# On the Approximation Ratio of the Path Matching Christofides Algorithm

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# The Traveling Salesman Problem

## Definition

The *traveling salesman problem (TSP)* is the following optimization problem. Given a complete graph $G$ with edge costs, find a Hamiltonian cycle of minimum cost in $G$.

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The *metric traveling salesman problem ($\Delta$-TSP)* is the TSP restricted to instances satisfying the triangle inequality

$$c(\{v, w\}) \leq c(\{v, u\}) + c(\{u, w\}) \quad \forall u, v, w \in V(G).$$

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The *$\beta$-metric traveling salesman problem ($\Delta_\beta$-TSP)* is the TSP restricted to instances satisfying the $\beta$-triangle inequality

$$c(\{v, w\}) \leq \beta \cdot \Big(c(\{v, u\}) + c(\{u, w\})\Big) \quad \forall u, v, w \in V(G).$$

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**Input:** A complete, weighted, metric graph.

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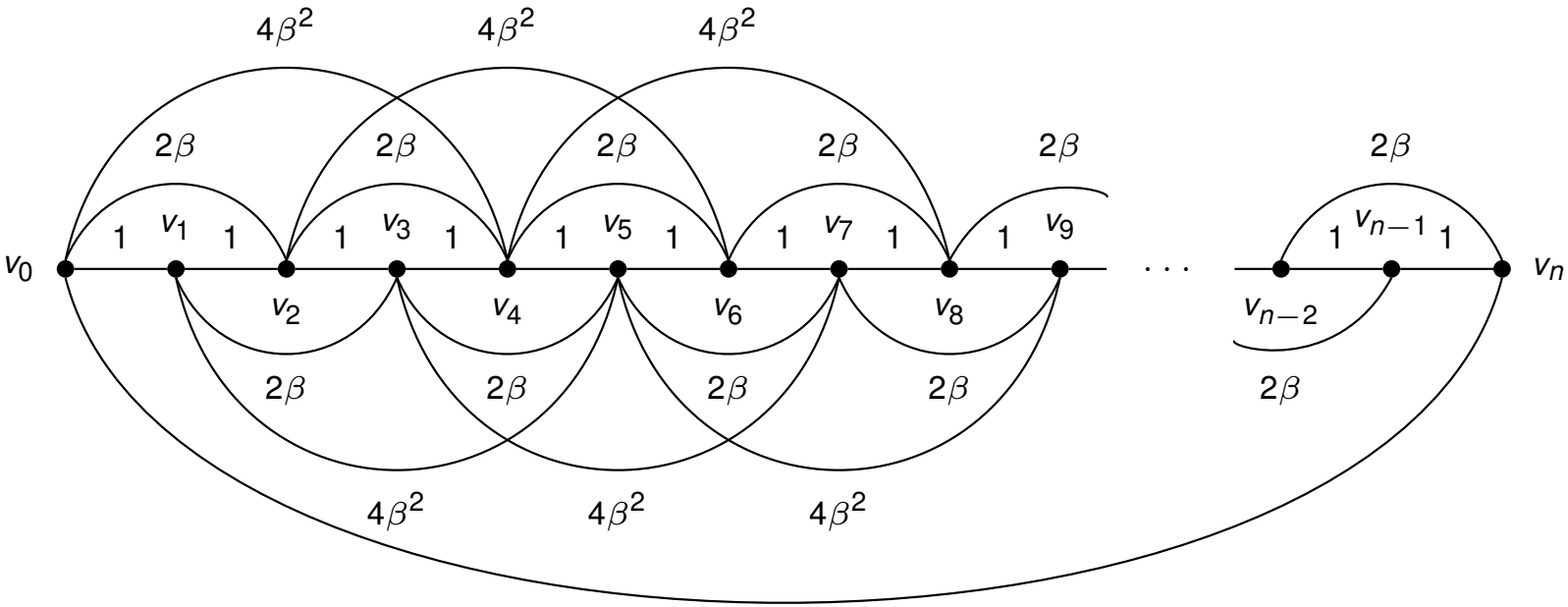


$n \cdot \beta^{\log n}$

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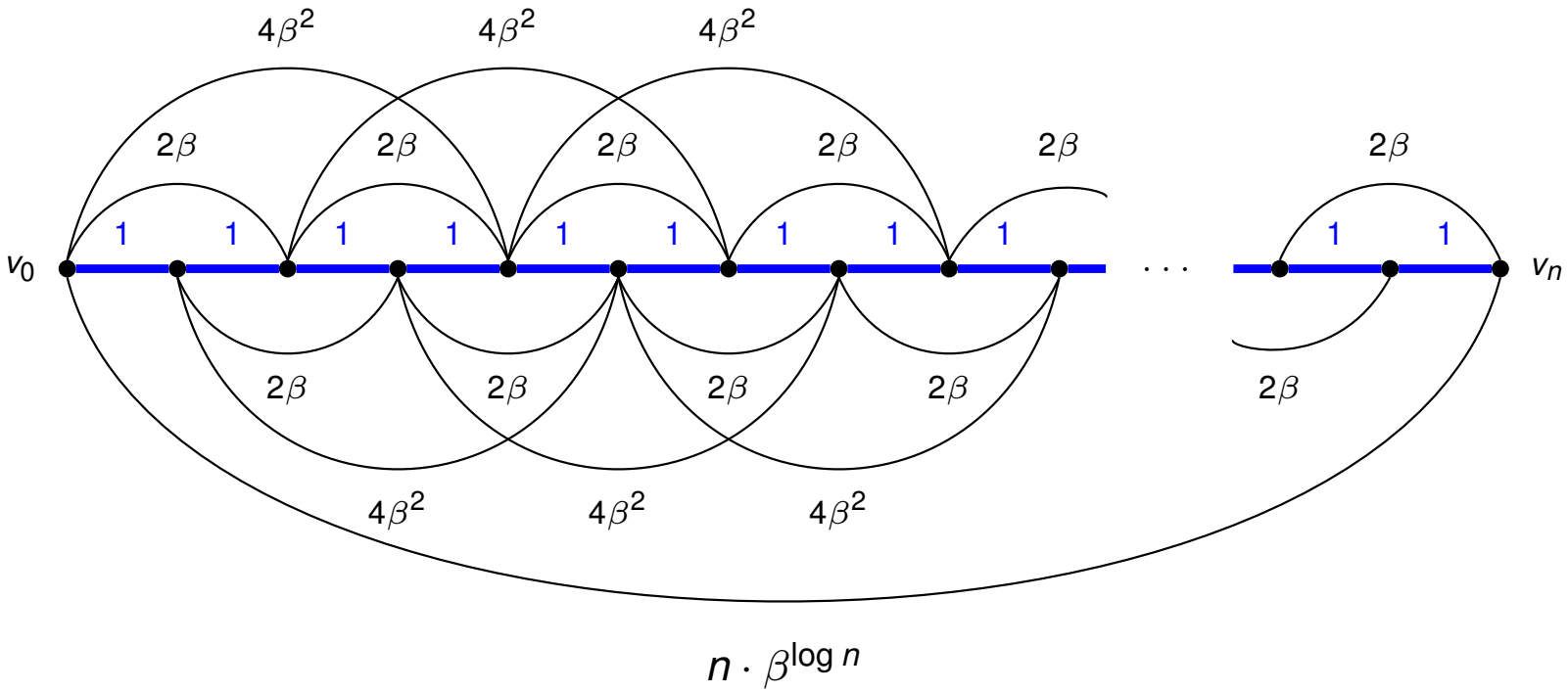


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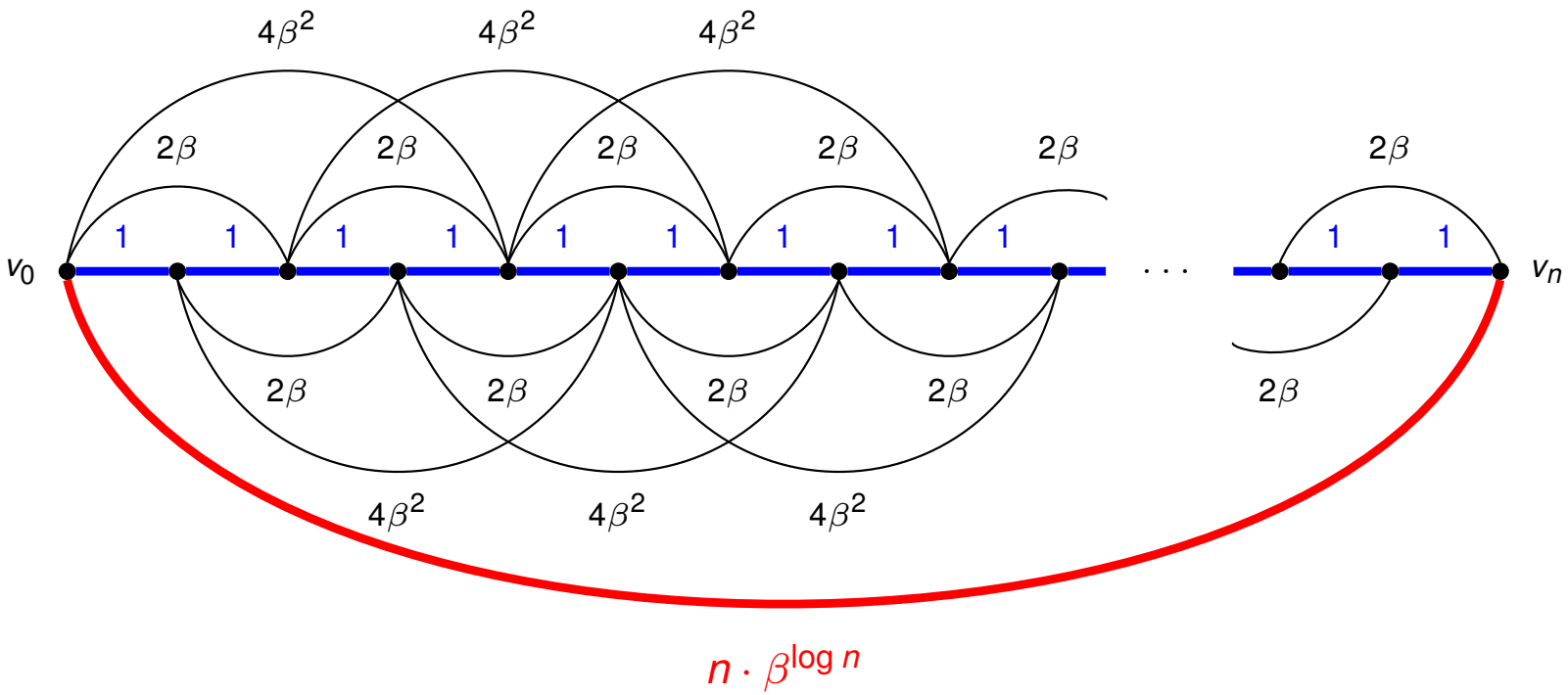


1. Minimum spanning tree $T$. 2. Minimum perfect matching $M$.

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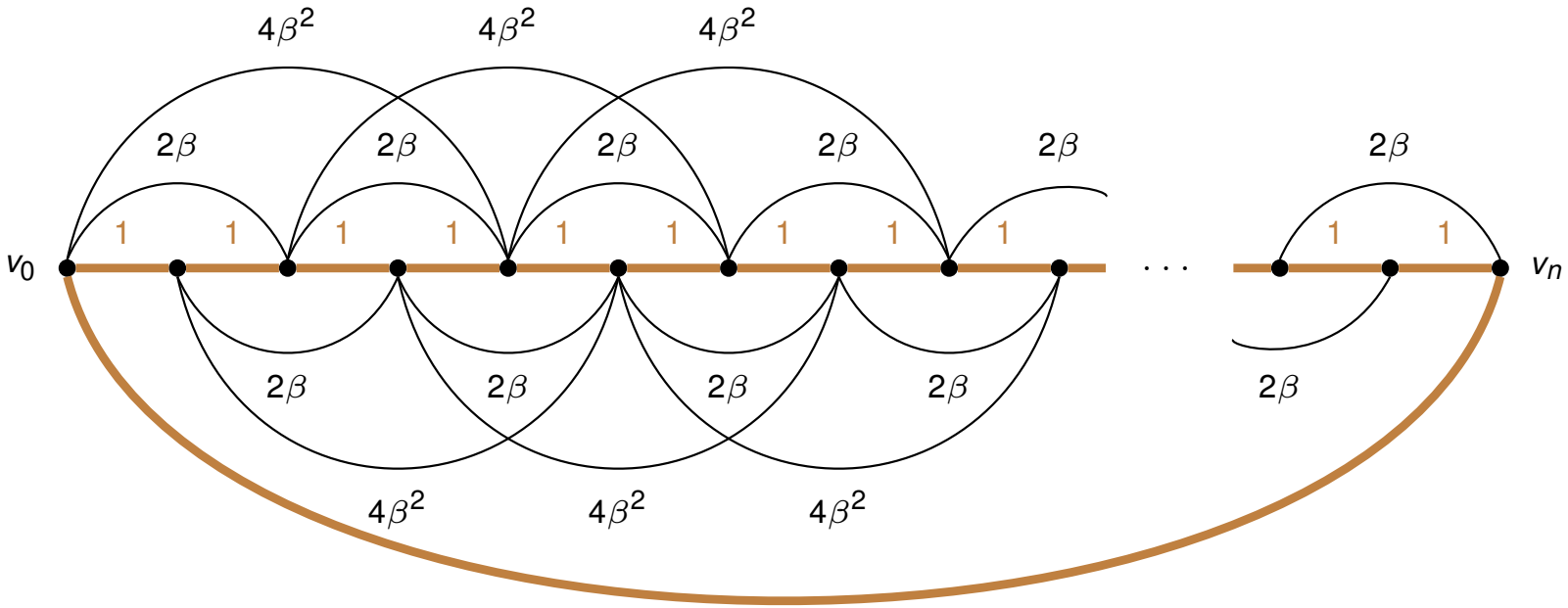


$n \cdot \beta^{\log n}$

1. Minimum spanning tree $T$. 2. Minimum perfect matching $M$.
3. Eulerian cycle $E$.

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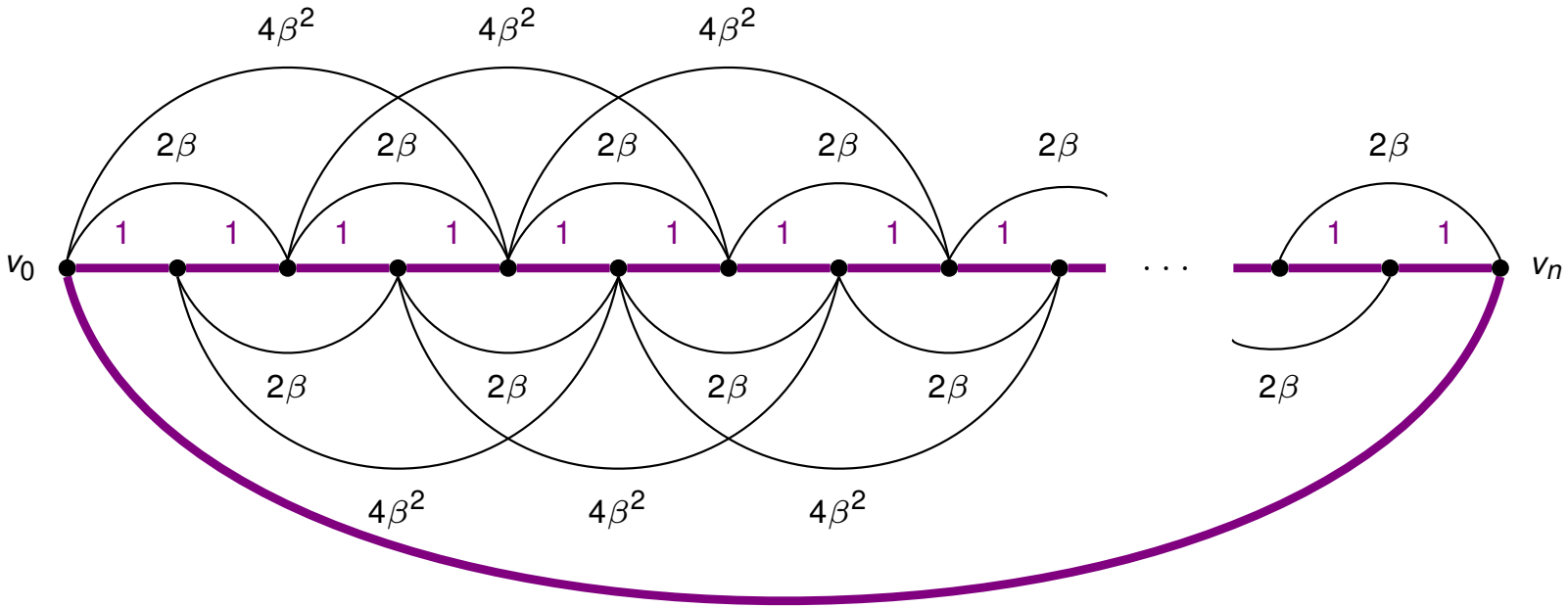


