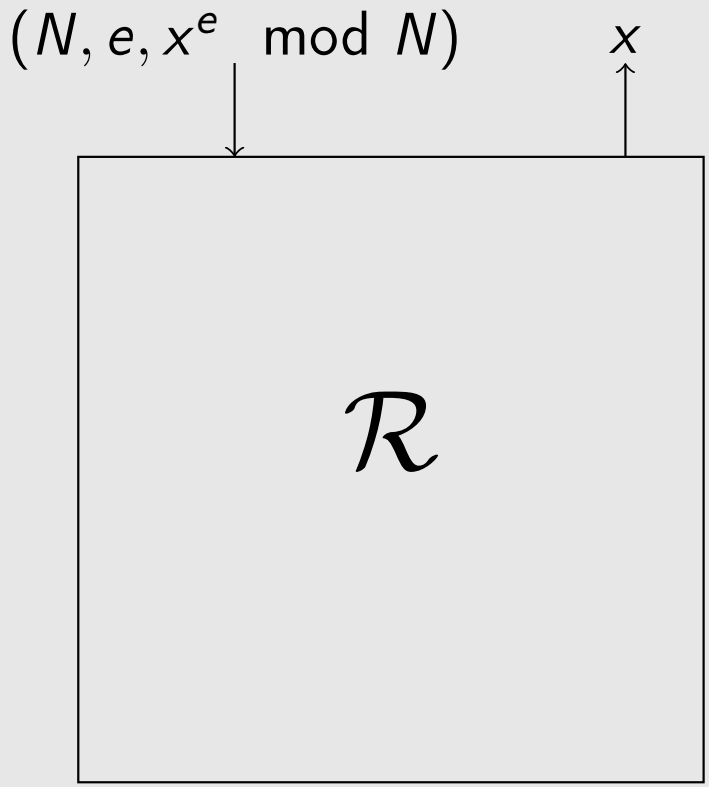

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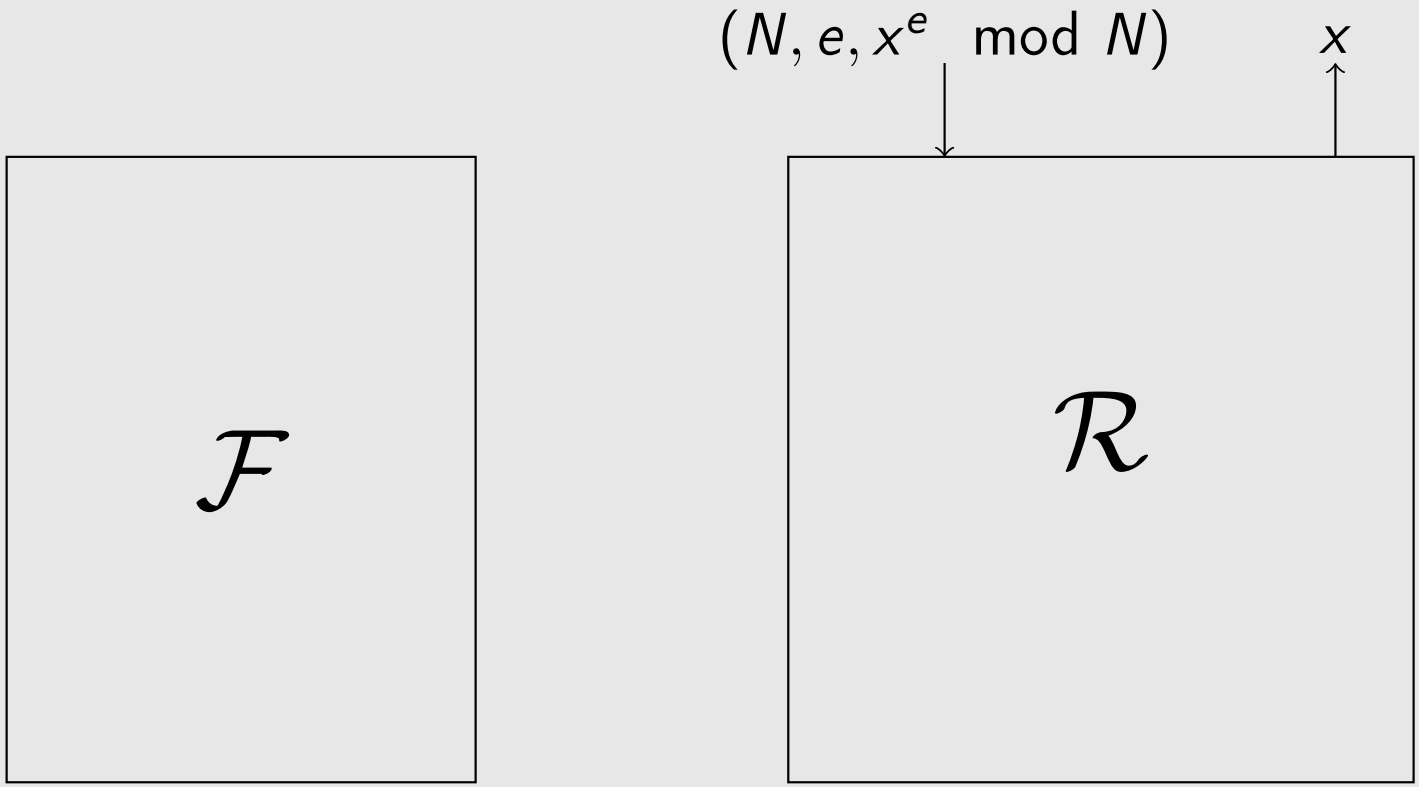

