

The Equational Theory of Weak Complete Simulation Semantics over BCCSP

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Outline

- 1 Motivation and Introduction
- 2 Preliminaries
- 3 From Weak Simulation to Weak Complete Simulation
 - Definitions and properties
 - Axiomatizations
- 4 Weak Complete Simulation: ω -completeness
 - Weak Ready Simulation: definitions and properties
 - Weak Ready Simulation: axiomatization of \sqsubseteq_{RS} when A is infinite
 - Weak Complete Simulation: non existence of complete axiomatizations
- 5 Conclusions and (some) future work

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Motivation and Introduction

- We want to extend the previous results on (non)-axiomatizability of process semantics to the weak case.
- In order to do it we translate the unification effort to clarify the concrete ltbt-spectrum, to the weak case.
- In this paper we study the weak complete simulation semantics.
- All the results for the concrete case remain the same, although the existent axiomatizations have to be adequately extended, and we have to develop careful proofs for all the results.

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Preliminaries

- Syntax of $\text{BCCSP}(A_\tau)$

$$p ::= x \mid \mathbf{0} \mid \alpha.p \mid p_1 + p_2 \quad \alpha \in A_\tau = A \cup \{\tau\}$$

- Operational semantics

- $p \xrightarrow{\alpha} q \quad (\alpha \in A_\tau)$

- $p \xRightarrow{\alpha} p \quad (\alpha \in A_\tau)$ (We abstract internal moves)

- Semantics defined by a preorder \succsim
and its kernel \approx .

- Equational semantics: Axioms defining $=$ or \leq
Sound axiomatizations.

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- Complete axiomatizations
 - Ground completeness
 - Completeness
 - ω -completeness.
- Axiomatization of the weak bisimulation equivalence.

$$\begin{array}{ll}
 B_1 & x + y = y + x \\
 B_2 & (x + y) + z = x + (y + z) \\
 B_3 & x + x = x \\
 B_4 & x + \mathbf{0} = x \\
 W_1 & \alpha x = \alpha \tau x \\
 W_2 & \tau x = \tau x + x \\
 W_3 & \alpha(\tau x + y) = \alpha(\tau x + y) + \alpha y
 \end{array}$$

$$BW = \{B_1, B_2, B_3, B_4, W_1, W_2, W_3\}$$

- Unification of the equational semantics (concrete case)
Use of conditional axioms.

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Weak Simulation: definitions and properties

- \lesssim_S , is the largest relation satisfying that whenever $p \lesssim_S q$,

If $p \xrightarrow{\alpha} p'$ then there exists some q' such that
 $q \xRightarrow{\alpha} q'$ and $p' \lesssim_S q'$.

- \approx_S is the induced equivalence.
- The preorder \lesssim_S is a precongruence over $T(A_\tau)$.

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Weak Complete Simulation: definitions and properties

- p must terminate ($p \Downarrow$) iff there does not exist any $a \in A$ such that $p \xrightarrow{a}$.
- \preceq_{CS} , is the largest relation satisfying that whenever $p \preceq_{CS} q$,
 - If $p \xrightarrow{\alpha} p'$ then there exists some q' such that $q \xrightarrow{\alpha} q'$ and $p' \preceq_{CS} q'$.
 - If $p \Downarrow$ then $q \Downarrow$.
- \approx_{CS} is the induced equivalence.
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Weak Complete Simulation: definitions and properties

- $\tau 0 \not\lesssim_{CS} 0$ but $\tau 0 + a \not\lesssim_{CS} a$.
- \sqsubseteq_{CS} , is the largest precongruence over $\mathbb{T}(A_\tau)$ included in \lesssim_{CS} .
- $p \lesssim_{CS} q ::= p \approx_{CS} q$ and
 Whenever $p \xrightarrow{\tau} p'$ with $p' \Downarrow$, there exists q' such that
 $q(\xrightarrow{\tau})^+ q'$ and $q' \Downarrow$.
- \approx_{CS} is the kernel of \lesssim_{CS} .
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Weak Simulation: axiomatization

$$(S) \quad x \leq x + y$$

$$(\tau e) \quad \tau x = x$$

PROPOSITION

The set of equations

$$E_{S \leq} = \{B_1, B_2, B_3, B_4, S, \tau e\}$$

is sound and ground-complete for BCCSP(A_τ) modulo \approx_S .

$W_1 - W_3$ are not needed here, because of the presence of (τe) .

Weak Complete Simulation: ground-complete axiomatization

$$(CS_{\tau}) \quad (x \Downarrow \Leftrightarrow y \Downarrow) \Rightarrow x \leq x + y$$

$$(CS_{\tau e}) \quad \tau(ax + y) = ax + y$$

$$(CS_{\tau e})' \quad x \Downarrow \Rightarrow \tau x = x$$

PROPOSITION

The set of equations

$$E_{CS \leq}^c = BW \cup \{CS_{\tau e}, CS_{\tau}\},$$

where CS_{τ} is conditional, is sound and ground-complete for $BCCSP(A_{\tau})$ modulo \lesssim_{CS} .