$n \cdot \beta^{\log n}$

1. Minimum spanning tree $T$. 2. Minimum perfect matching $M$.
3. Eulerian cycle $E$. 4. Hamiltonian cycle $H$.

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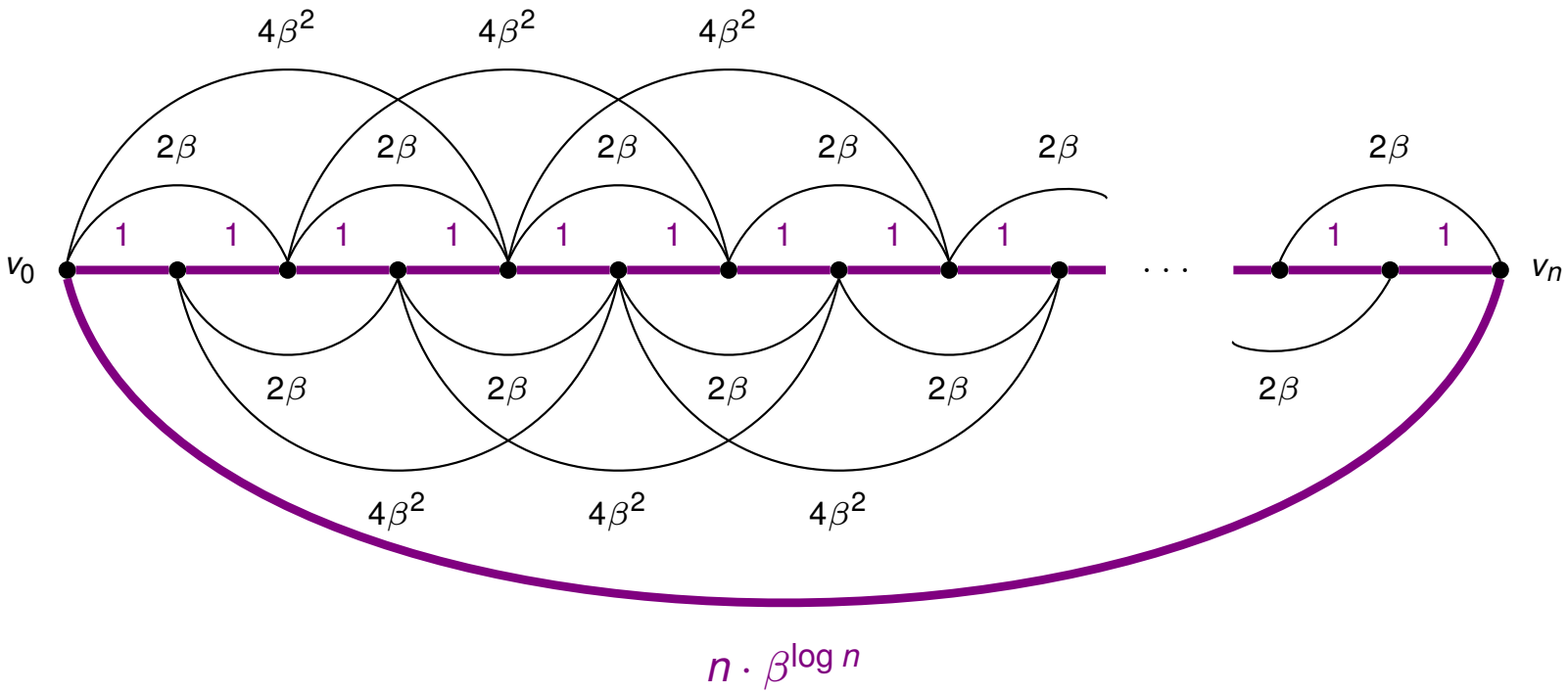


Overall cost of $H$: $n + n \cdot \beta^{\log n} = n + n \cdot n^{\log \beta}$

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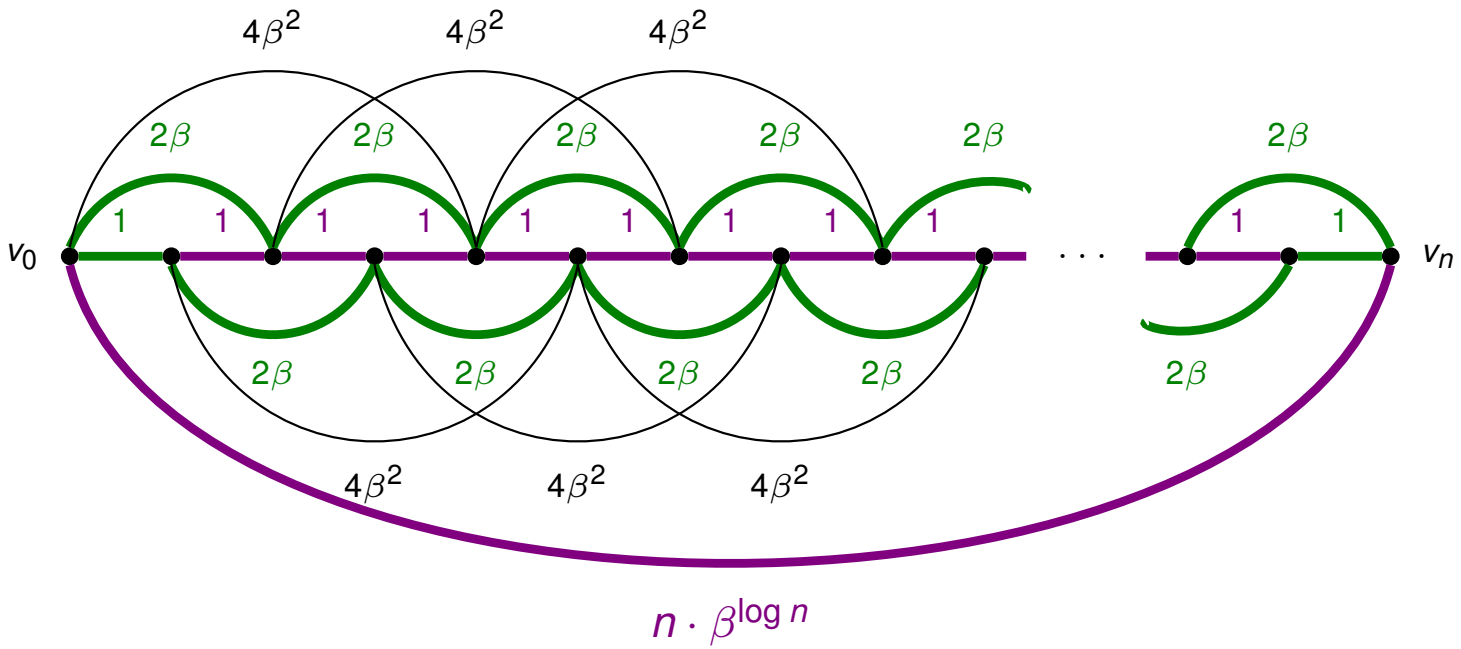


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Overall cost of optimal solution: $(n-1) \cdot 2\beta + 2$

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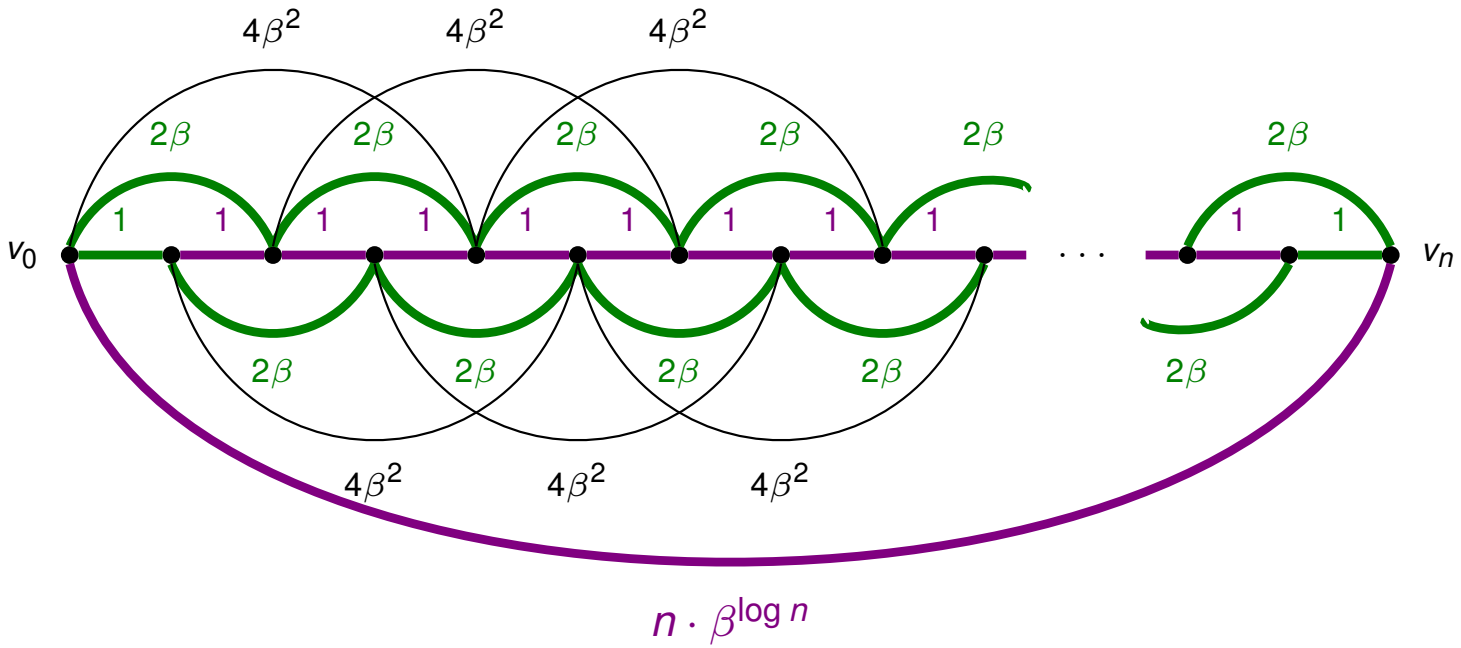


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Overall cost of optimal solution: $(n-1) \cdot 2\beta + 2$

Approximation ratio: $\frac{n+n \cdot n^{\log \beta}}{(n-1) \cdot 2\beta + 2} \geq n^{\log \beta}/2\beta$

# Motivation

- PMCA is a stable generalization of Christofides algorithm for $\beta \geq 1$ (approximation ratio of $\frac{3}{2}\beta^2$)
- PMCA as good as Christofides algorithm for $\Delta$-TSP
- Worst-case example: lower bound, insight into how to improve the algorithm
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# Basic Idea of the PMCA

Take Christofides algorithm, but compute a *path matching* instead of a normal matching in step 2.

*Path matching:* Two vertices are not necessarily matched by an edge, but by a path.

# First Approach

So can we just do that?

**Input:** A complete, weighted, $\beta$-metric graph.

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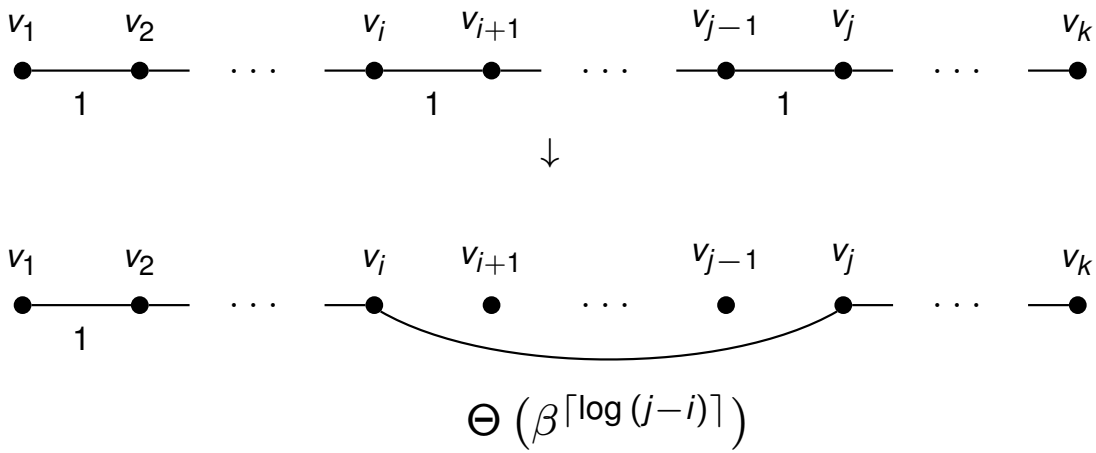
Yes, but . . .

# Two Problems

1. much more conflicts:
   - MST vs. MST
   - MST vs. path matching
   - path matching vs. path matching
2. cannot just bypass vertices arbitrarily



# The Path Matching Christofides Algorithm

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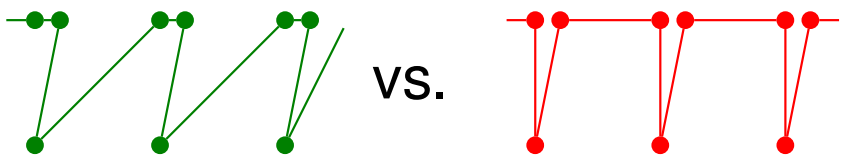
# Goal

**Theorem**

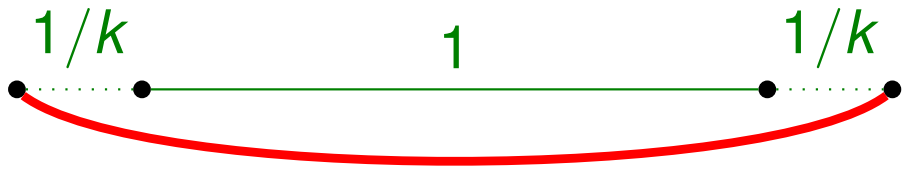
*The upper bound of $\frac{3}{2}\beta^2$ on the approximation ratio of the PMCA is tight.*

# Two Tricks

- $\frac{3}{2}$:

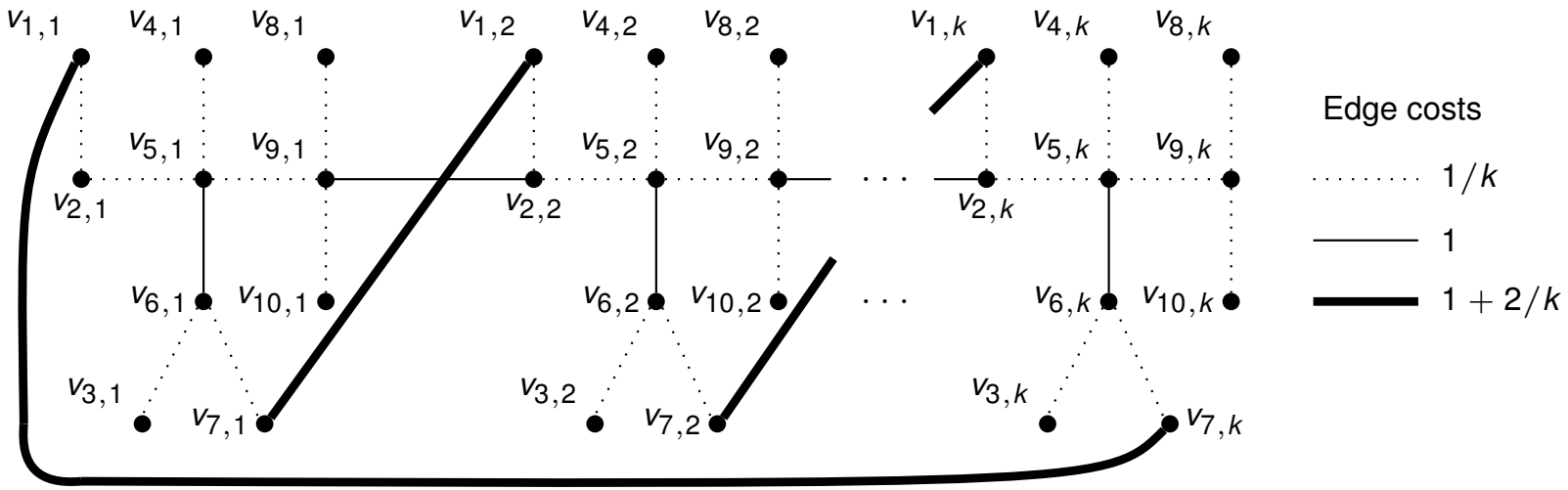



- $\beta^2$:




$$\beta \cdot (\beta \cdot (1 + 1/k) + 1/k) \geq \beta^2$$

# The Graph


$v_{1,1}$
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$v_{7,k}$
Edge costs
$1/k$
1
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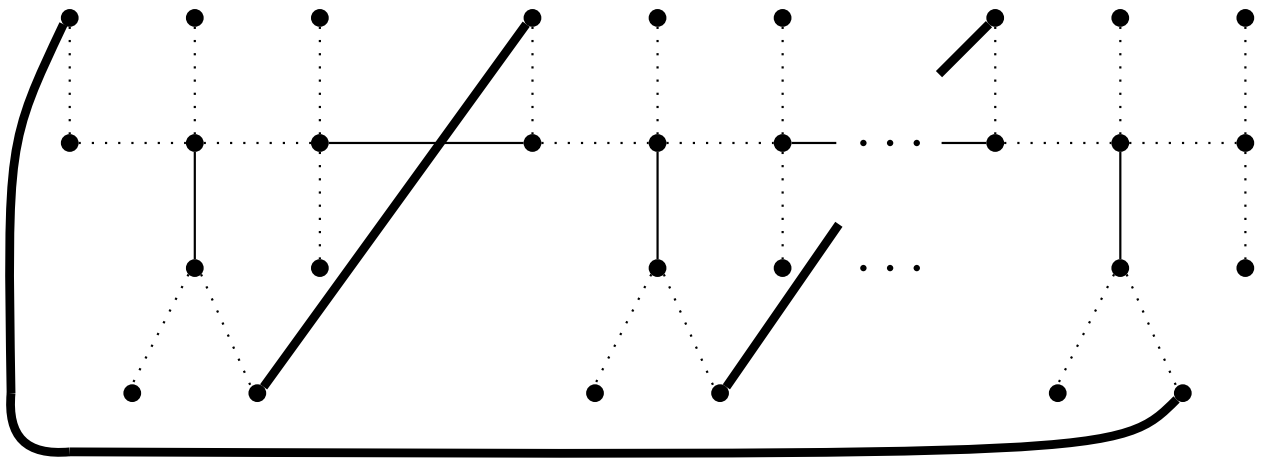
# Step 1: Minimum Spanning Tree $T$