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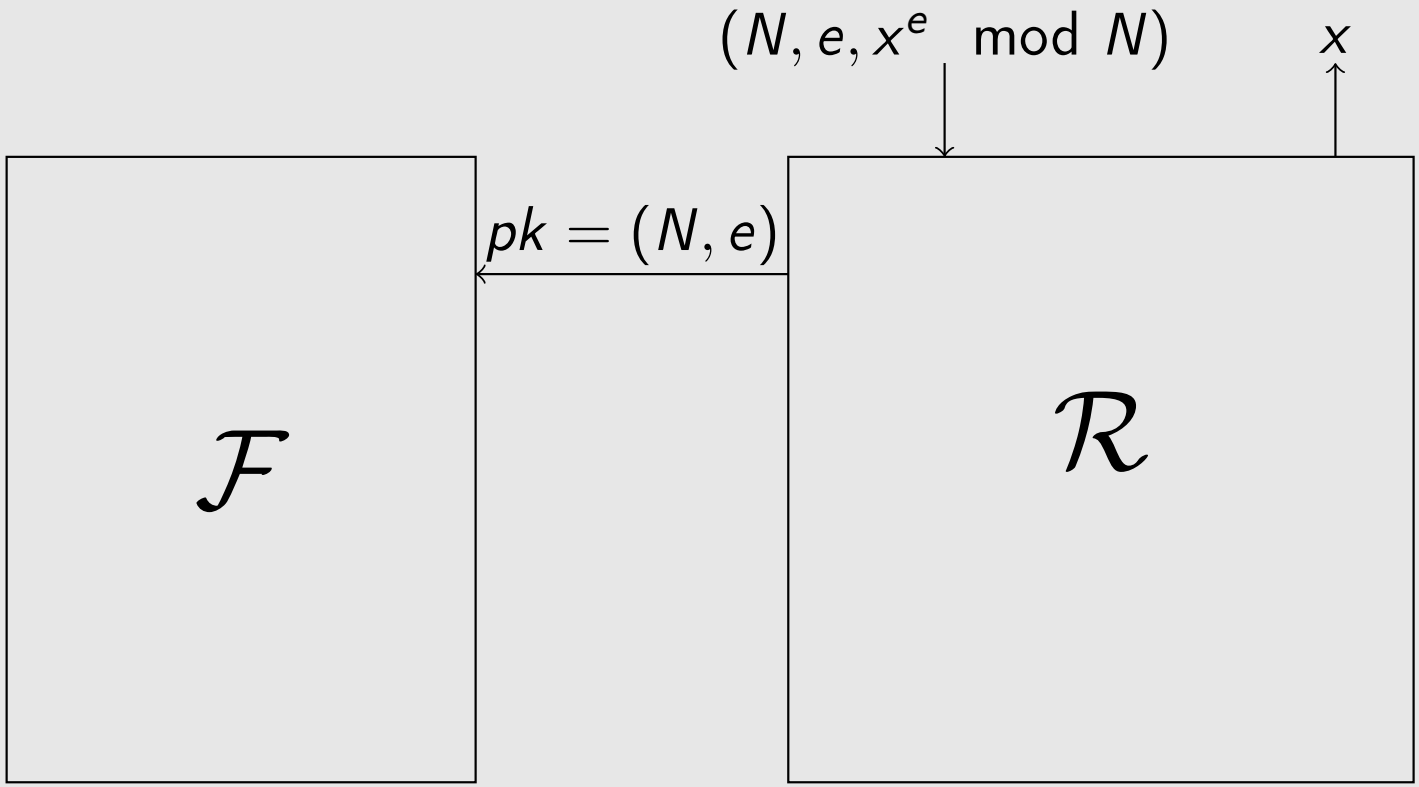

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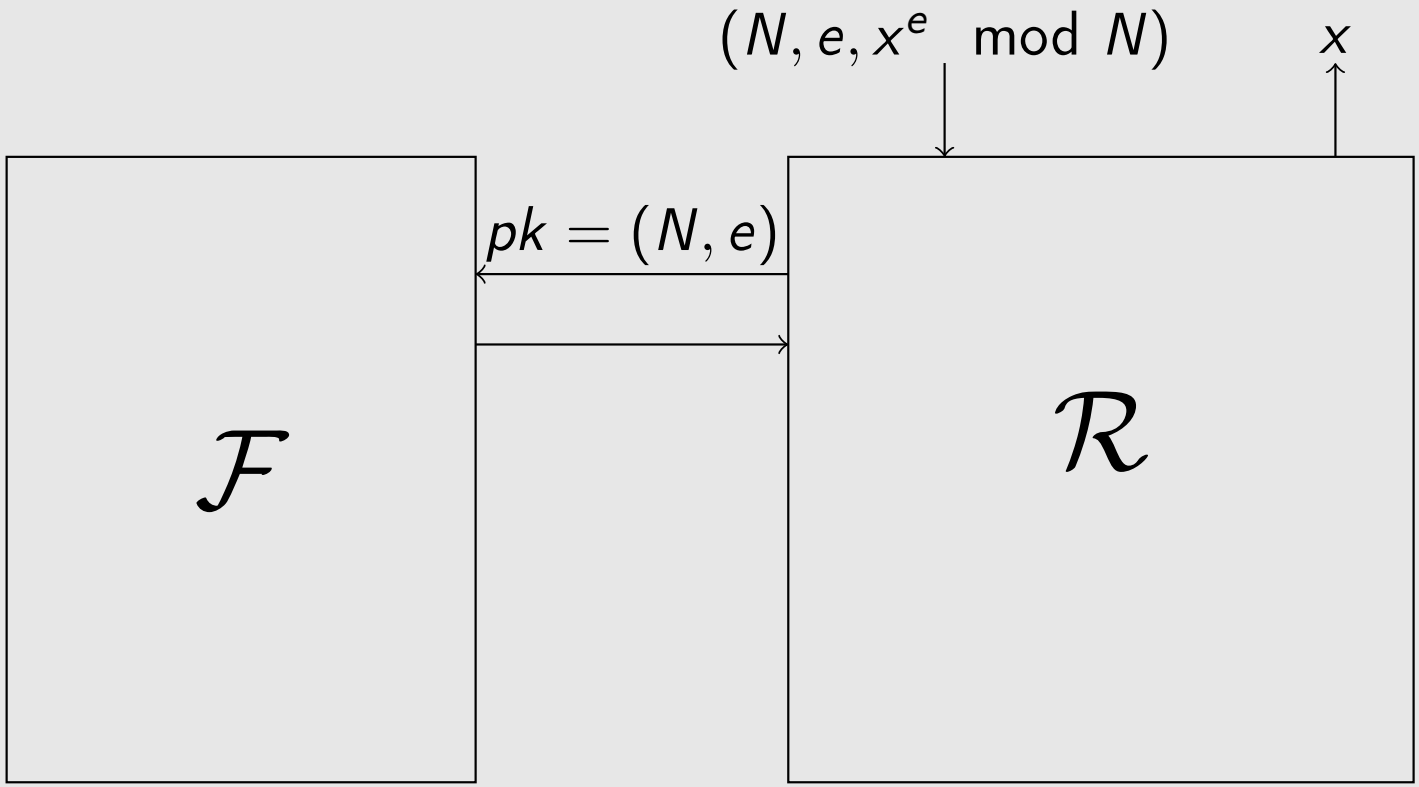

