

Compact Closed Categories and Frobenius Algebras

4

Reasoning about Natural Language Meaning

Mehrnoosh Sadrzadeh, EPSRC Career Acceleration Research Fellow

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Communal work with: Clark, Coecke, Grefenstette, Kartsaklis,
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Universities of Oxford, Cambridge, and Montpellier



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Translate

From: Persian



To: English

Translate

Persian

English

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X

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Darya wrote to my family



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Analysis

Google is good with words, but bad with sentences.

Definition. Meaning of a sentence of n words ' $w_1 w_2 \dots w'_n$ ' is the set (bag) of the meanings of its words:

$$\{ \llbracket w_1 \rrbracket, \llbracket w_2 \rrbracket, \dots, \llbracket w_n \rrbracket \}.$$

Example: meaning of '*A vampire killed a man*' is

$$\{ \llbracket \text{vampire} \rrbracket, \llbracket \text{kill} \rrbracket, \llbracket \text{man} \rrbracket \} = \{ \llbracket \text{man} \rrbracket, \llbracket \text{kill} \rrbracket, \llbracket \text{vampire} \rrbracket \}$$

which is the same as meaning of '*a man killed a vampire*'.

Existing Theories of Meaning

- Review two orthogonal approaches to meaning:
 - 1- Meaning is Use!
 - 2- Meaning is Grammatical Structure!
- Show how to merge them and attempt to solve the problem.

Meaning is Use!

Firth/Harris: ‘you know a word from its context’.

Contexts in which a word often appears determine its meaning.

yamayama **blood** yama **vampire** yamayama **dead** yamayama
blood yamayama yama ... yamayama yama **dead** yamayama
yamayamayama **blood** yamayamayama yama **vampire** ya-
mayama **dead** yamayamayama **vampire** yamayamayama
yama **blood** yamayamayama yama

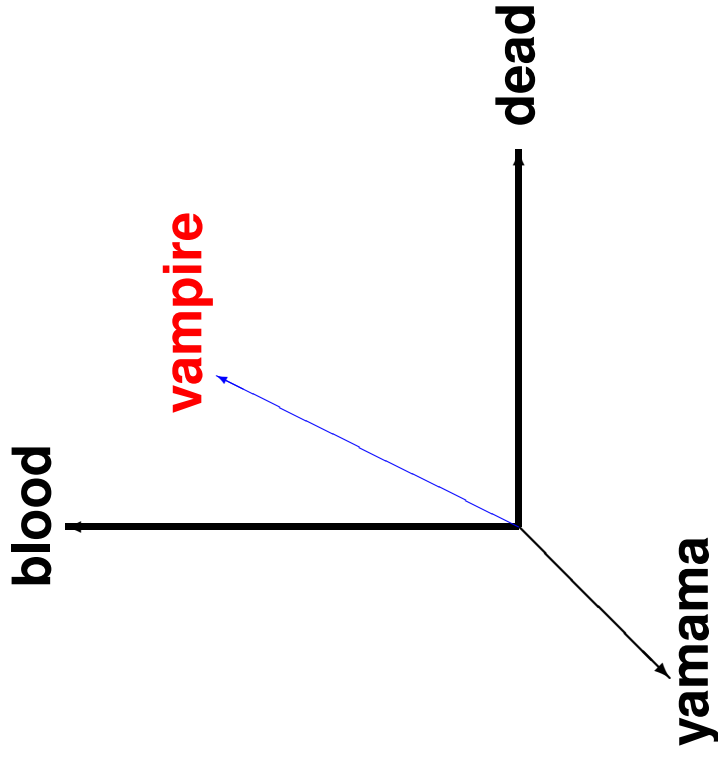
If you do not know the meaning of **vampire**, but you know the meanings of **blood** and **dead**, you can guess what would **vampire** mean.

Formalization

Vector Spaces/Distributional Semantics (e.g. Spärk-Jones, Schütze).

Context words = basis vectors,

Meanings of words = vectors, window of length k .

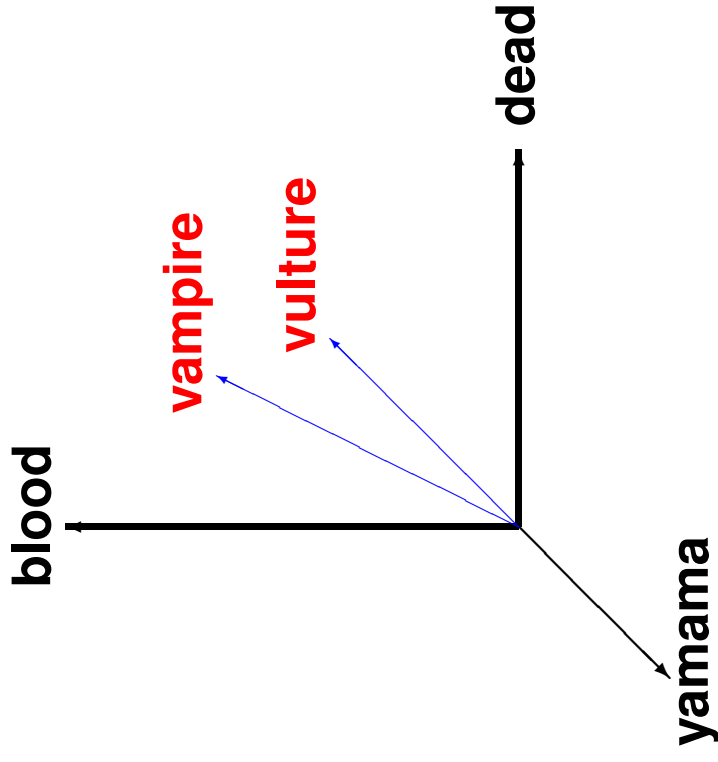


Applications

Good model of similarity between meanings of words.