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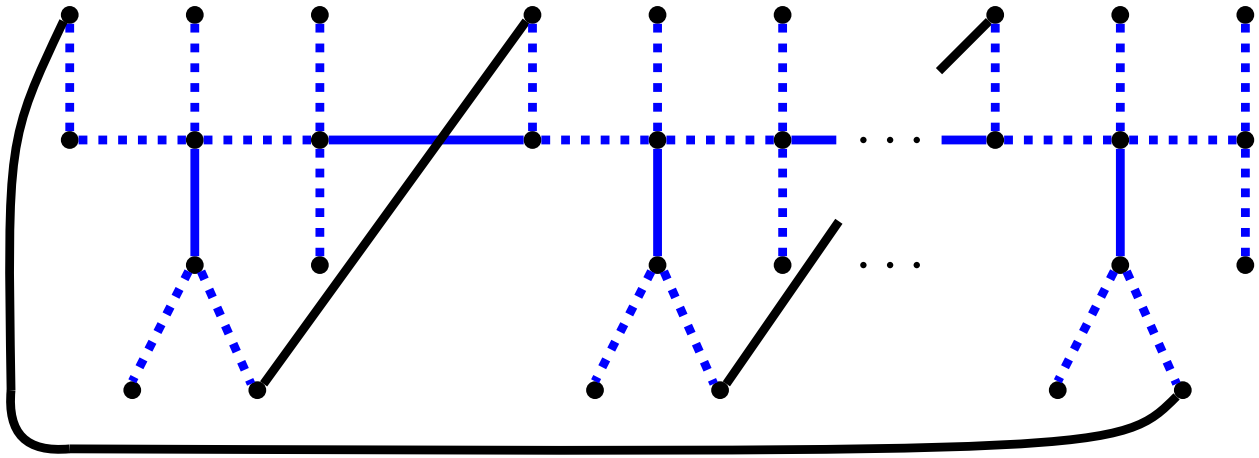

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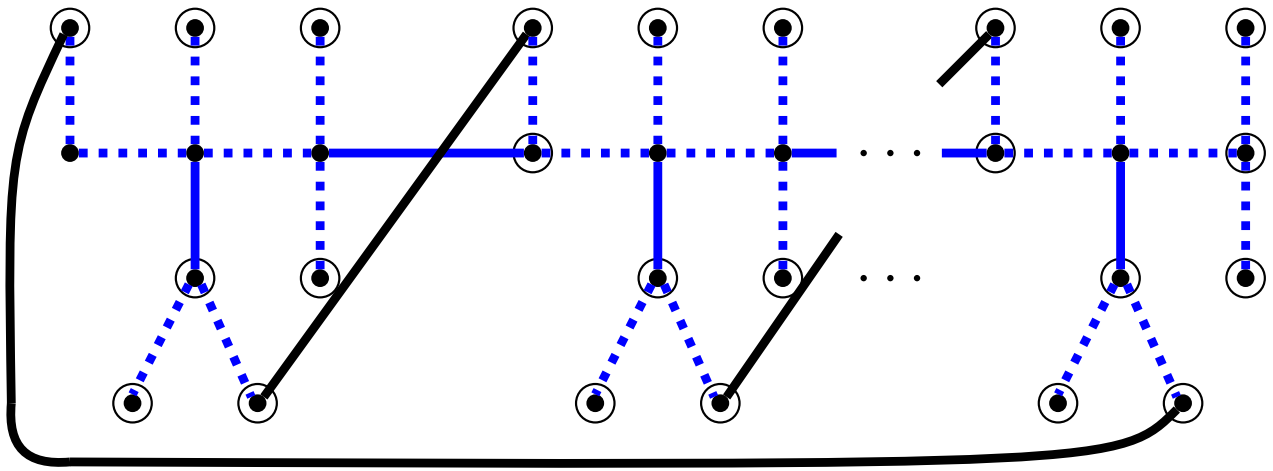
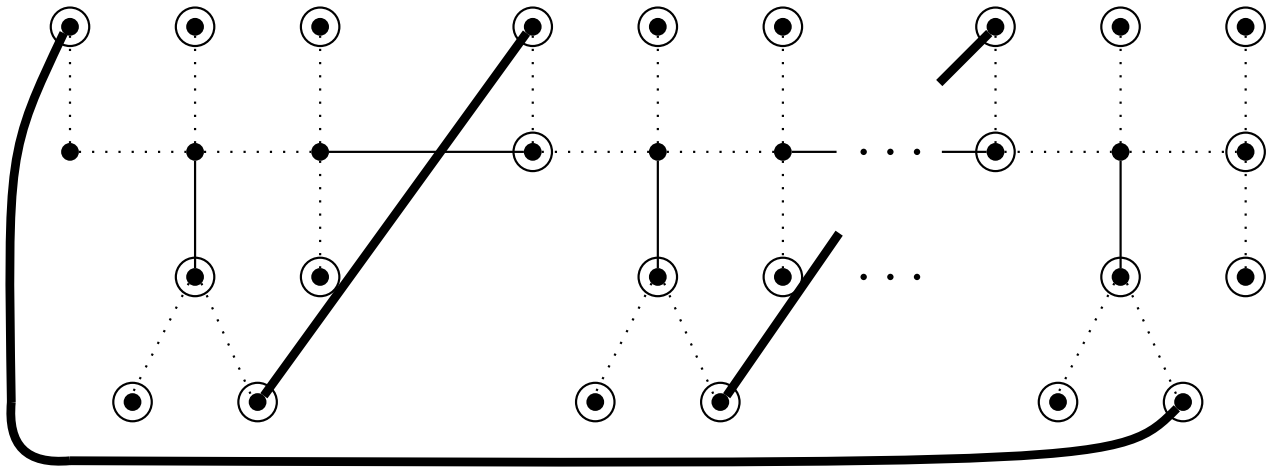


$\odot$ $=$ $U$ (set of odd-degree vertices)

# Step 2: Minimum Path Matching $M$ for $U$



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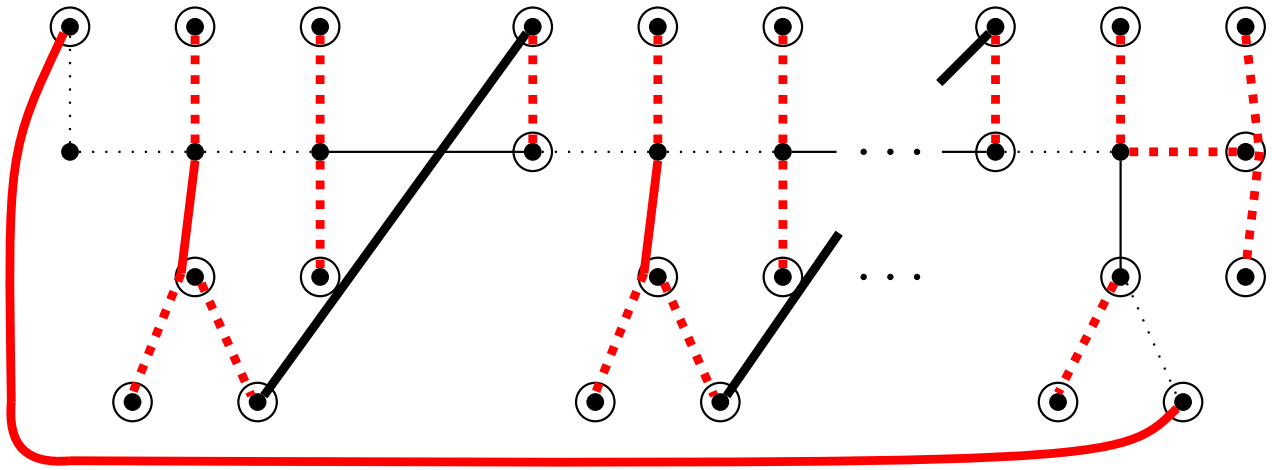
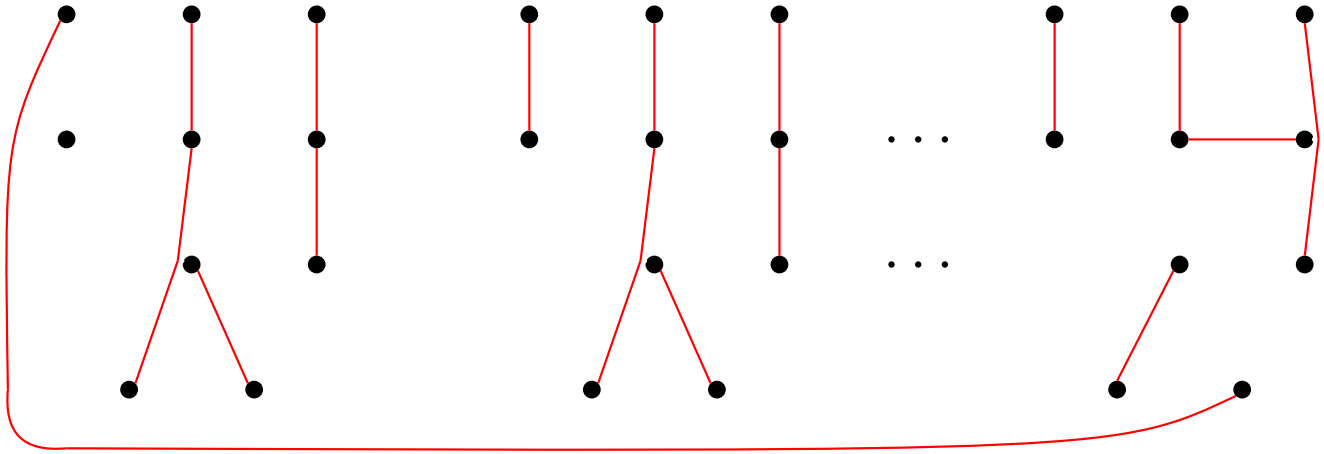


# Step 3: Conflict Resolution in $M$



**Part I: Finding a path with only one conflict**

1. Start with an arbitrary path $p$ with at least one conflict.
2. If $p$ has more than one conflict, let $v$, $w$ be the first and last.
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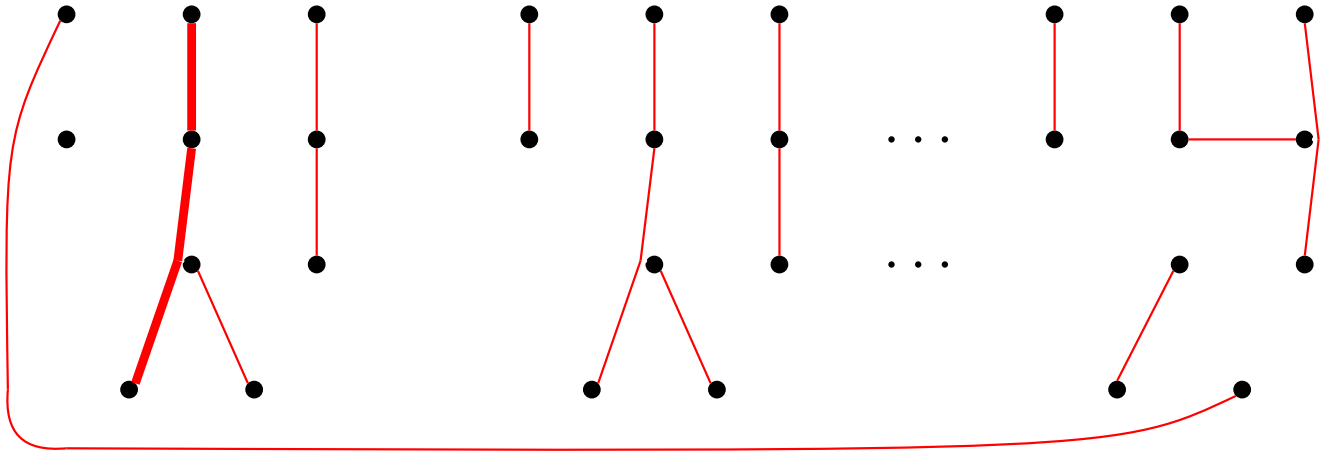

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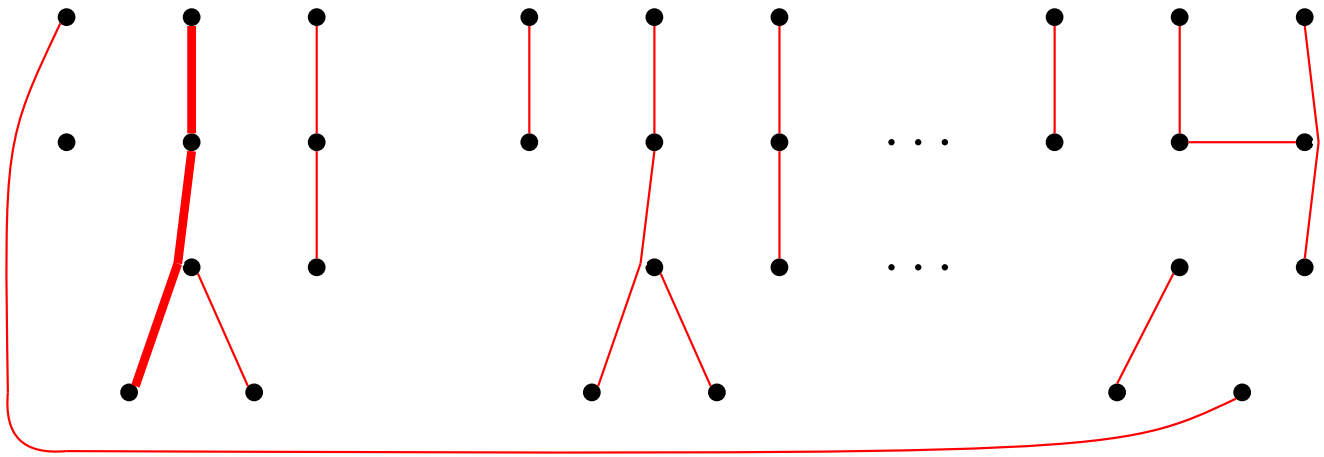

**Part II: Resolving a conflict**

Situation: We have a path $p$ with only one conflict $v$

- If $v$ is internal to $p$, bypass $v$ in $p$.
- If $v$ is an endpoint of $p$, do a certain transformation.

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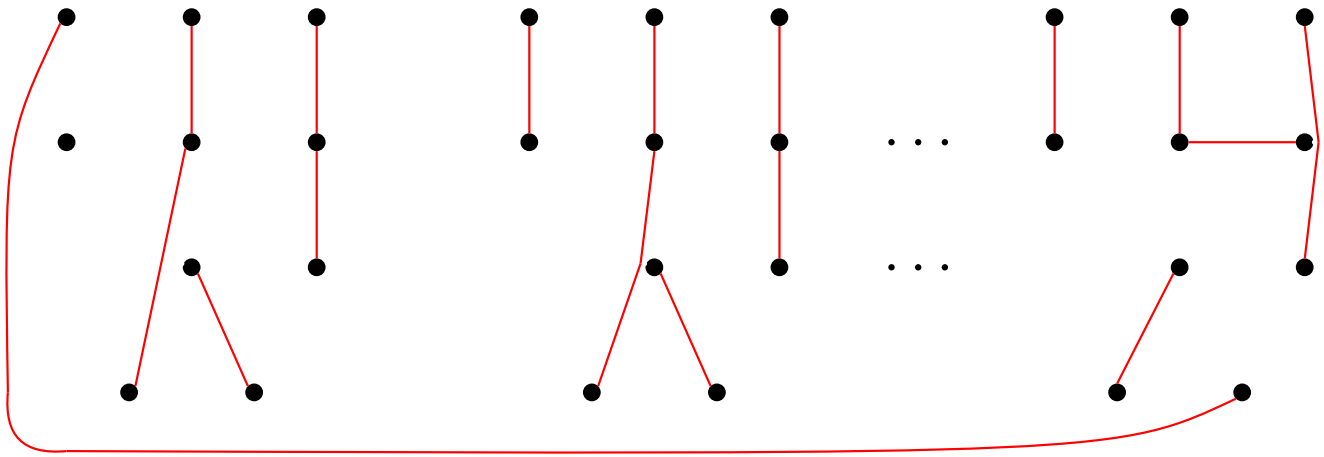