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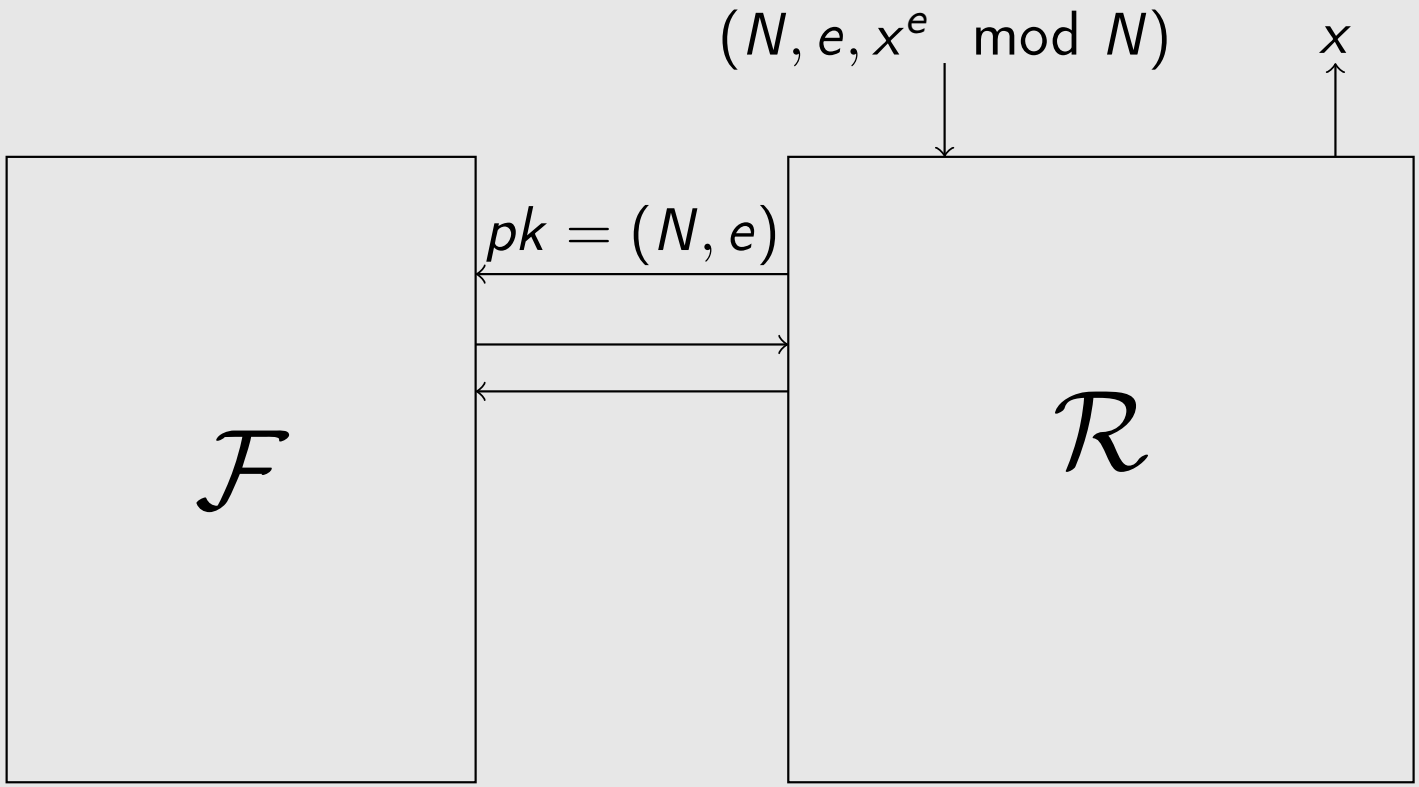

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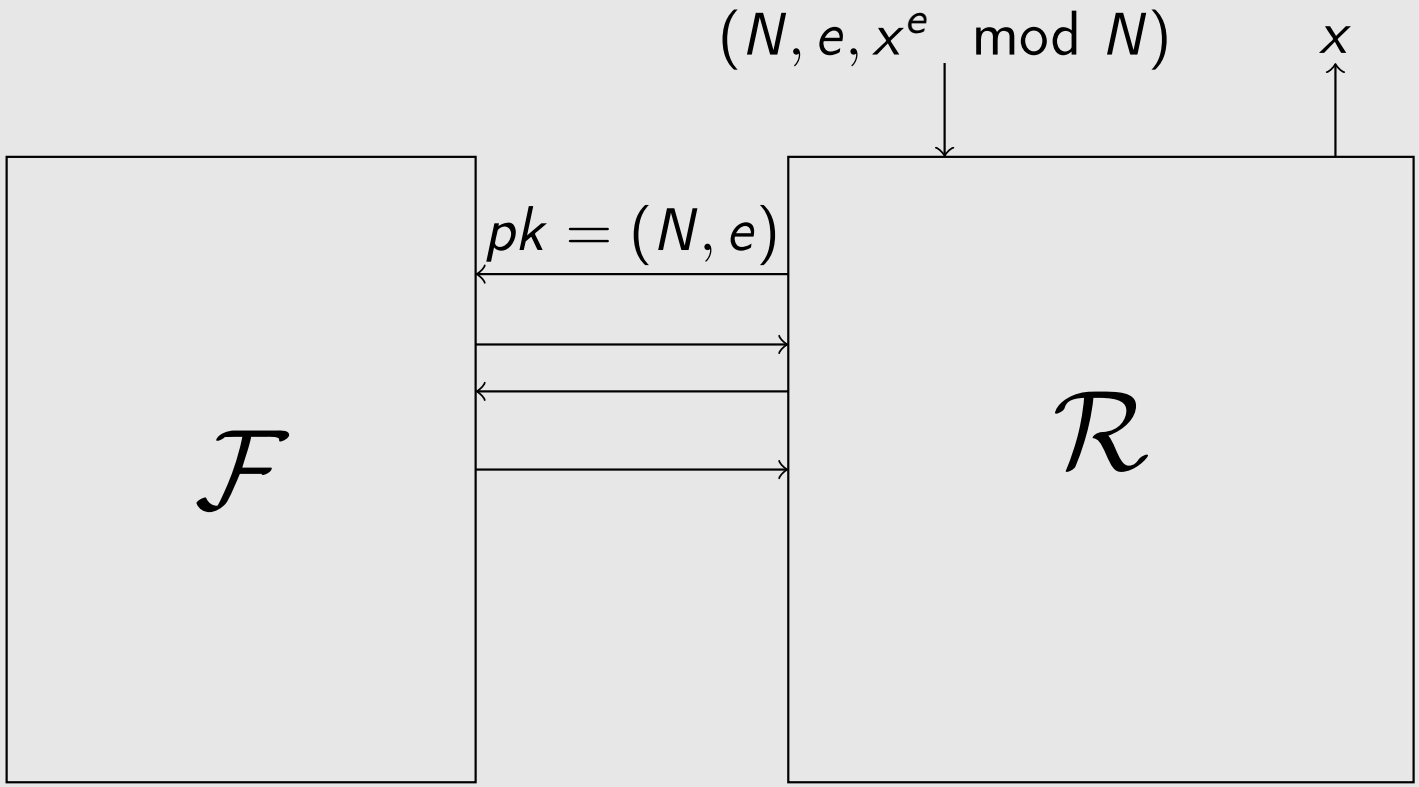