Weak Complete Simulation: ground-complete axiomatization

$$\begin{array}{ll}
 (CS) & ax \leq ax + y \\
 (\tau N) & \mathbf{0} \leq \tau \mathbf{0}
 \end{array}
 \qquad
 (\tau g) \quad x \leq \tau x$$

PROPOSITION

The set of unconditional inequations

$$E_{CS\leq} = BW \cup \{CS_{\tau e}, CS, \tau N\}$$

is sound and ground-complete for BCCSP(A_τ) modulo \lesssim_{CS} .

Weak Simulation: axiomatization of \approx_S

$$(SE) \quad a(x + y) = a(x + y) + ay \quad (a \in A)$$

PROPOSITION

The set of equations

$$E_{S=} = \{B_1, B_2, B_3, B_4, SE, \tau e\}$$

is sound and ground-complete for BCCSP(A_τ) modulo \approx_S .

Weak Complete Simulation: axiomatization of \approx_{CS}

$$(CSE_{\tau}) \quad (x \Downarrow \Leftrightarrow y \Downarrow) \Rightarrow a(x + y) = a(x + y) + ax$$

PROPOSITION

The set of conditional equations

$$E_{CS=}^c = BW \cup \{CS_{\tau e}, CSE_{\tau}\}$$

is sound and ground-complete for BCCSP(A_{τ}) modulo \approx_{CS} .

Weak Complete Simulation: axiomatization of \approx_{CS}

$$(CSE) \quad a(bx + y + z) = a(bx + y + z) + a(bx + z)$$

PROPOSITION

The set of unconditional equations

$$E_{CS=} = BW \cup \{CS_{\tau e}, CSE\}$$

is sound and ground-complete for BCCSP(A_{τ}) modulo \approx_{CS} .

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Weak Ready Simulation: definitions and properties

- $I^*(t) = \{a \mid a \in A \text{ and } t \xrightarrow{a} t' \text{ for some } t'\}$.
- Weak ready simulation \lesssim_{RS} , defined by weak simulations whose related pairs $(p, q) \in R$ satisfy $I^*(p) = I^*(q)$.
- \lesssim_{RS} is not a precongruence
 $\tau a \lesssim_{RS} a$, however, $\tau a + b \not\lesssim_{RS} a + b$.
- \sqsubseteq_{RS} is the largest precongruence included in \lesssim_{RS} .

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Weak Ready Simulation: definitions and properties

DEFINITION

The preorder relation \lesssim_{RS} between processes is defined as follows:

We have $p \lesssim_{RS} q$ iff

- for any $\alpha \in A_\tau$ such that $p \xrightarrow{\alpha} p'$, there exists some q' such that $q \xrightarrow{\tau} \xrightarrow{\alpha} \xrightarrow{\tau} q'$ with $p' \lesssim_{RS} q'$, and
- $I^*(p) = I^*(q)$.

We denote the kernel of \lesssim_{RS} by \approx_{RS} .

If $|A| = \infty$ then we have $\sqsubseteq_{RS} = \lesssim_{RS}$.

Axiomatization of \sqsubseteq_{RS} when A is infinite

- (RS_τ) $I^*(x) = I^*(y) \Rightarrow x \leq x + y.$

PROPOSITION

The set of equations

$$E_{RS \leq}^c = BW \cup \{RS_\tau\},$$

in which RS_τ is conditional, is sound and ground-complete for \lesssim_{RS} over the language $BCCSP(A_\tau)$.

- (RS_τ) can be replaced by the unconditional axioms

$$\begin{array}{ll} (RS) & ax \leq ax + ay \\ (\tau g) & x \leq \tau x \end{array}$$

thus getting $E_{RS \leq}$

Weak Ready Simulation: complete axiomatization

- Completeness:
If $|A| = \infty$ then $E_{RS\leq}$ is complete.

THEOREM

If $|A| < \infty$, no finite set of sound inequations over $BCCSP(A_\tau)$ modulo \lesssim_{RS}^F can prove all of the sound inequations in the family

$$a^n x \leq a^n \mathbf{0} + \sum_{b \in A} a^n (x + b) \quad (n \geq 1).$$

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Complete Simulation: non existence of complete axiomatizations

PROPOSITION

For each $n \geq 0$, the equation

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is sound modulo ready simulation equivalence, and therefore modulo the kernel of \lesssim_{CS} .

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THEOREM

No finite set of sound (in)equations over $\text{BCCSP}(A_\tau)$ modulo \lesssim_{CS} can prove all of the sound equations in the family

$$a^n x + a^n \mathbf{0} + a^n (x + a) = a^n \mathbf{0} + a^n (x + a) \quad (n \geq 1).$$

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- Now we are working on the linear-time part of the spectrum, where the definition of the semantics is not still totally clear, because of the subtleties produced by the mixed roles of internal actions (abstraction / non-determinism).

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THANKS!