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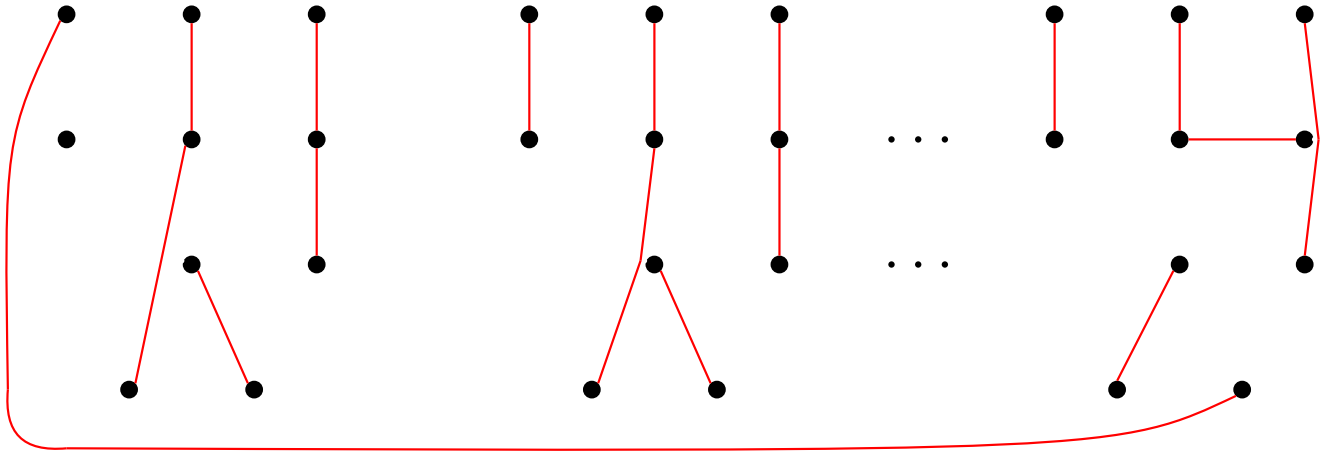
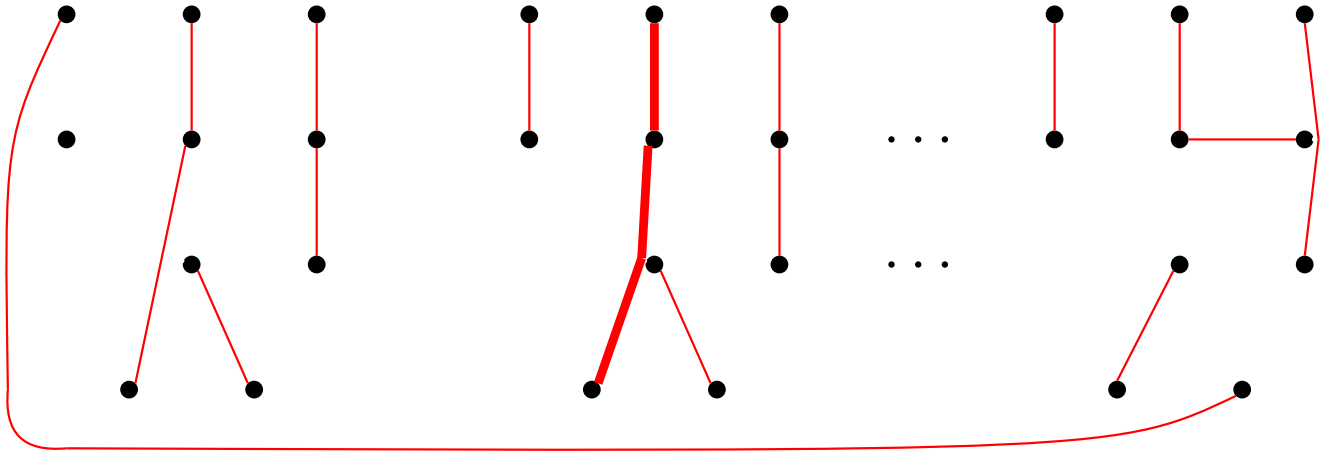


**Part I: Finding a path with only one conflict**

1. Start with an arbitrary path $p$ with at least one conflict.
2. If $p$ has more than one conflict, let $v$, $w$ be the first and last.
3. Pick as new $p$ one of the paths $p_v$, $p_w$ that was not previously picked.

# Step 3: Conflict Resolution in $M$



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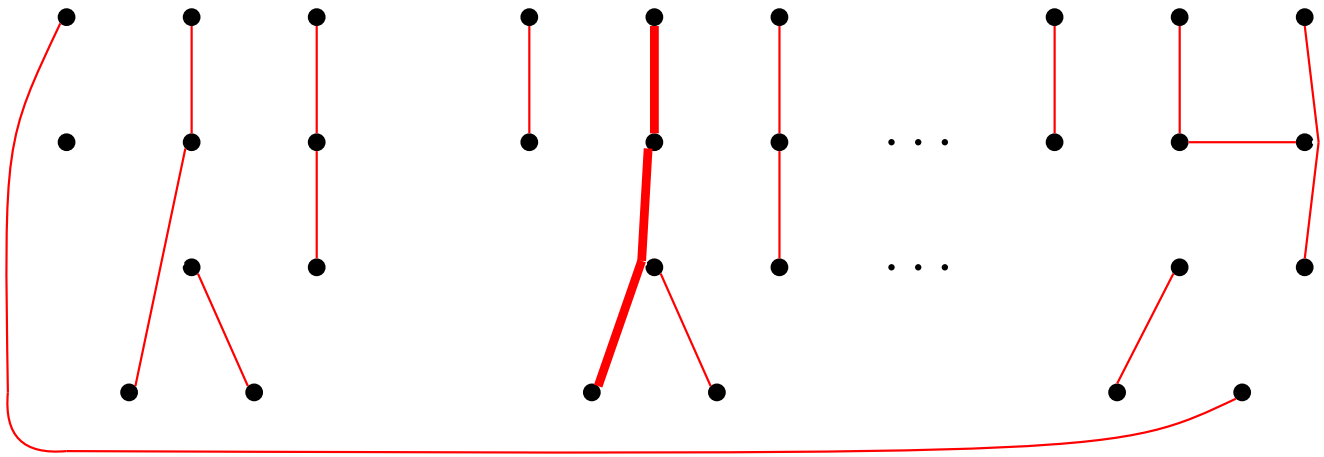

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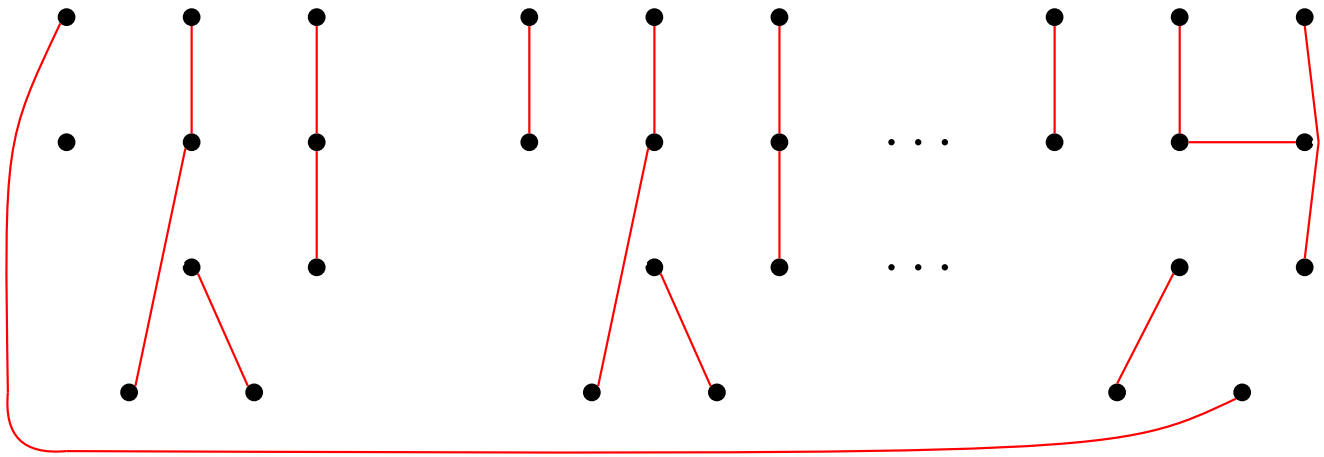

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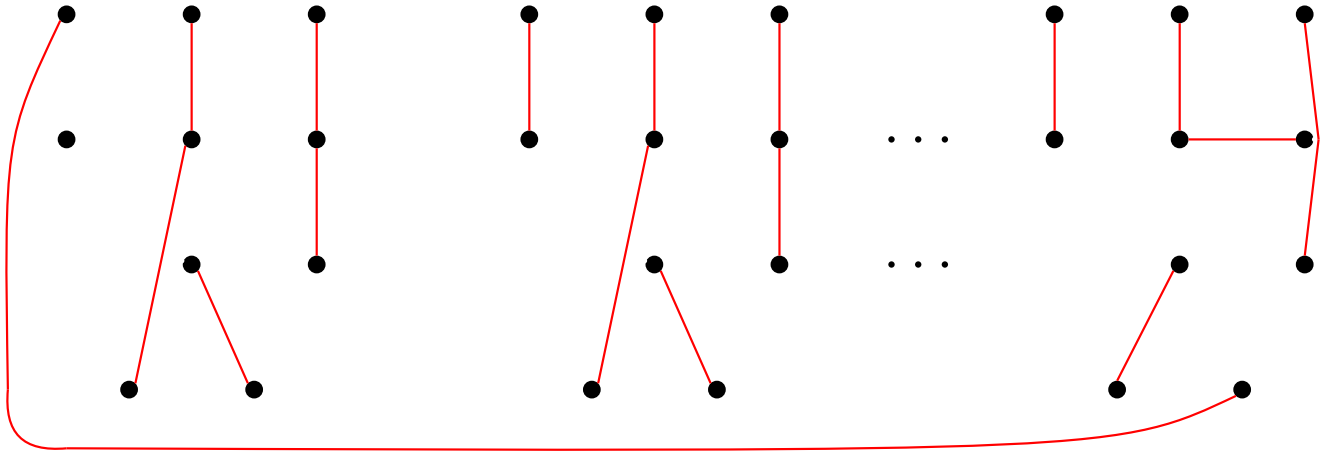

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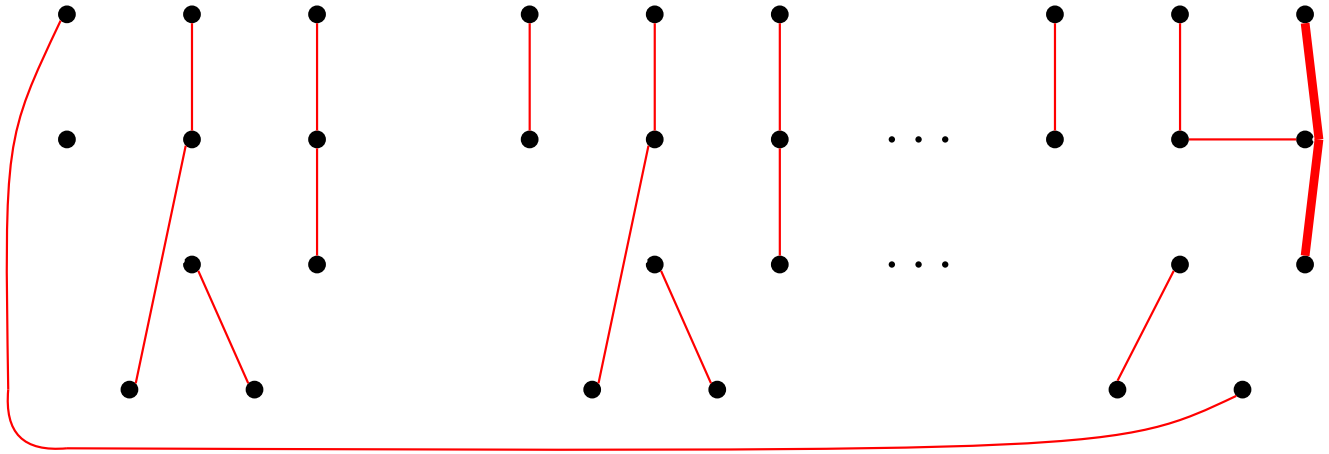

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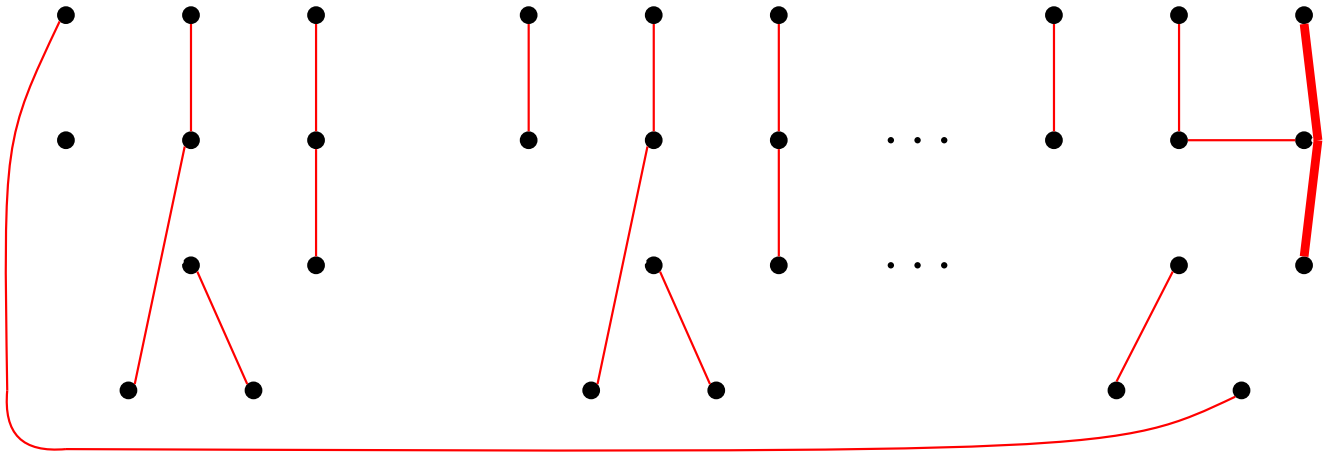

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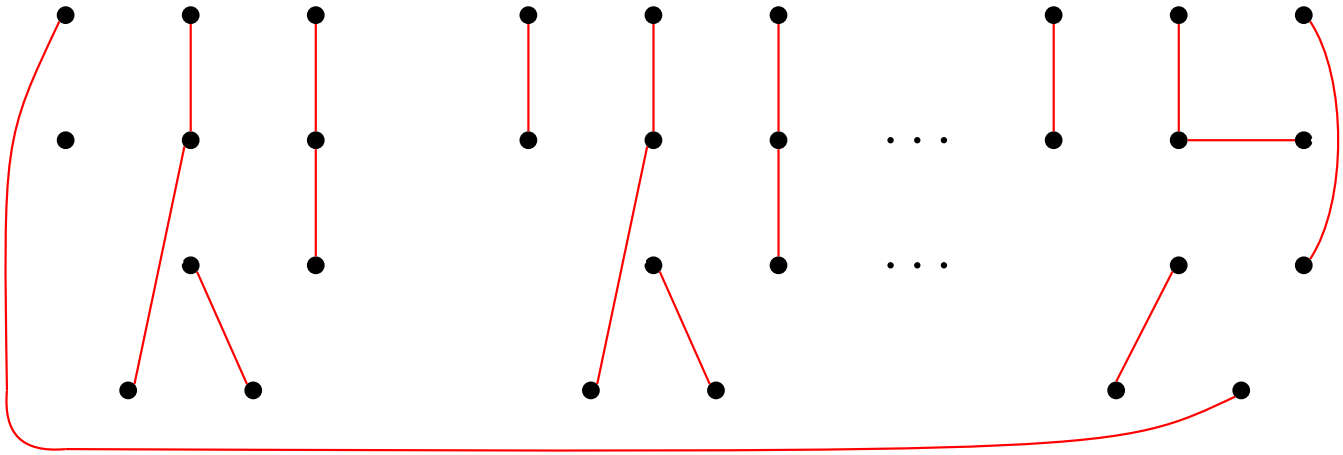

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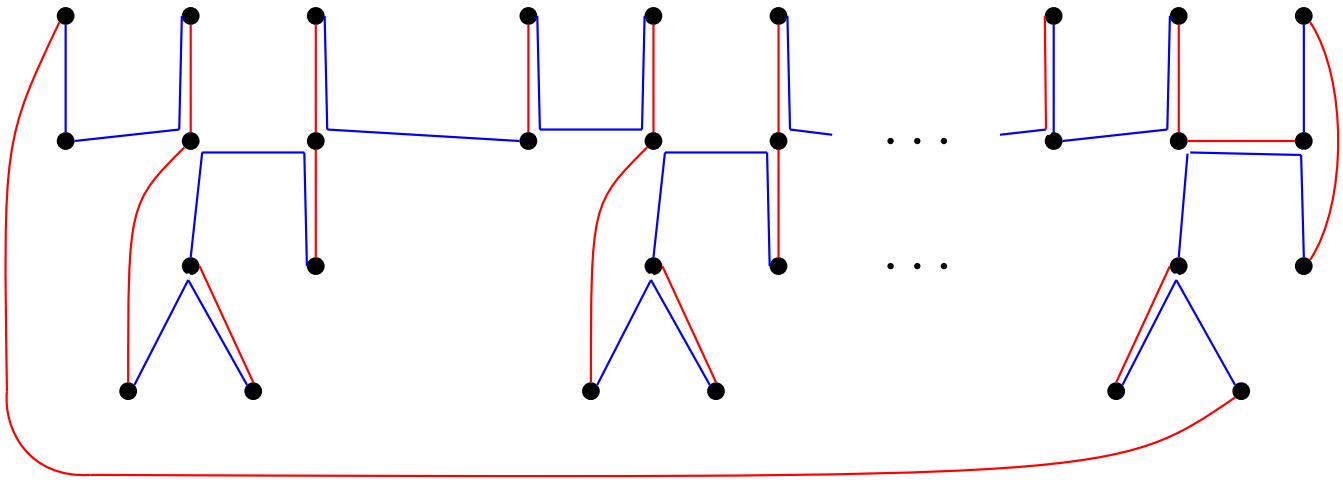
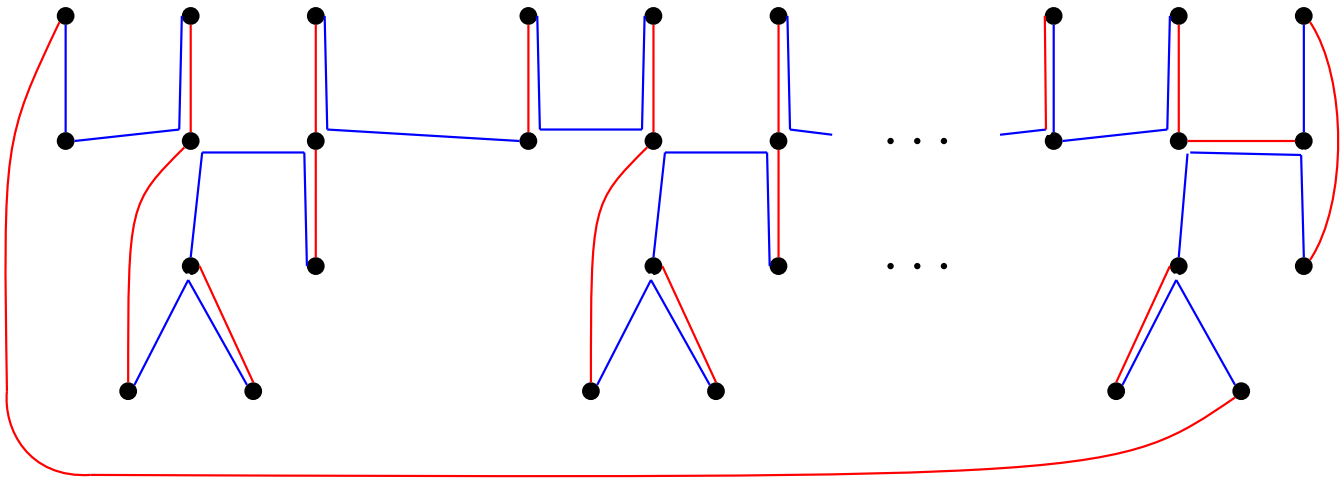
# Step 4: Eulerian Cycle $E$