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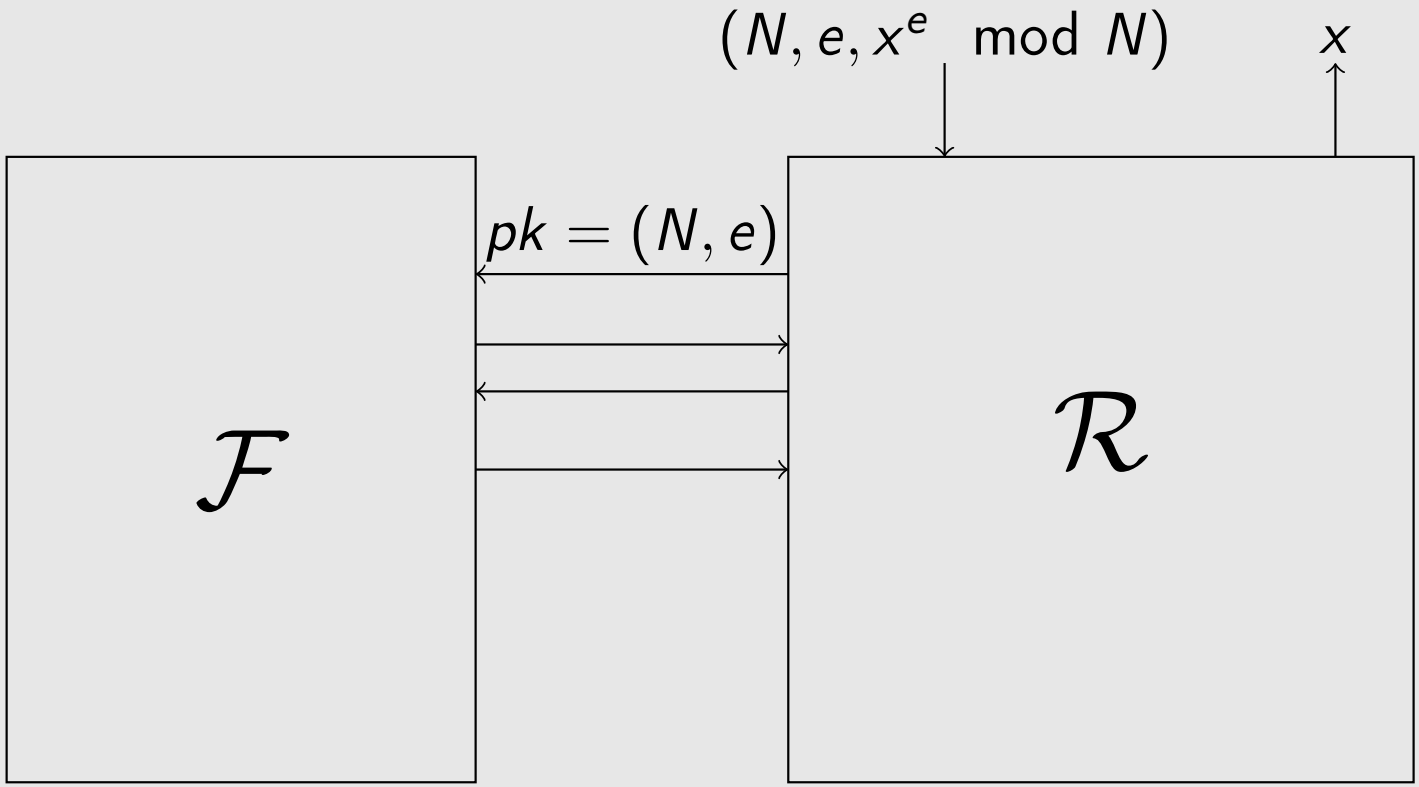

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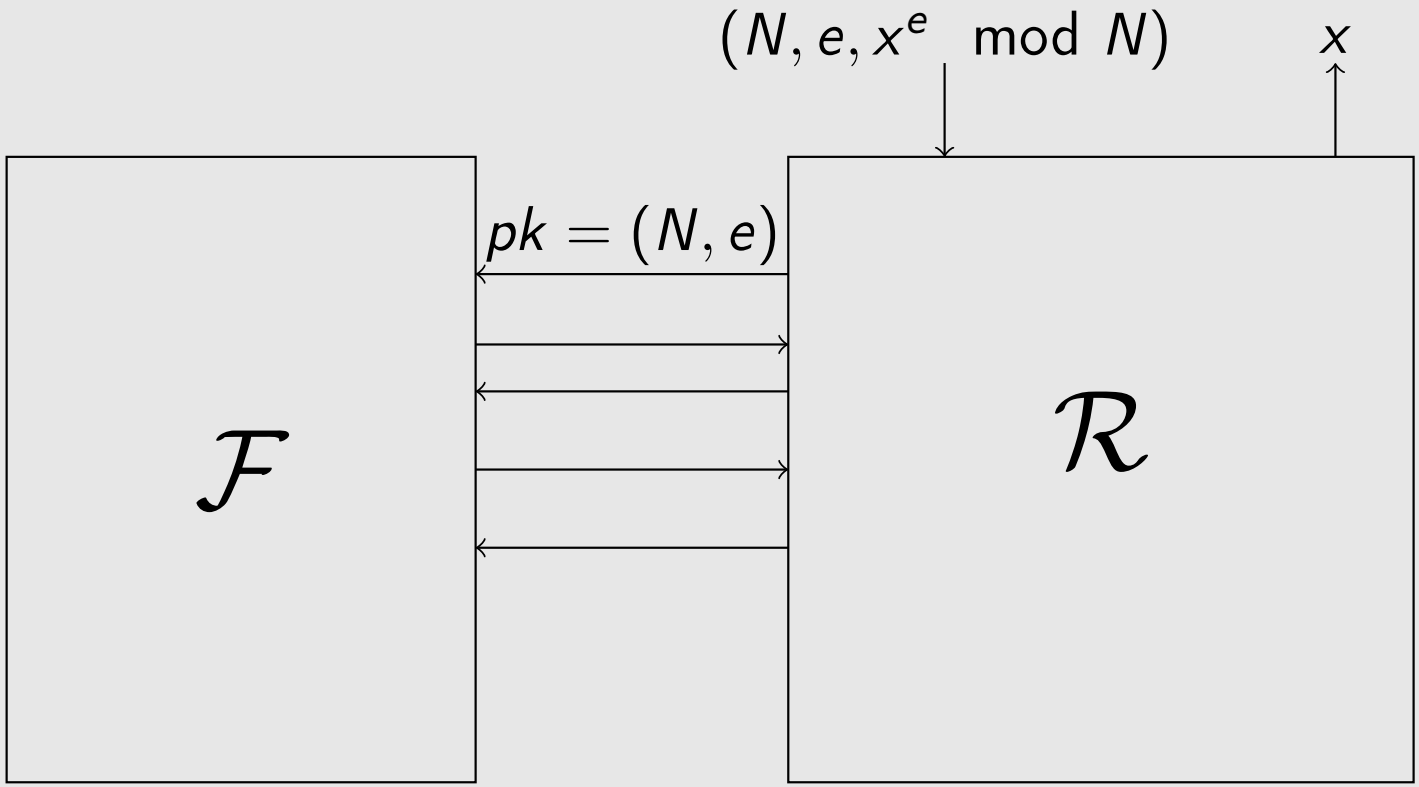