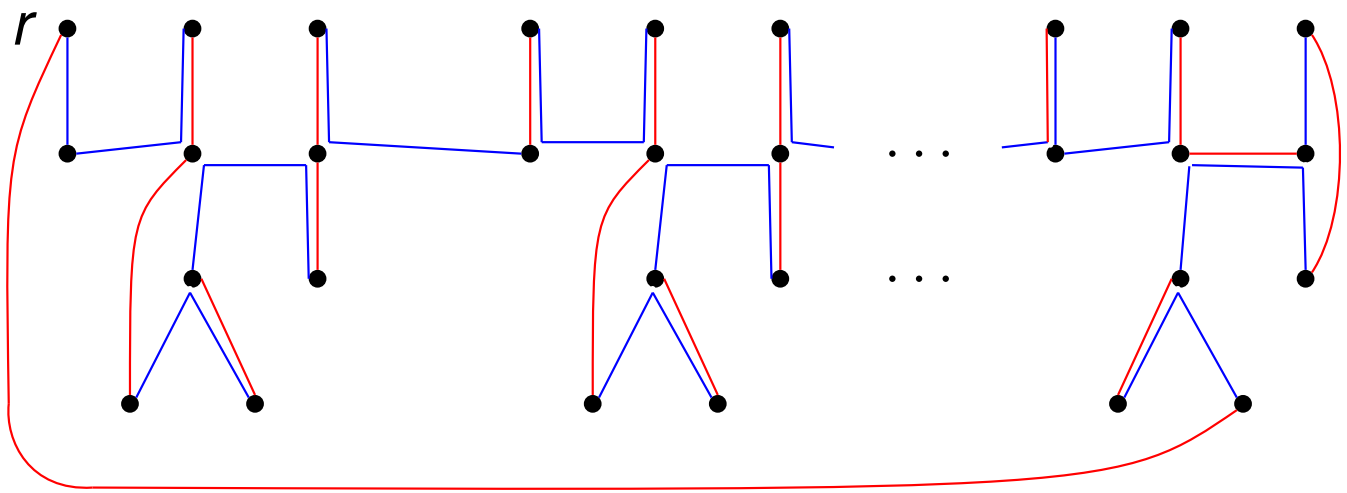
Combine $T$ and $M'$ to get $E$

# Step 5: Conflict Resolution in the Paths of $T$



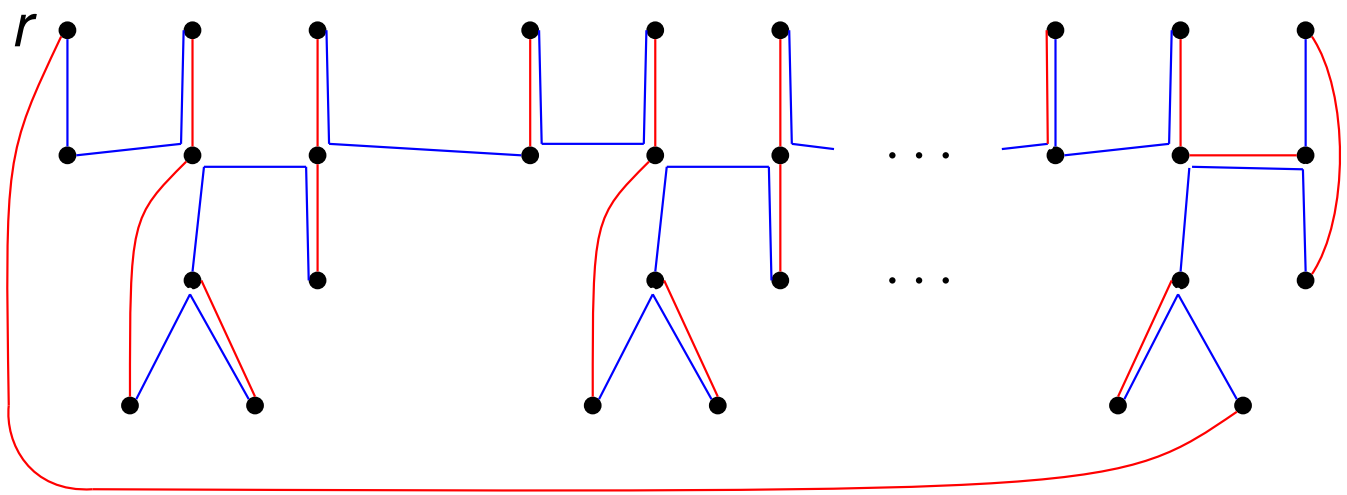
Goal: Every vertex has blue degree $\leq 3$

# Step 5: Conflict Resolution in the Paths of $T$




1. Pick a root vertex $r$ (arbitrarily).

# Step 5: Conflict Resolution in the Paths of $T$




1. Pick a root vertex $r$ (arbitrarily).
2. In every blue path $p$, bypass the vertex $v$ closest to $r$ in $T$ if $v$ is internal to $p$ and if blue-deg$(v) \geq 4$.

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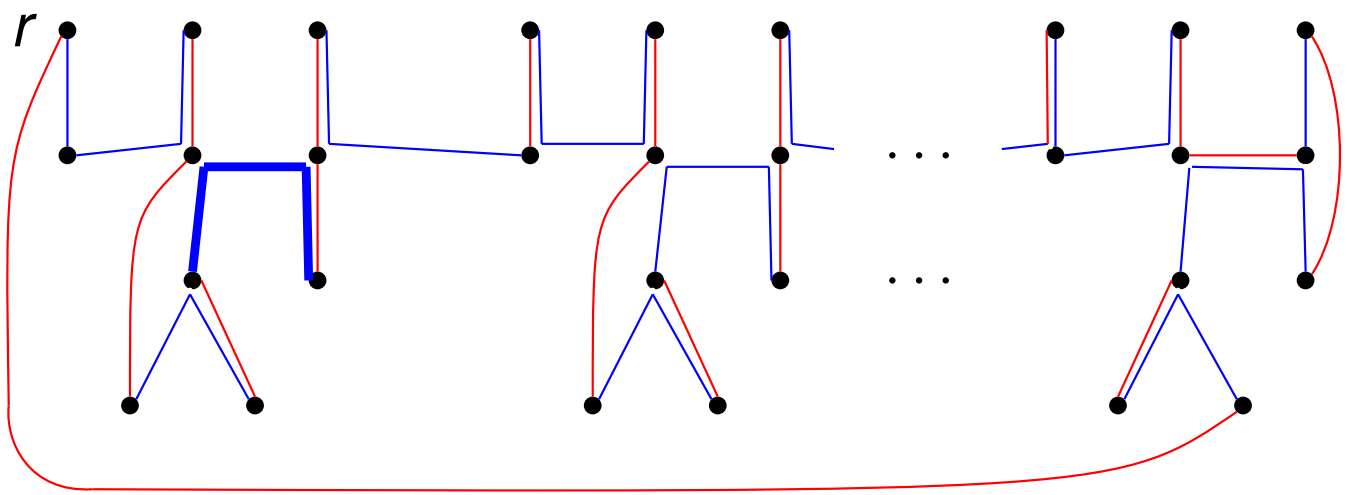


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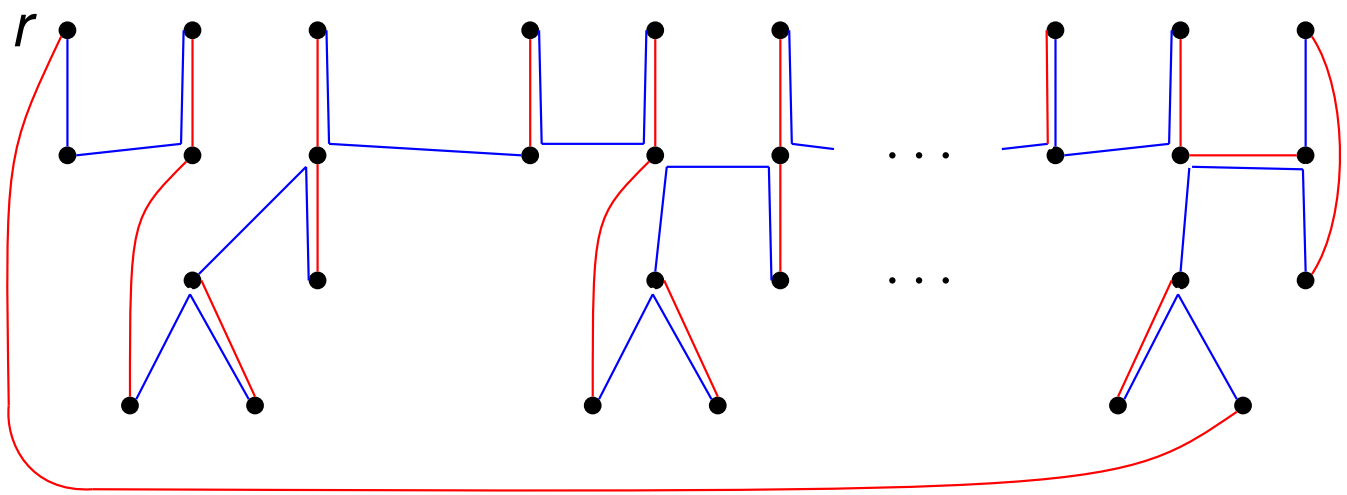


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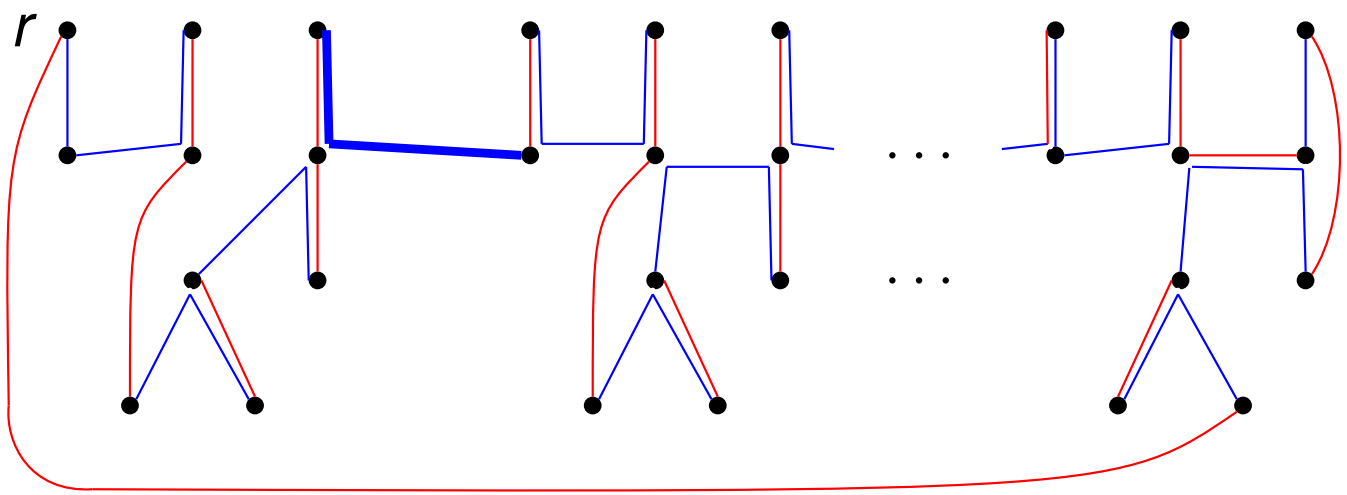


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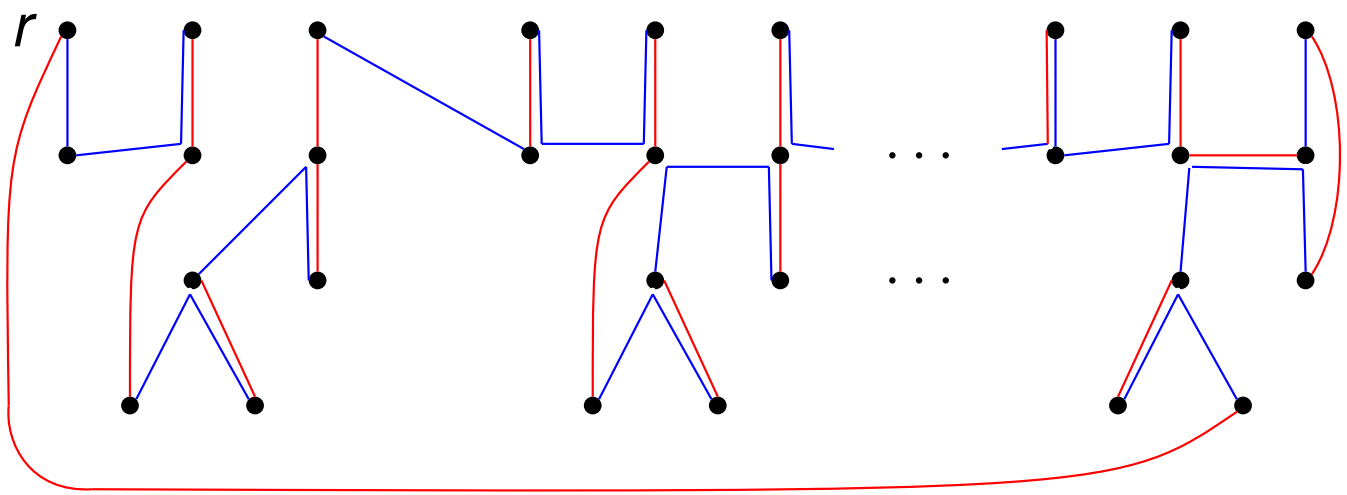


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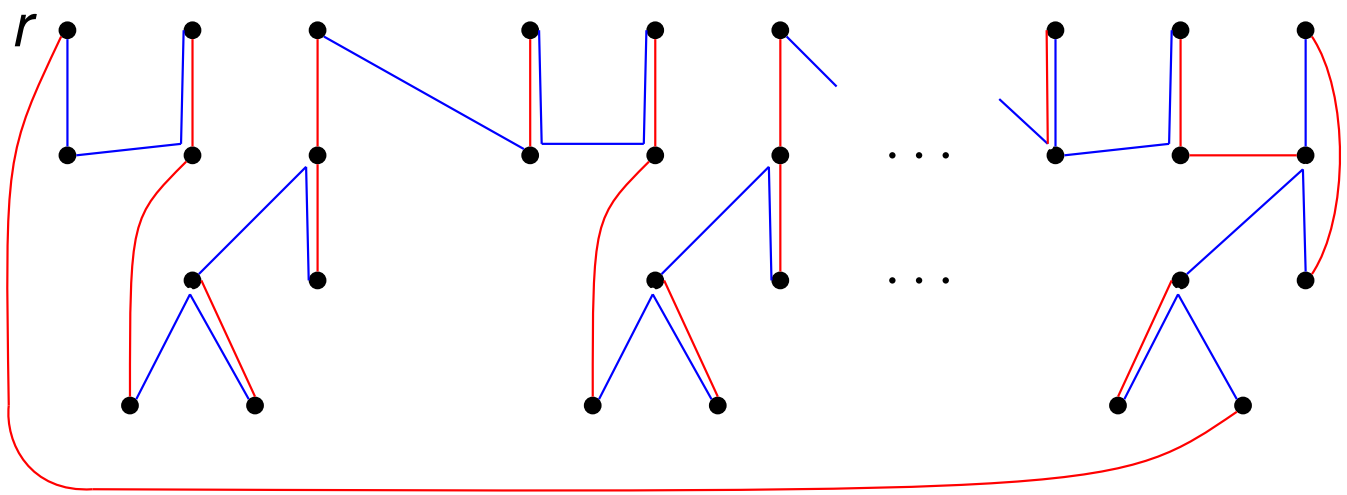


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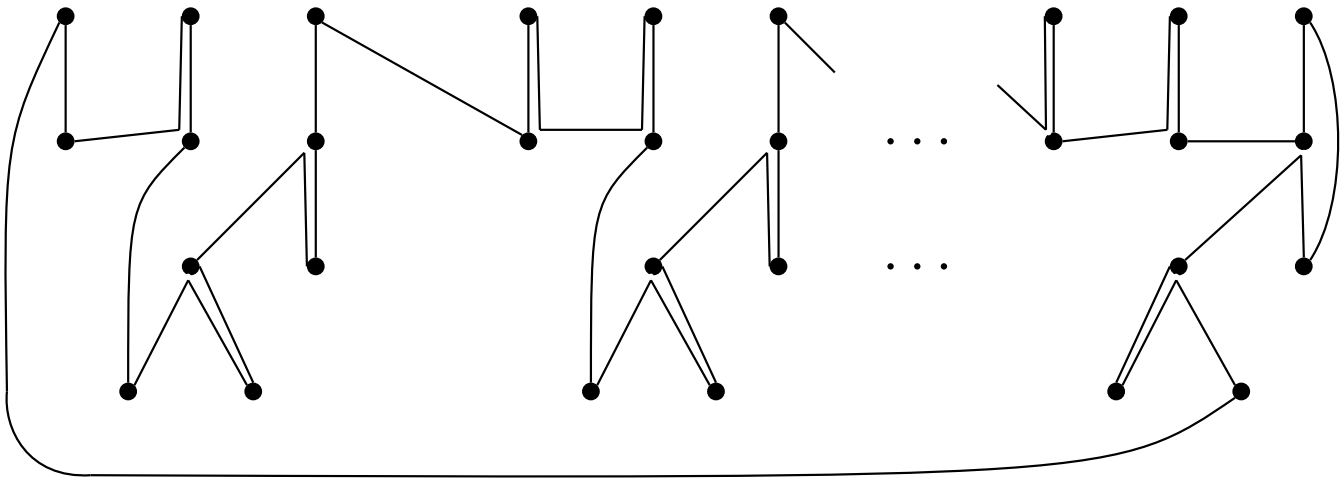
# Step 6: Resolution of Remaining Conflicts



1. Bypass an arbitrary conflict $v$.

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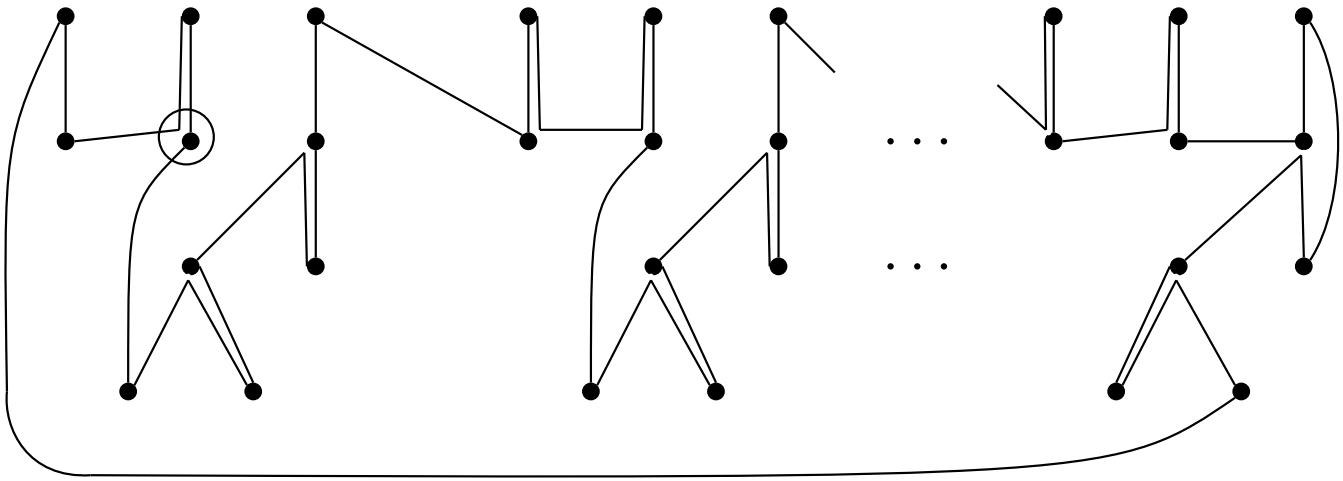

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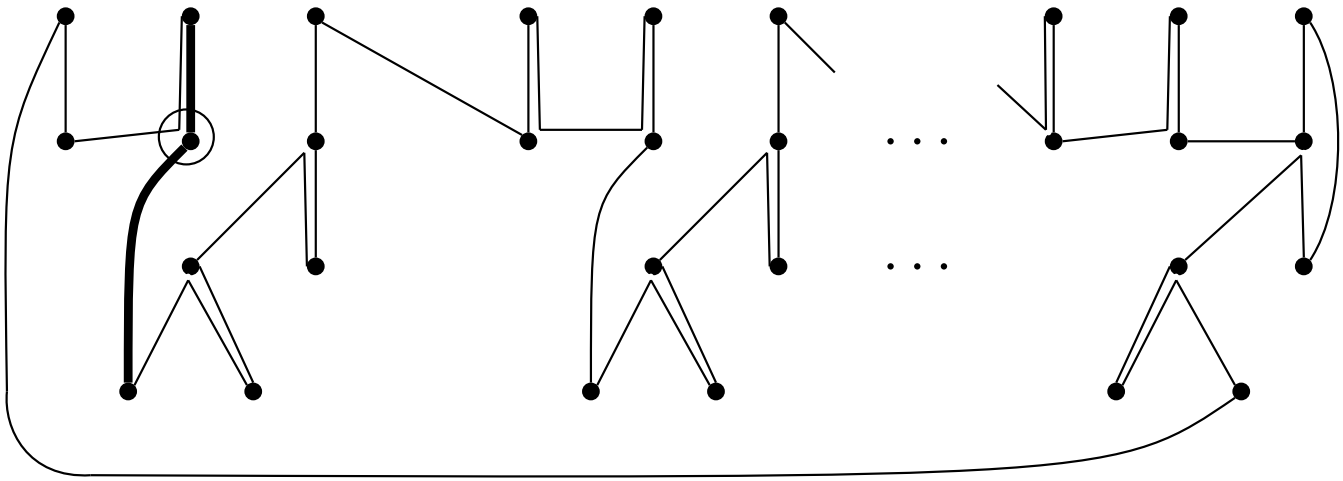

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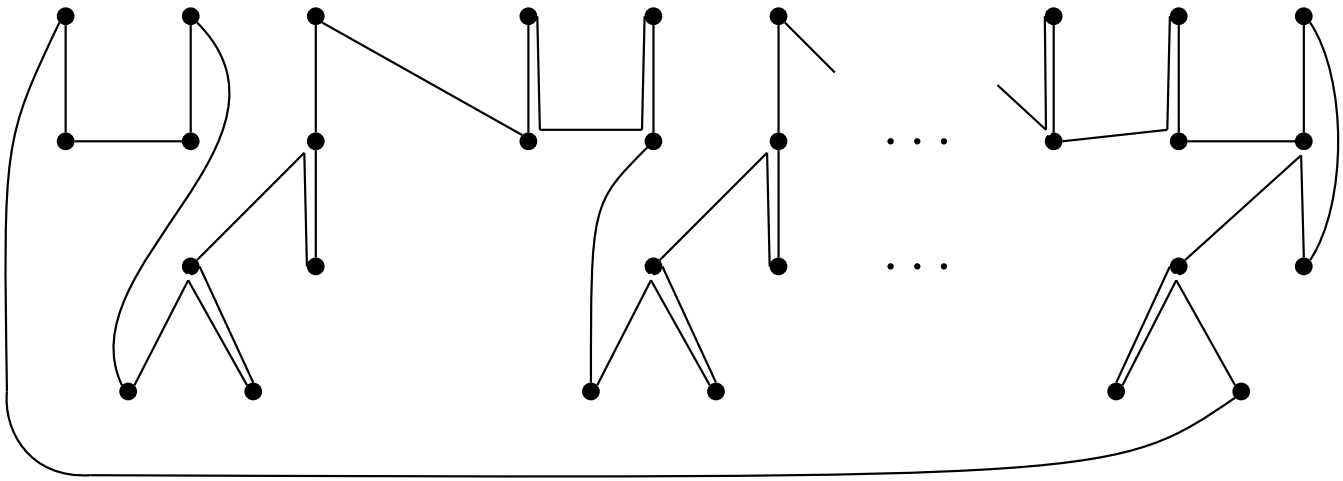

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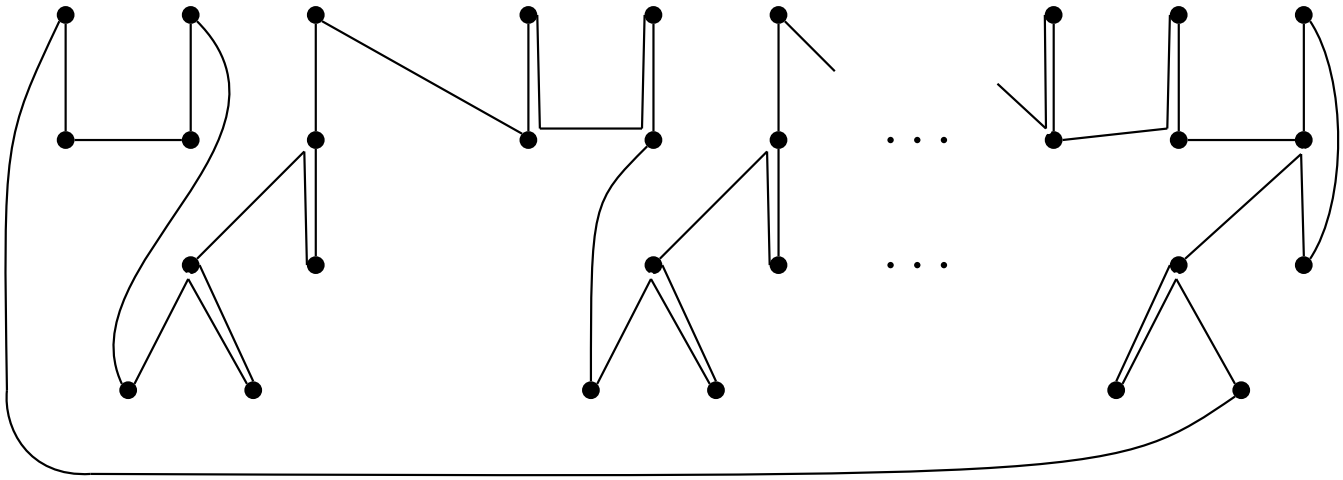

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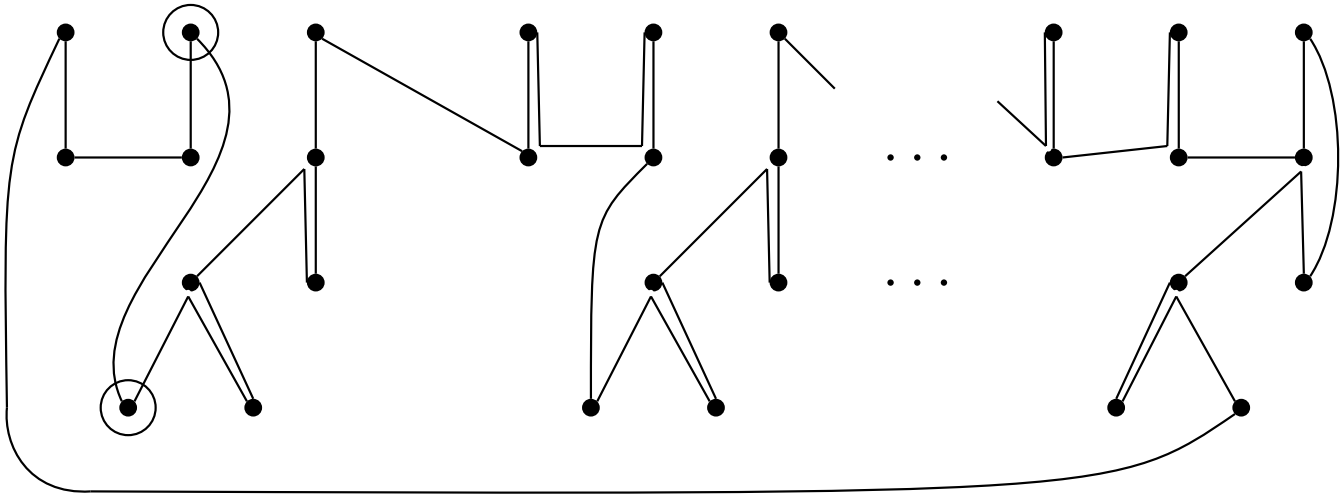

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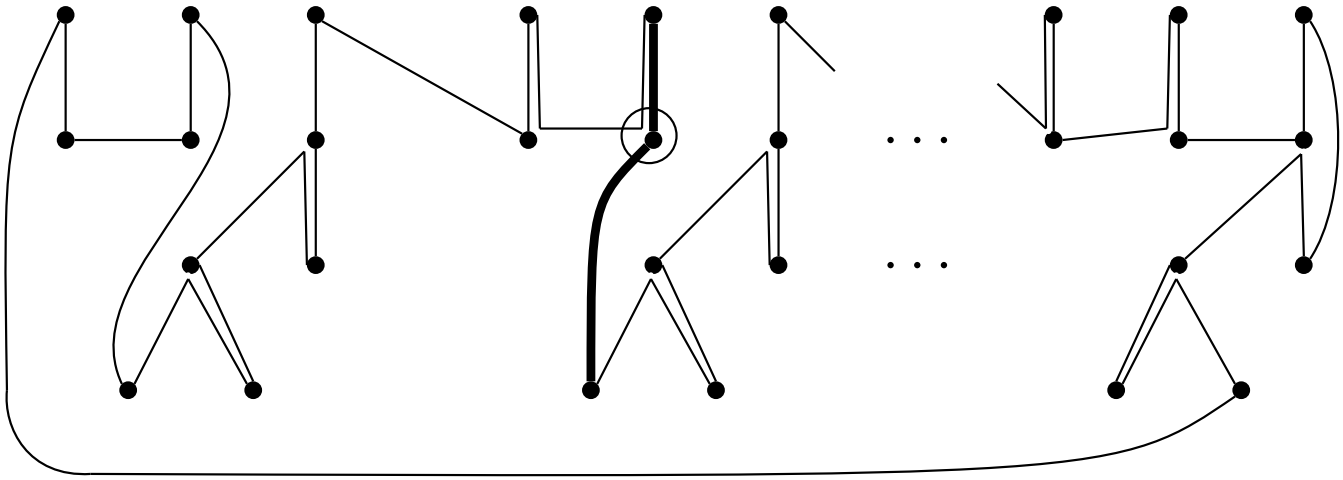

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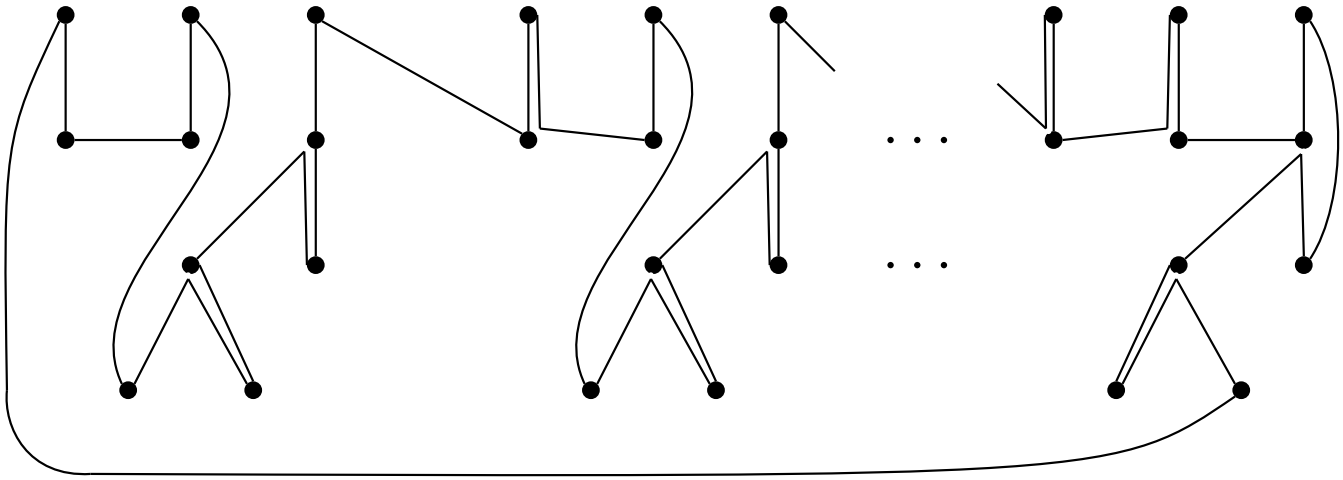

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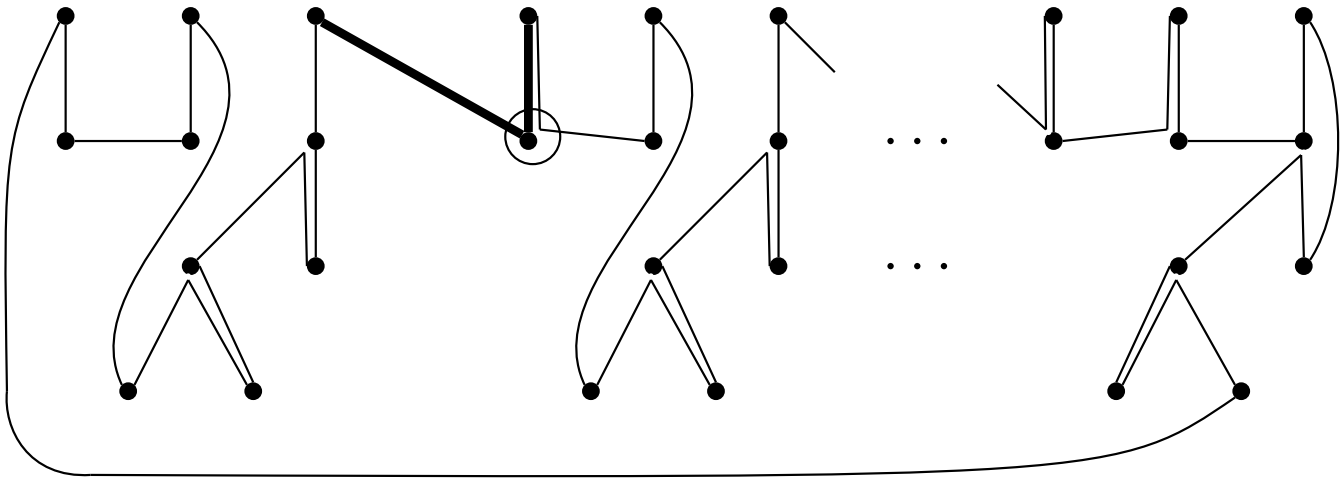