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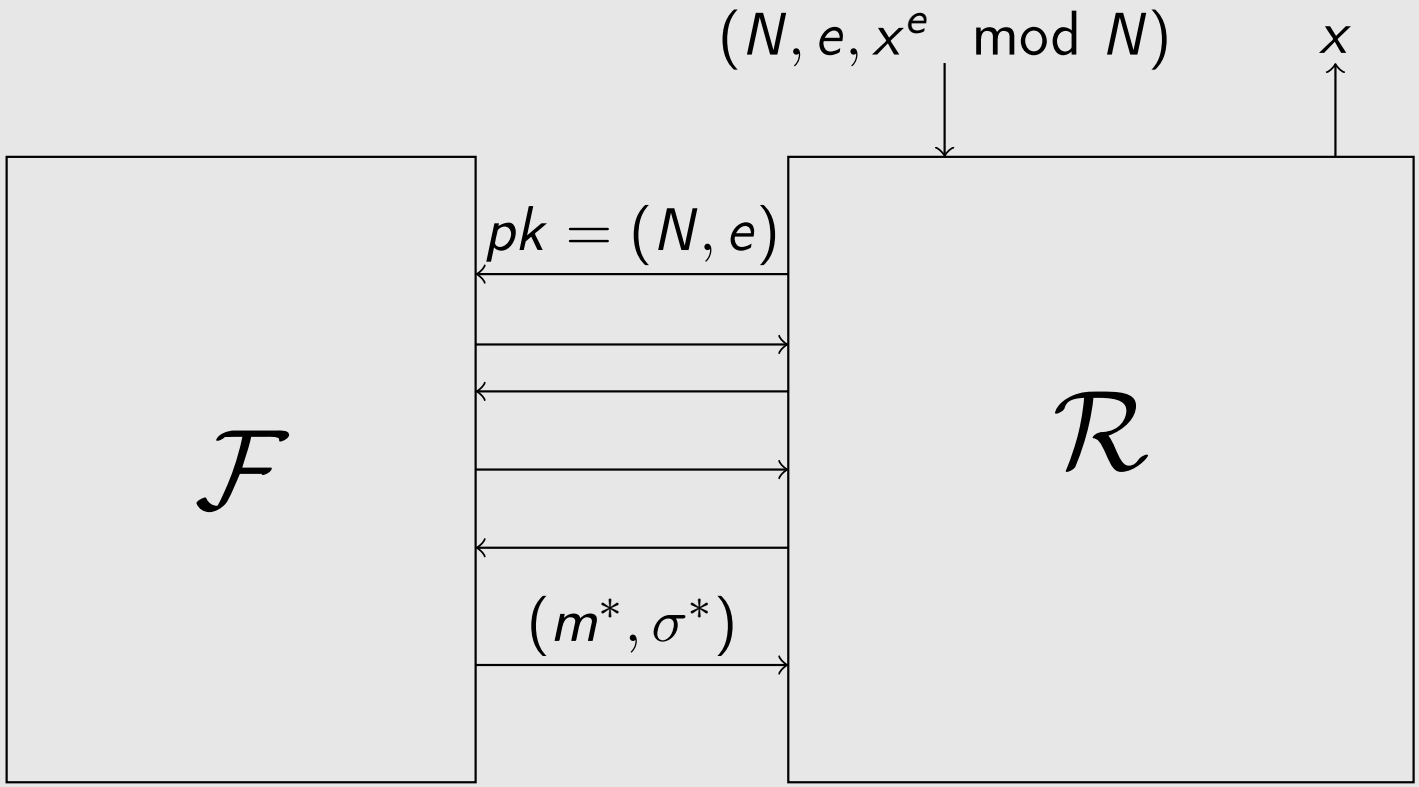

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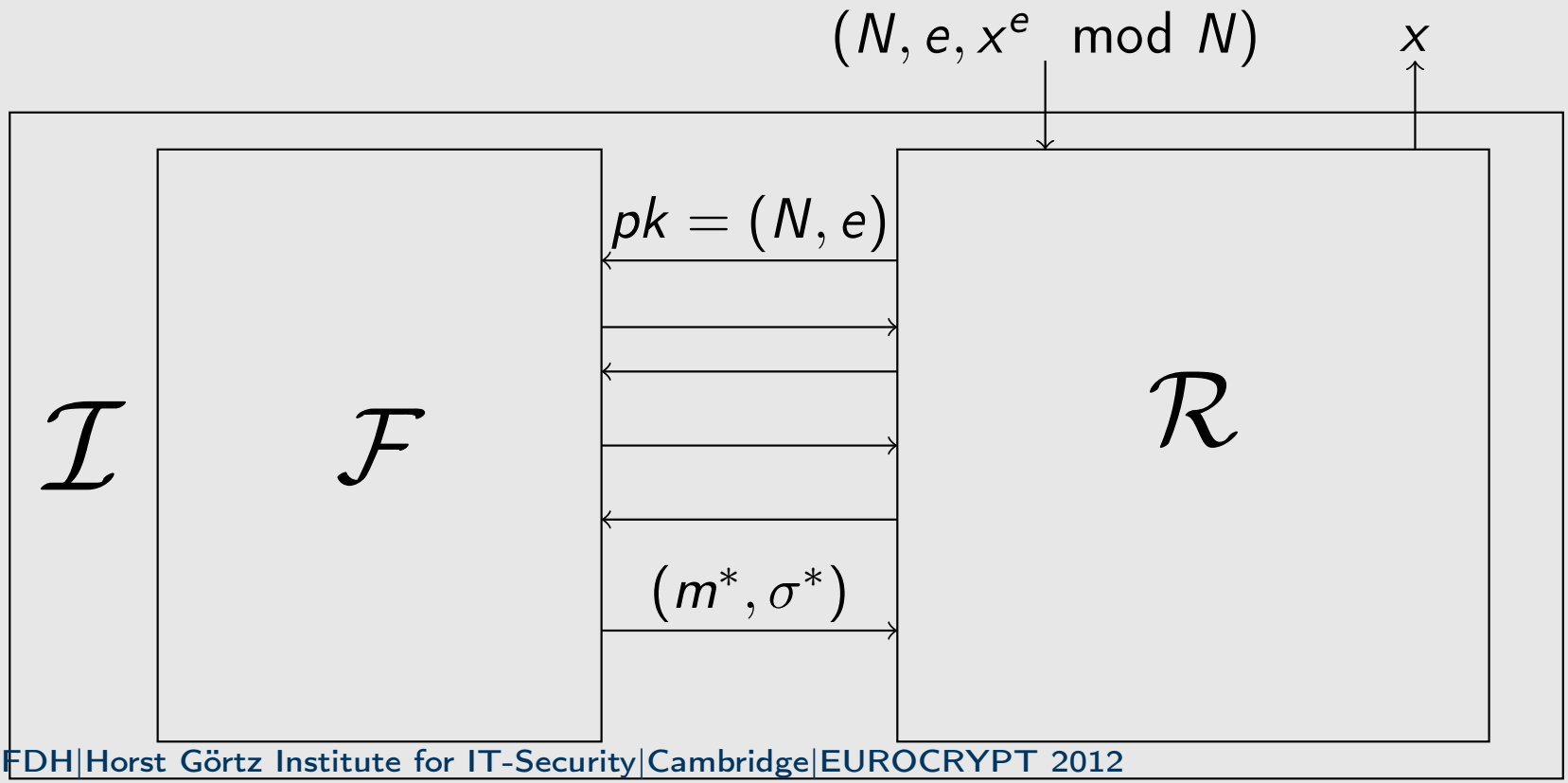