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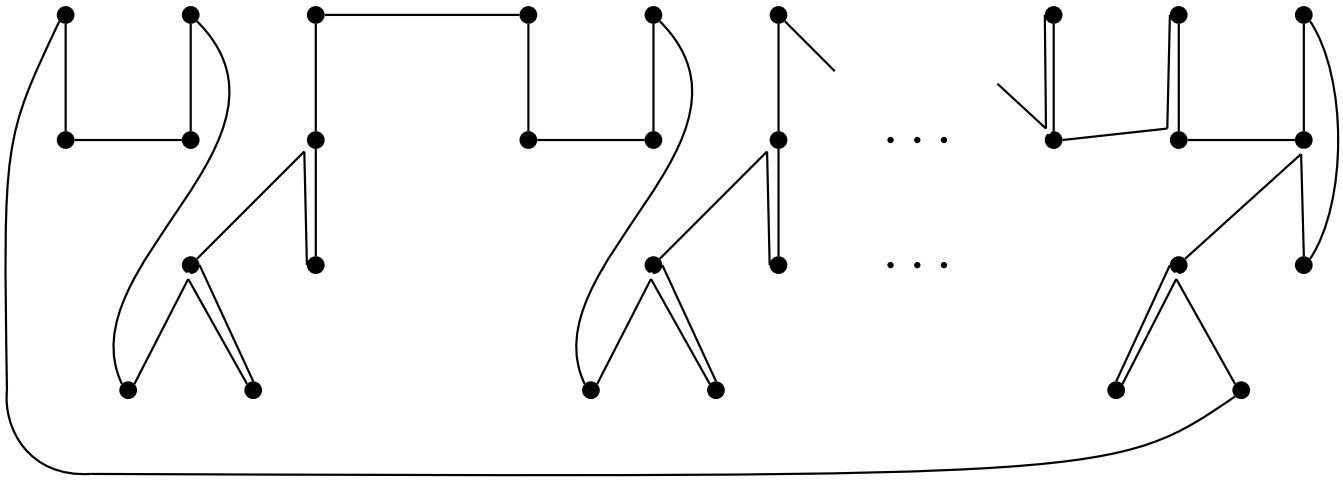

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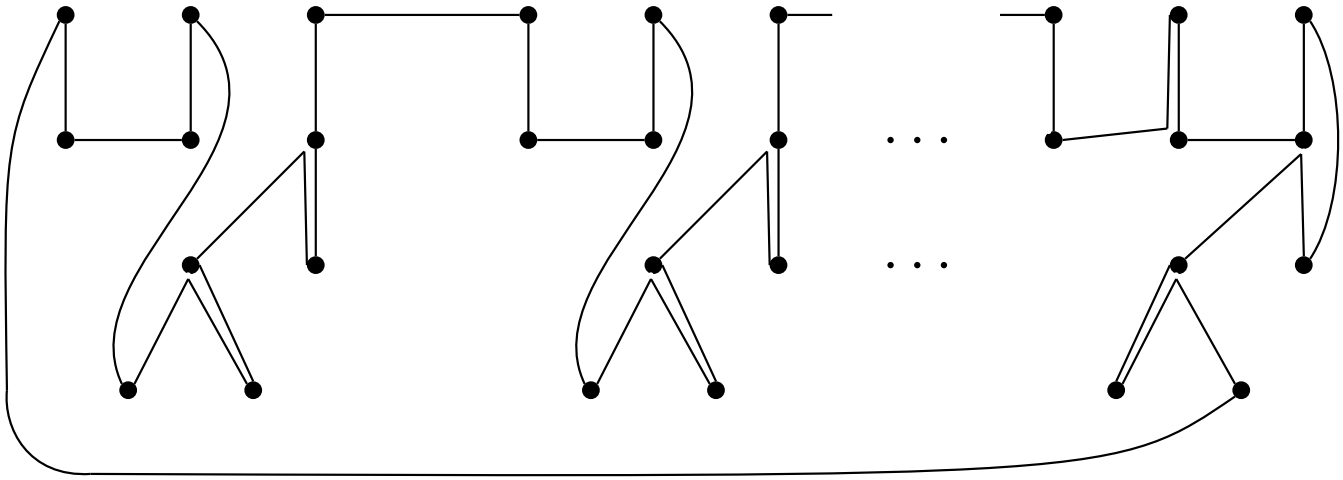

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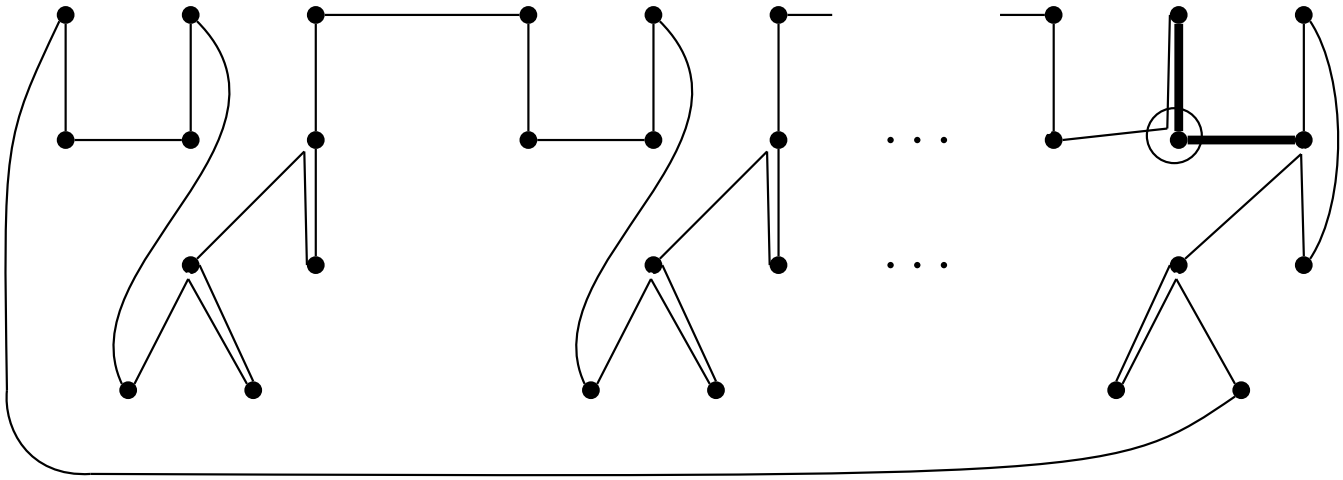

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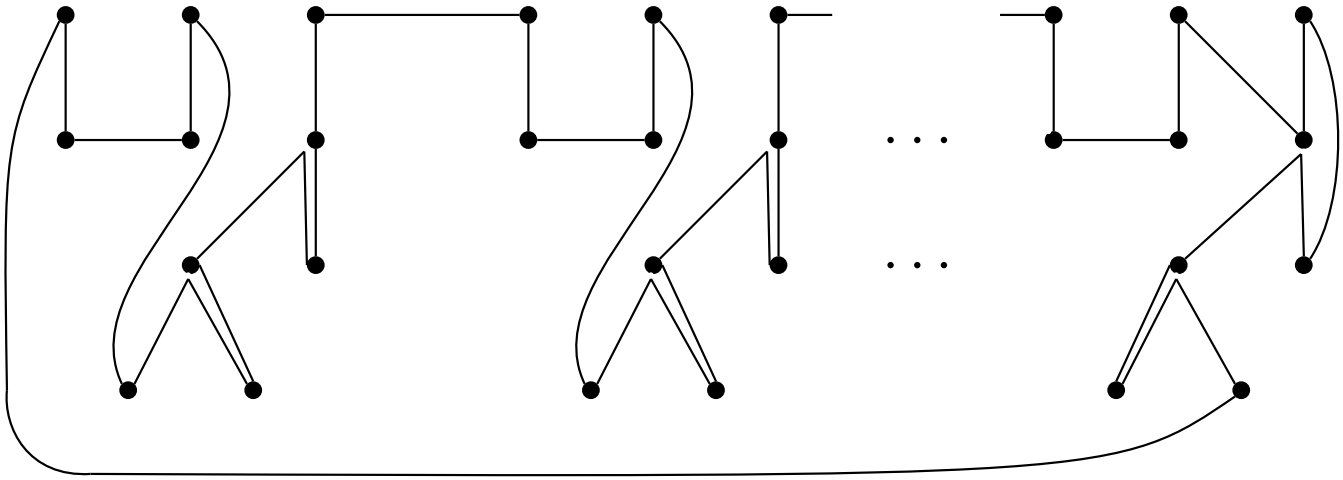

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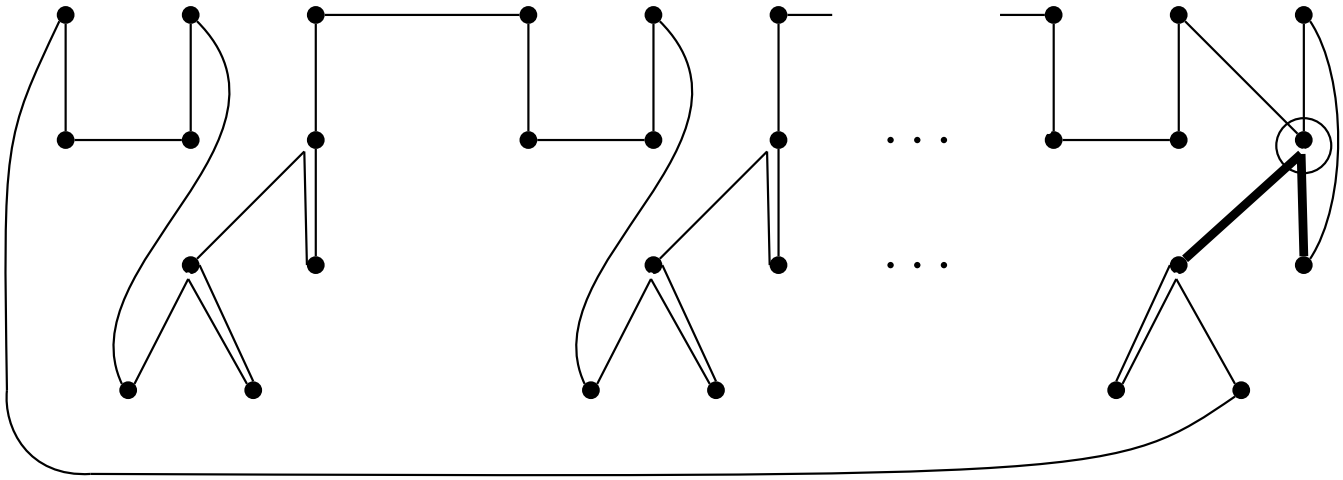

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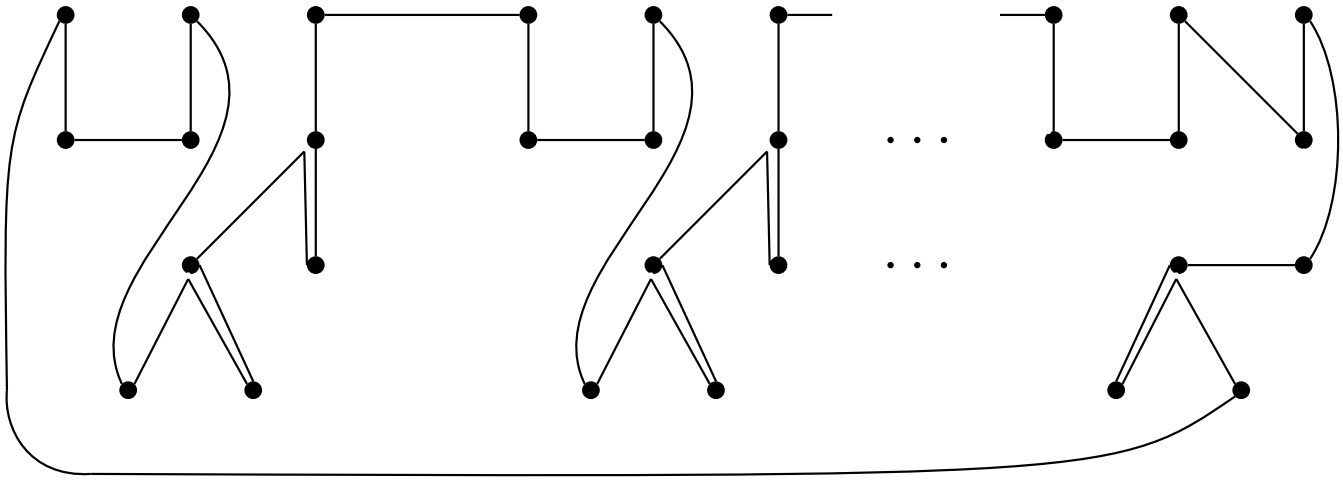

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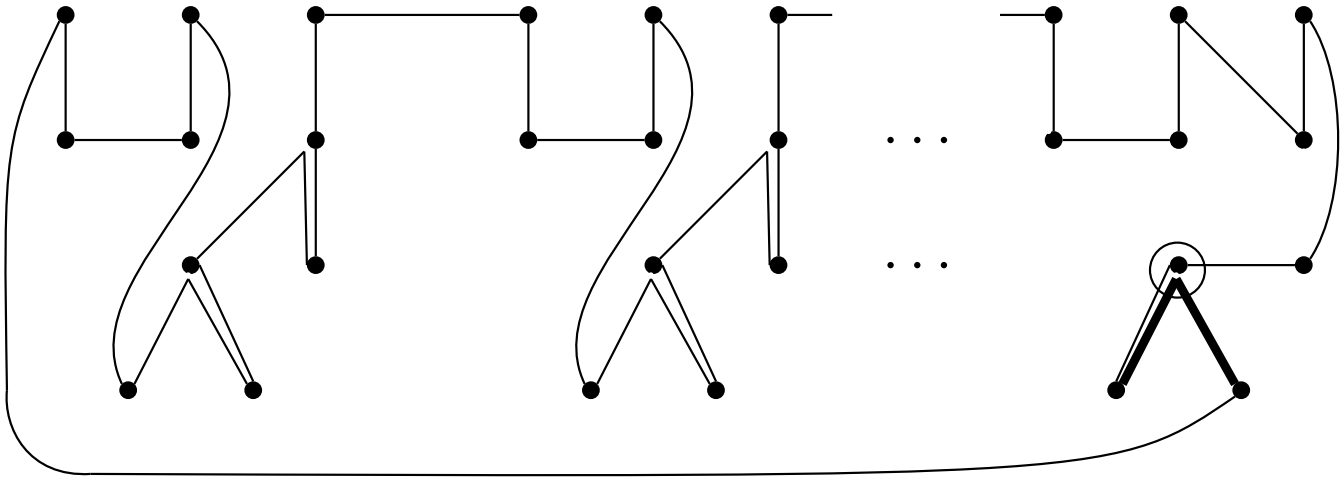

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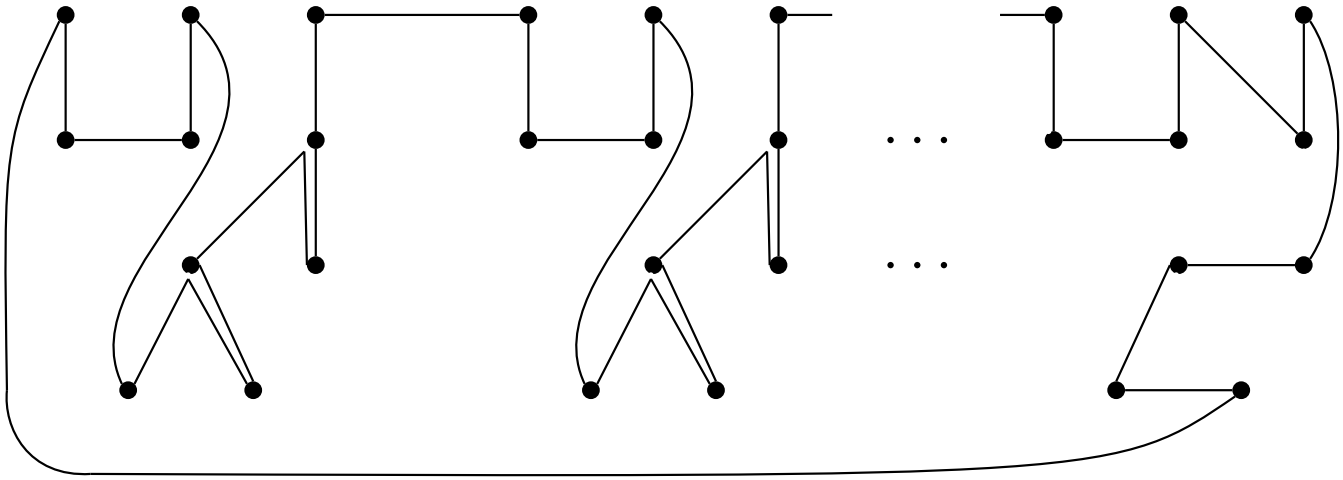