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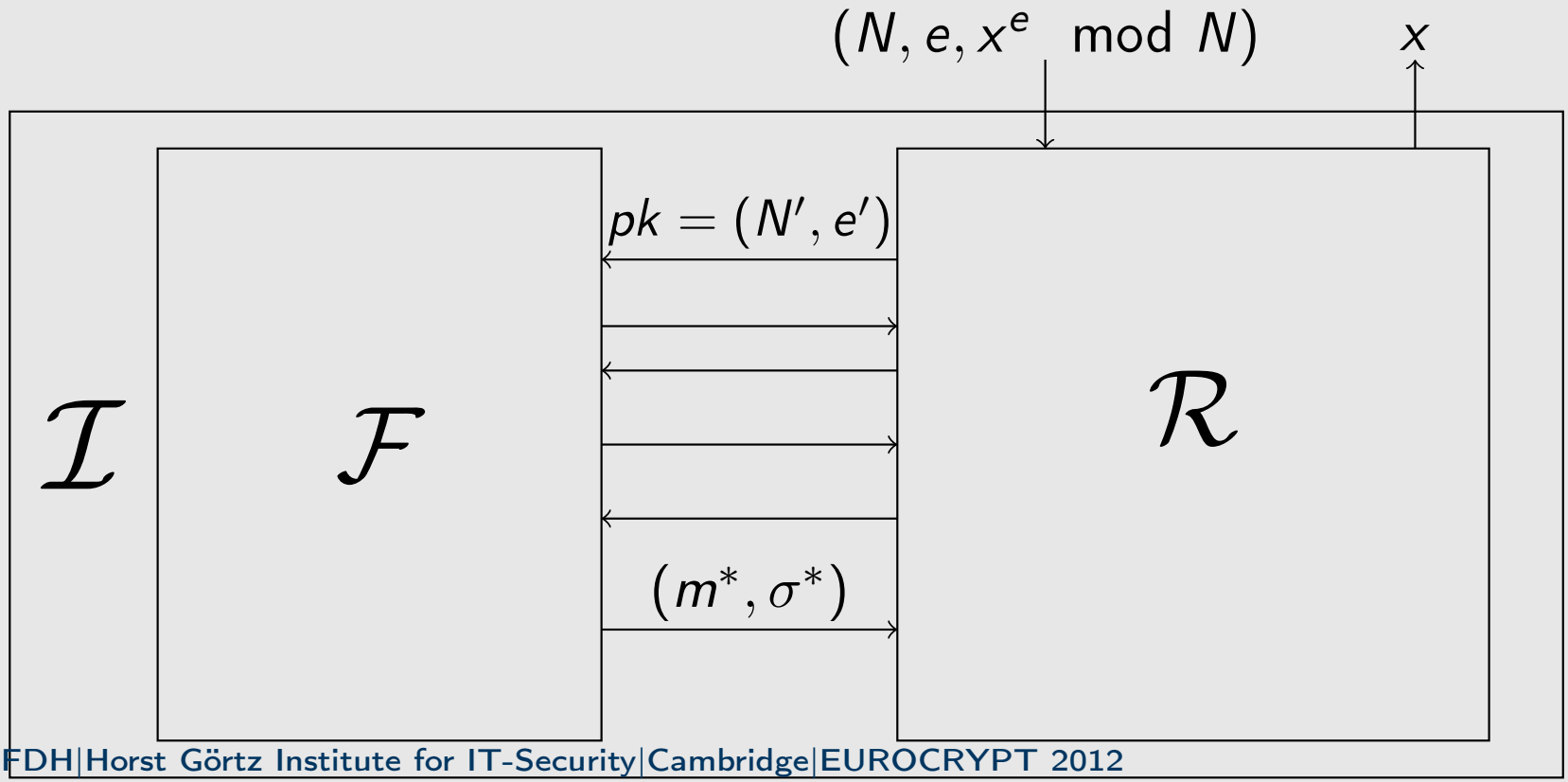

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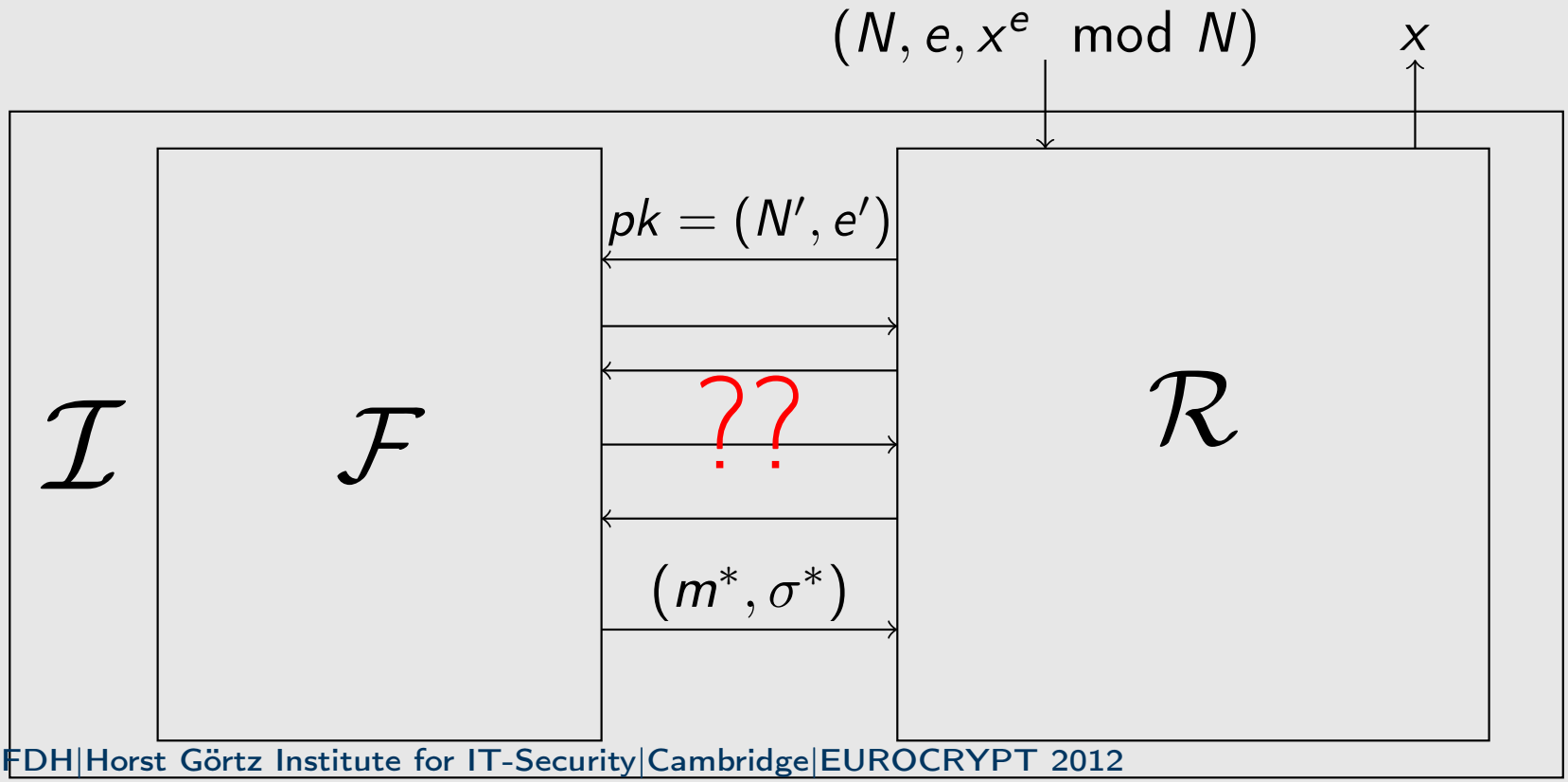