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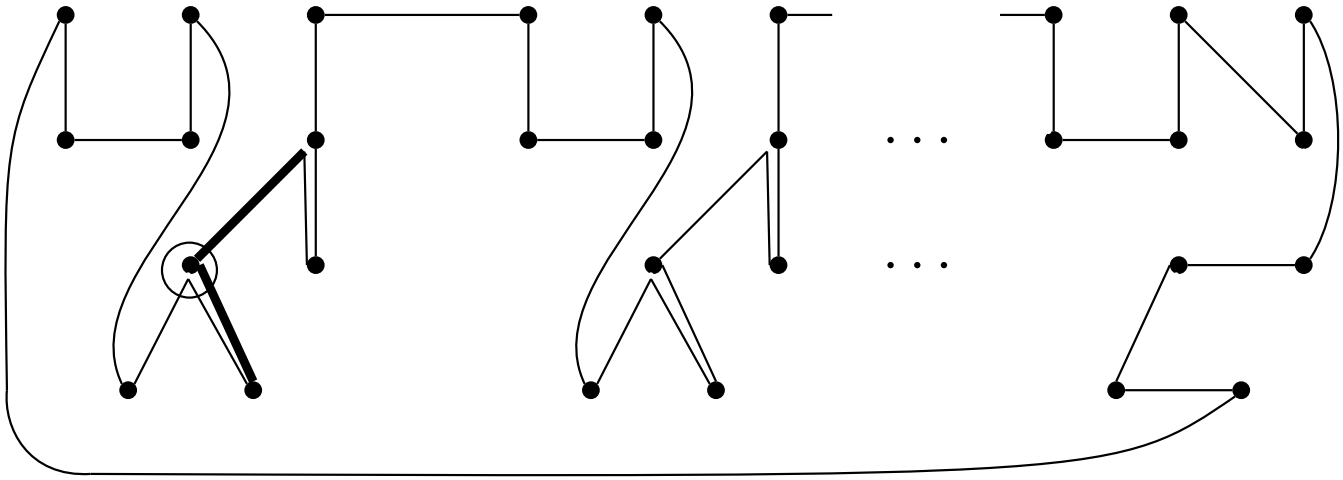

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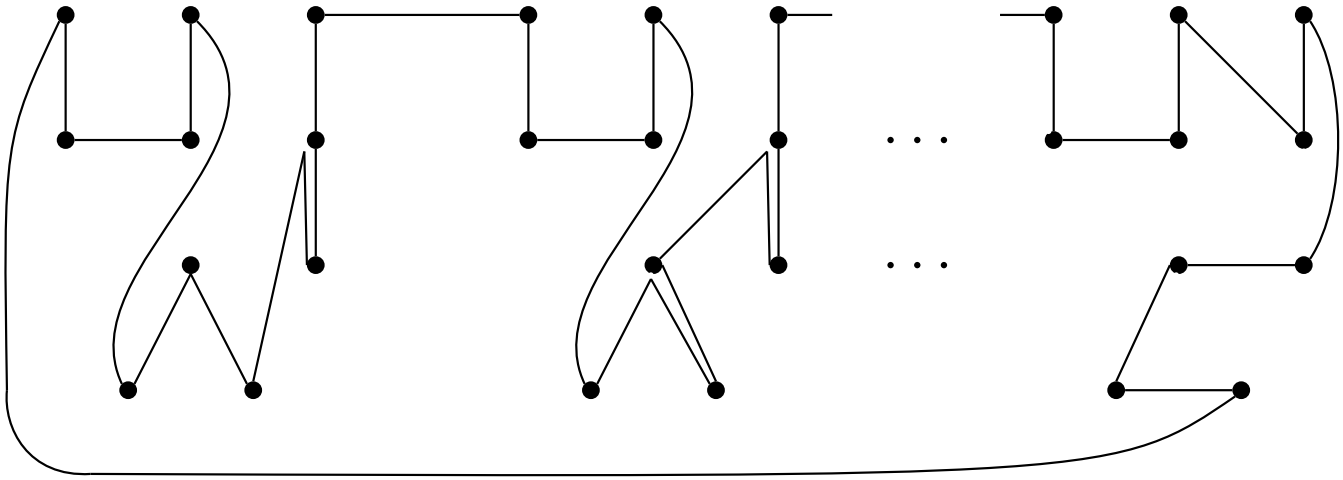

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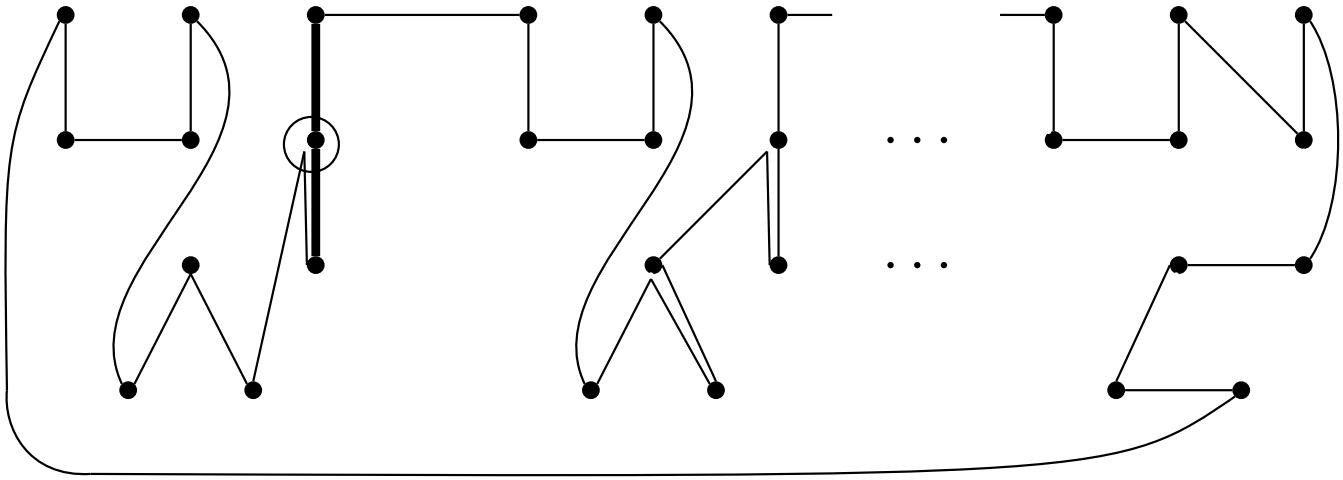

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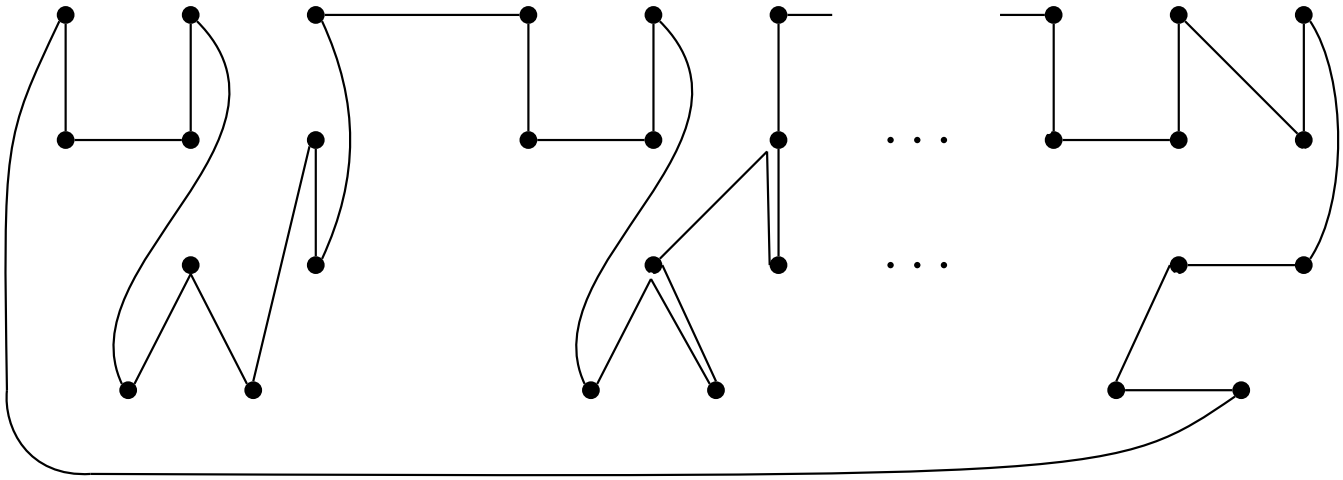

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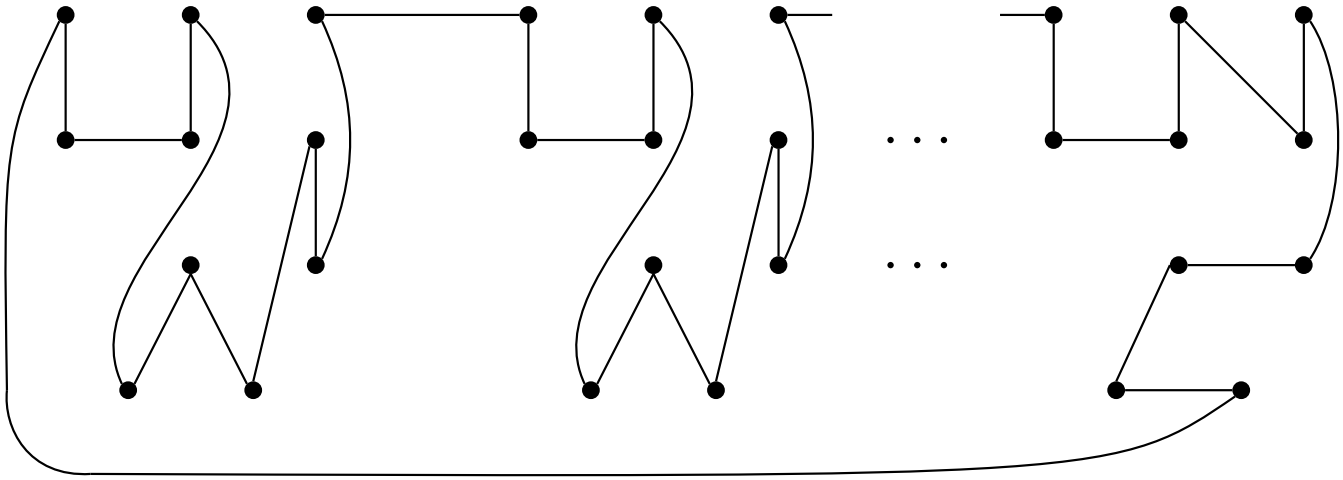
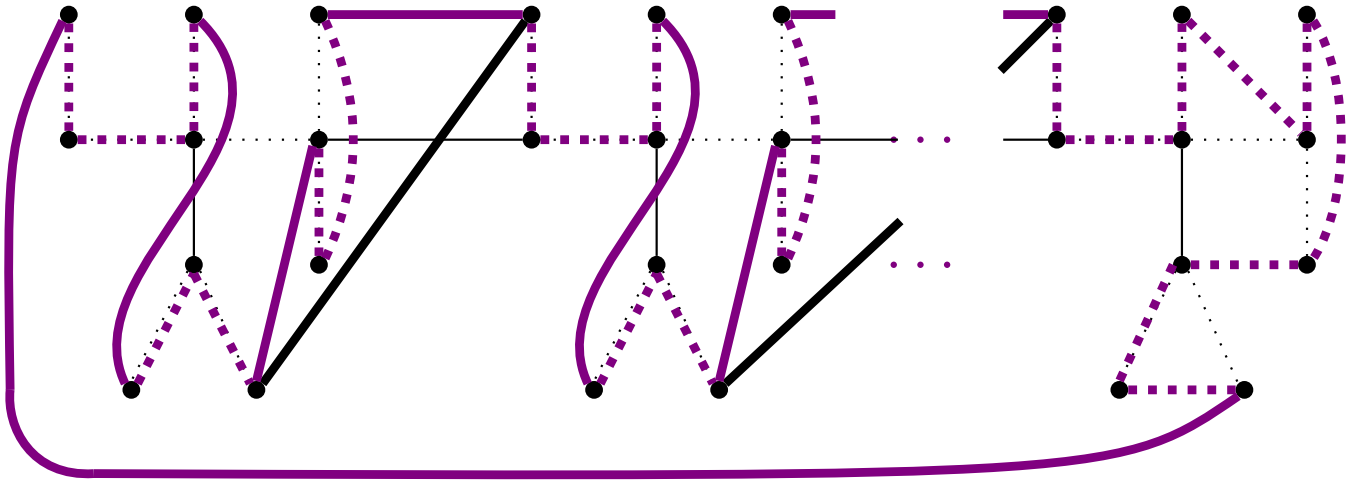
2. While there are conflicts: If $v$ has conflicts as neighbors, let $w$ be one of them. Let $v := w'$ and bypass $v$. Else let $v$ be an arbitrary conflict and bypass $v$.

# Step 6: Resolution of Remaining Conflicts



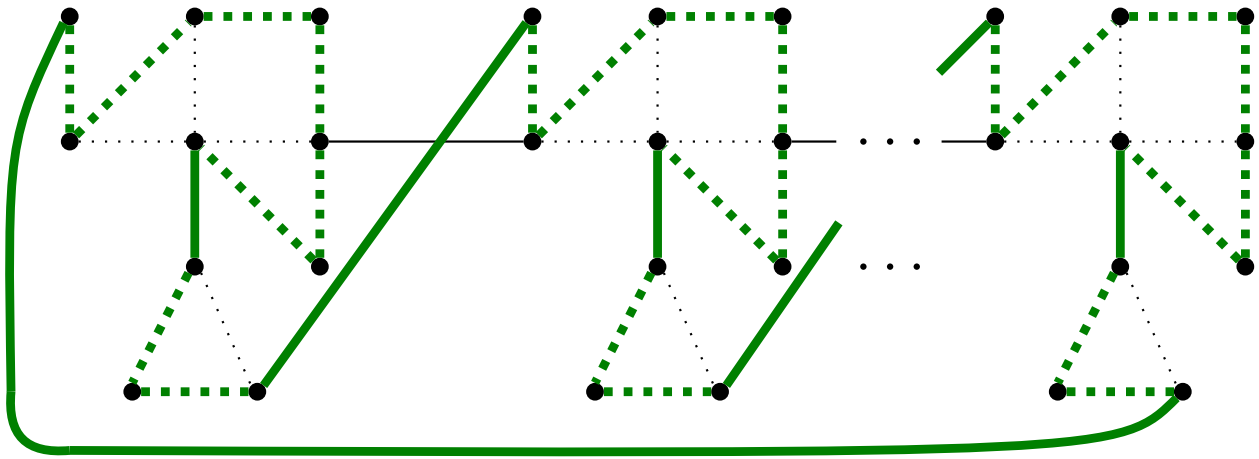
1. Bypass an arbitrary conflict $v$.
2. While there are conflicts: If $v$ has conflicts as neighbors, let $w$ be one of them. Let $v := w'$ and bypass $v$. Else let $v$ be an arbitrary conflict and bypass $v$.

# Cost of this Solution



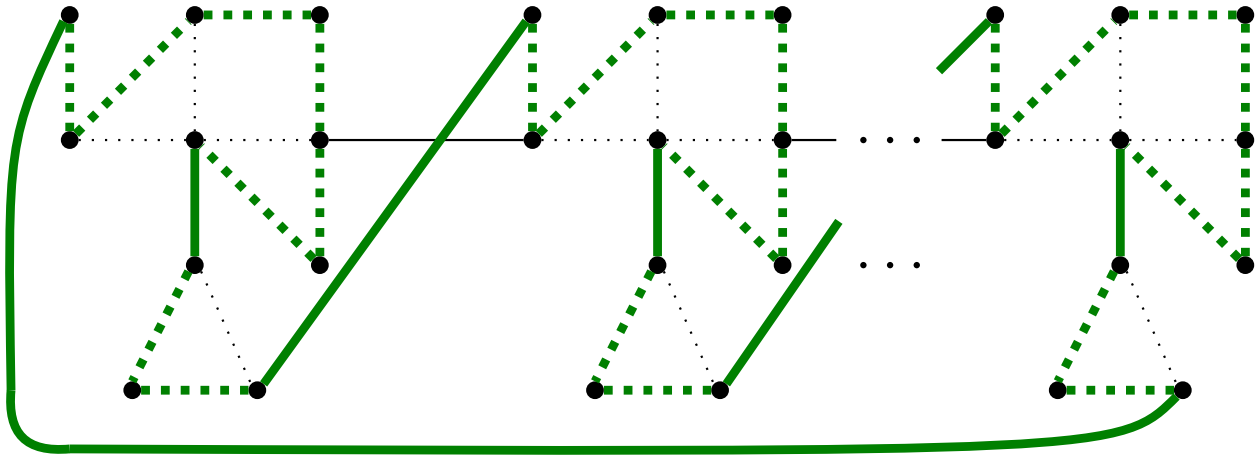
- Overall cost $\geq 3 \cdot (k-1) \cdot \beta^2$

# Cost of this Solution



- Overall cost $\geq 3 \cdot (k-1) \cdot \beta^2$
- Cost of this solution: $2k + 2\beta^2 + 7\beta + 6$

# Cost of this Solution



- Overall cost $\geq 3 \cdot (k-1) \cdot \beta^2$
- Cost of this solution: $2k + 2\beta^2 + 7\beta + 6$
- $\forall \varepsilon > 0 : \frac{3 \cdot (k-1) \cdot \beta^2}{2k+2\beta^2+7\beta+6} \geq \frac{3}{2}\beta^2 - \varepsilon$, for sufficiently large $k$

# The End

Thank you!