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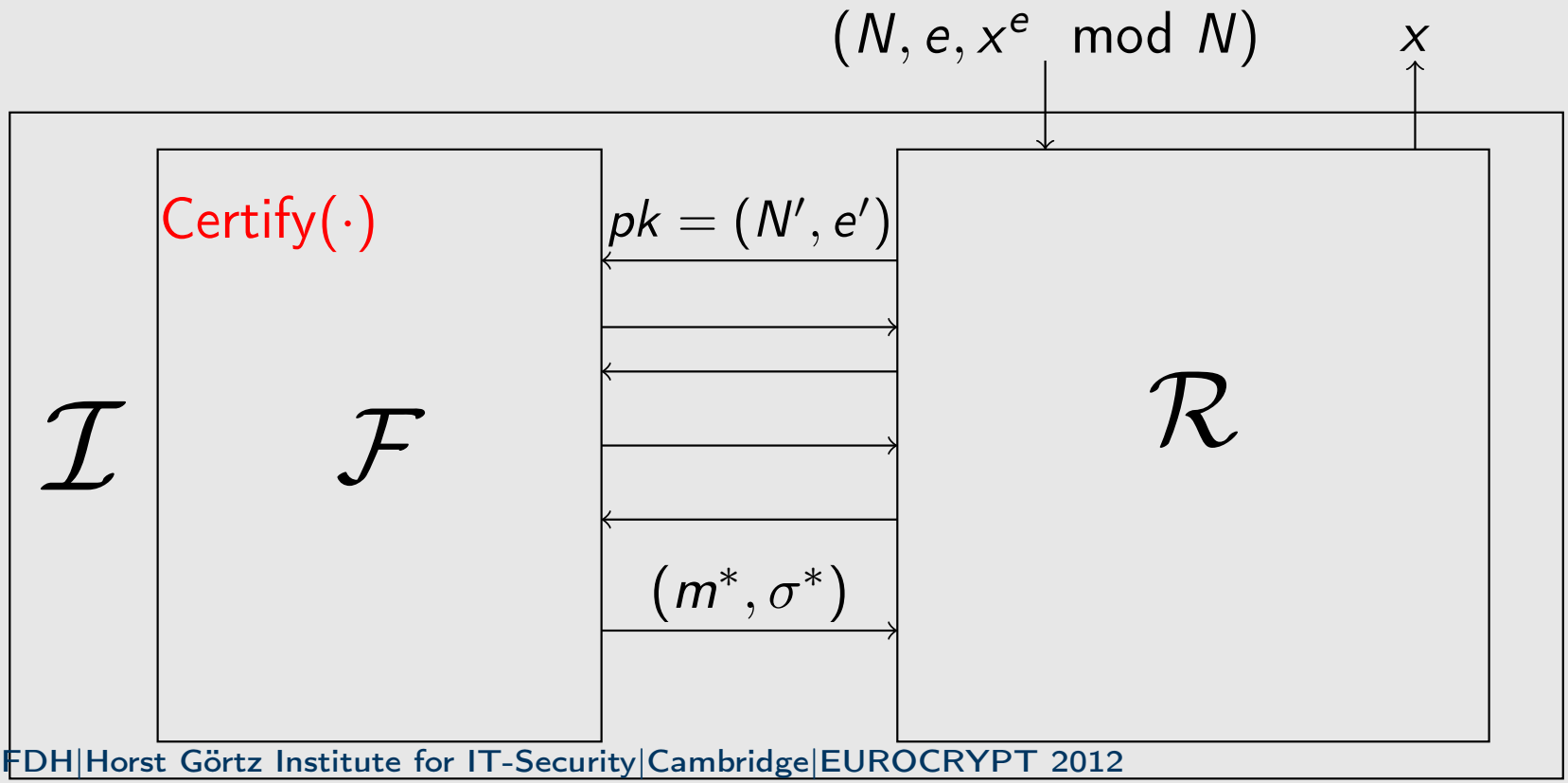

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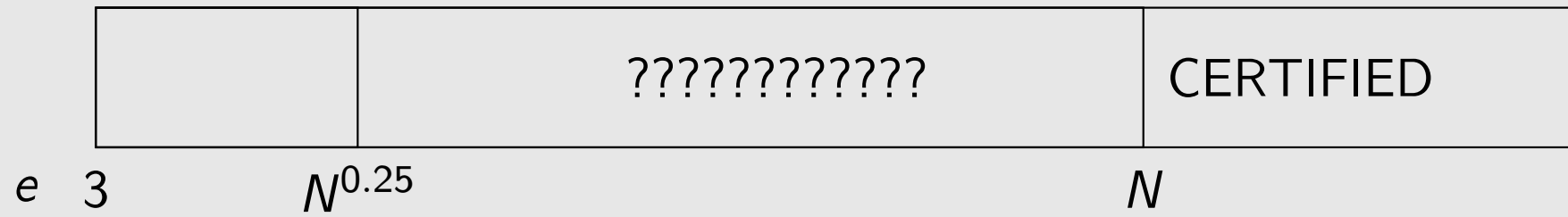

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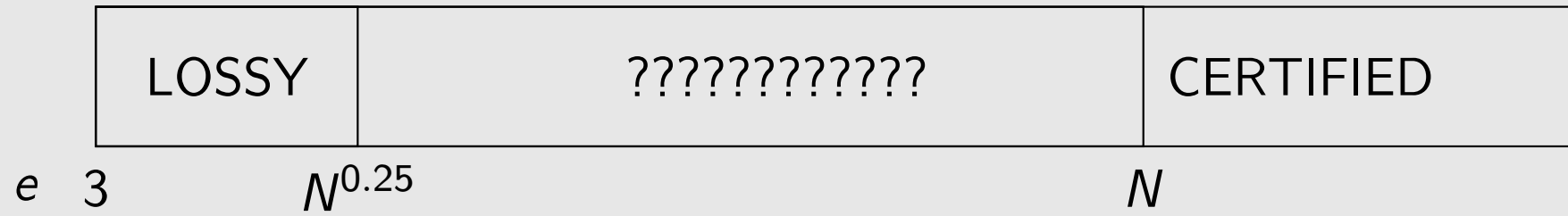

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- In particular for RSA, the lossy function is e-to-1.

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- Φ-Hiding states that given N and a prime $e < N^{0.25}$ it is hard to distinguish $e|\varphi(N)$ and $gcd(e, \varphi(N)) = 1$.

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If Φ-Hiding is (t', ε')-hard, then RSA-FDH is $(q_h, q_s, t, \varepsilon)$-secure, for any q_h, q_s, with $t \approx t', \varepsilon \approx 2\varepsilon'$.

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- The final security loss is approximately $2 = O(1)$.

Implications of our results

§ If we assume that solving Φ-Hiding is equivalent to inverting RSA, then:

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- We can show impossibility for any hard problem Π and any certified unique signature scheme Σ.

Theorem 4

If there is a reduction $\mathcal{R}$ from Σ to solving Π, with security loss less than q_s, then we can efficiently solve Π.

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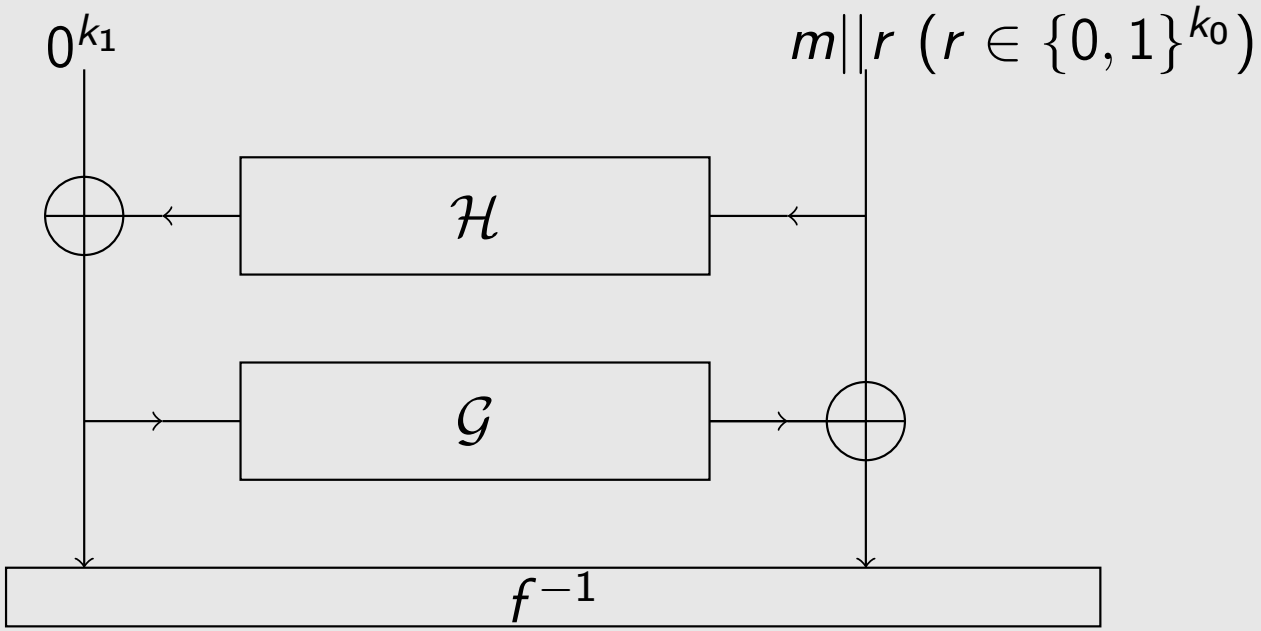

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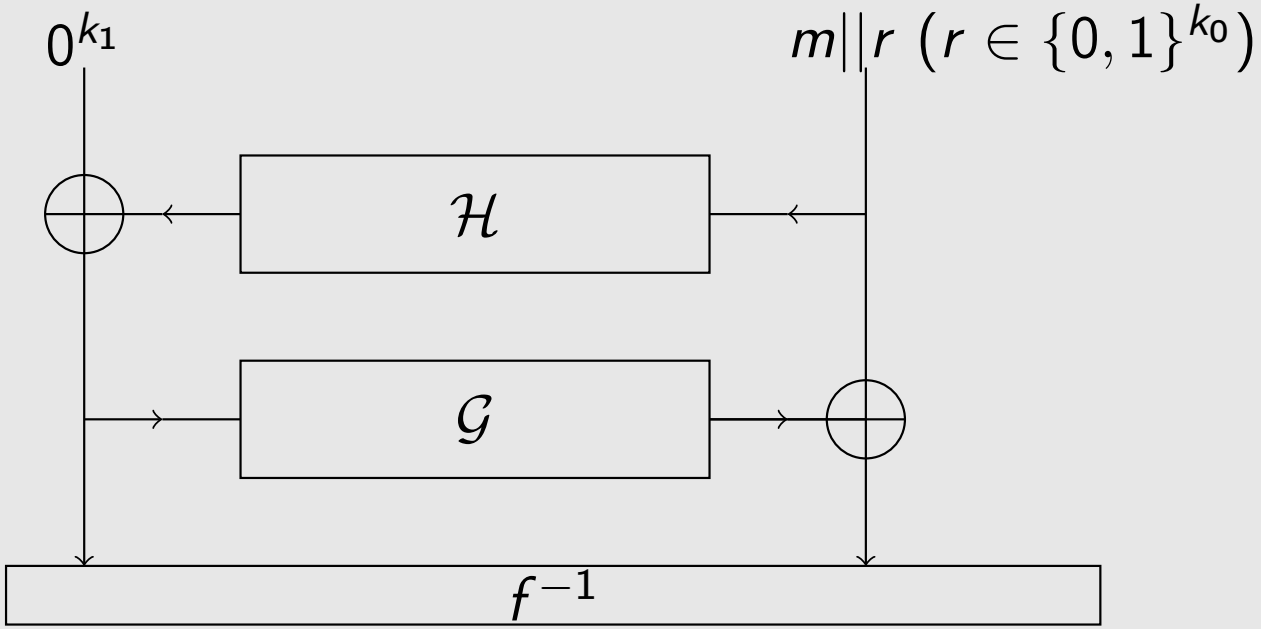
Theorem 5

If Φ-Hiding is (t', ε')-hard, then RSA-PSS-R is $(q_h, q_s, t, \varepsilon)$-secure, for any q_h, q_s, with $t \approx t', \varepsilon \approx 2 \cdot \varepsilon' + \frac{(q_h+q_s)^2}{2^{k_1}}$



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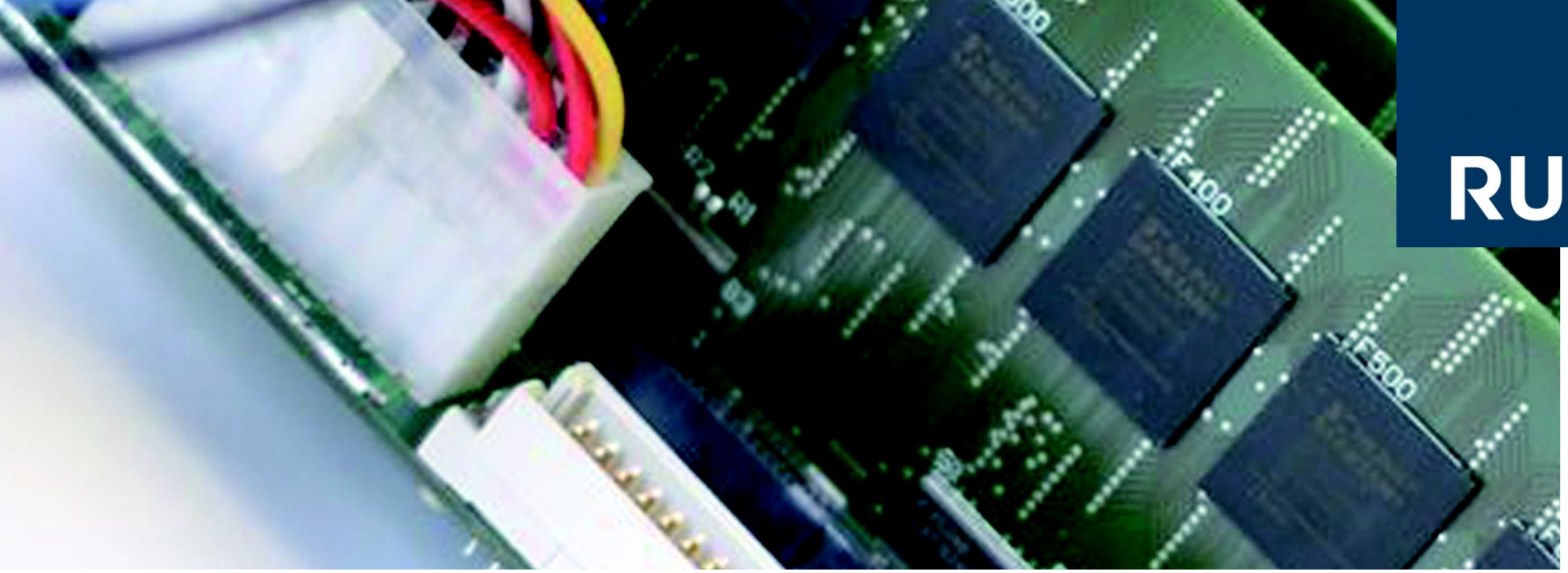

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Many thanks for your attention!

QUESTIONS?

hgi
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