Words that appear in the same contexts have similar meanings.

Reflected in the angle between their vectors.



Automated Thesaurus

Curran: enough philosophising, let's test this idea.

Introduction: launch, implementation, advent, addition, adoption, arrival, absence, inclusion, creation

Evaluation: assessment, examination, appraisal, review, audit, analysis, consultation, monitoring, testing, verification

Methods: technique, procedure, means, approach, strategy, tool, concept, practice, formula, tactic

Weaknesses

Some words are not synonyms, but appear in the same contexts:

black,

Some words appear in all contexts, treated as 'noise'.

and, or, who, whom

The model does not naturally extend to sentences.

Meaning is Structure!

One fixes a structure, e.g. ordered algebras; forms a dictionary by assigning elements of the algebra to words of a language, based on their grammatical roles.

Pregroup (Lambek/Harris). A partially ordered monoid, where every element has a left and a right adjoint:

$$(P, \leq, \otimes, \mathbf{1}, (-)^r, (-)^l)$$

$$p \otimes p^r \leq \mathbf{1} \leq p^l \otimes p \quad p^l \otimes p \leq \mathbf{1} \leq p \otimes p^l$$

Example. $B = \{n, s\}$, generate the free pregroup.

Pregroup Dictionary.

vampire: n blood: n drink: $n^r \otimes s \otimes n^l$

Meaning is Structure!

Meaning of a sentence becomes its grammatical structure, which is, how the grammatical roles of the words within the sentence interact with each other to produce a sentence.

Pregroup Algebraically. Meaning of a sentence $w_1 w_2 \cdots w_n$, where p_i is the pregroup type of each word w_i , is the following inequality:

$$p_1 \otimes p_2 \otimes \cdots \otimes p_n \leq s$$

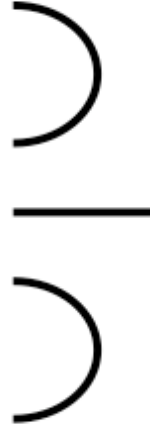
Example. Meaning of 'Vampires drink blood' is:

$$n \otimes (n^r \otimes s \otimes n^l) \otimes n \leq s$$

Interaction Diagrams

Vampires drink blood.

$n \quad n^r \quad s \quad n^l \quad n$



Vampires do not drink blood.

$n \quad n^r \quad s \quad j^l \quad \sigma \quad \sigma^r \quad j \quad j^l \quad \sigma \quad \sigma^r \quad j \quad n^l \quad n$



J. Lambek, 'From Word to Sentence', Polimetrica, 2008.

Weaknesses

Sentences with same grammatical structure have same meanings:

Vampires drink blood. = People go home.

Meanings of words is just their grammatical role:

blood = home: n , drink = go: $n^r \otimes s \otimes n^l$

Does not say anything about meanings of words.

Weakness-Overcoming Clinique

Weakness-Overcoming Clinique

Grammar
Compact Closed Category of
Grammatical Types and
Reductions

$$\rightleftarrows F \rightleftarrows$$

Meaning
Compact Closed Category of
Finite Dimensional Vector
Spaces and Linear maps

Quantization (Atiyah, Baez-Dolan, TQFT)

Weakness-Overcoming Clinique

Grammar
 Compact Closed Category of
 Grammatical Types and
 Reductions

$$\begin{array}{c} \rightrightarrows \\ F \\ \lleftarrow \end{array}$$

Meaning
 Compact Closed Category of
 Finite Dimensional Vector
 Spaces and Linear maps

Grammar	Strongly Monoidal Functor	Vector Spaces
$Preg$	F	$FVect_{\mathbb{R}}$
1	\rightrightarrows	\mathbb{R}
p, q	\rightrightarrows	V, W
p^r, p^l	\rightrightarrows	V^*
$p \otimes q$	\rightrightarrows	$V \otimes W$

Weakness-Overcoming Clinique

<p>Grammar</p> <p>Compact Closed Category of Grammatical Types and Reductions</p>	$\begin{array}{c} \Rightarrow \\ \Rightarrow \\ F \\ \Rightarrow \\ \Rightarrow \end{array}$	<p>Meaning</p> <p>Compact Closed Category of Finite Dimensional Vector Spaces and Linear maps</p>
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Grammar	Strongly Monoidal Functor	Vector Spaces
$Preg$	F	$FVect_{\mathbb{R}}$
1	\Rightarrow	\mathbb{R}
p, q	\Rightarrow	V, W
p^r, p^l	\Rightarrow	V^*
$p \otimes q$	\Rightarrow	$V \otimes W$
$p \leq q$	\Rightarrow	$V \rightarrow W$
$p \otimes p^r \leq 1$	\Rightarrow	$V \otimes V^* \rightarrow \mathbb{R} :: \text{inner product}$
$1 \leq p^r \otimes p$	\Rightarrow	$\mathbb{R} \rightarrow V \otimes V^* :: \text{Bell pair}$

Meaning of a Sentence

Definition. Meaning of a sentence of n words

$$w_1 w_2 \cdots w_n$$

where each word has type p_i
and the sentence has grammatical structure

$$p_1 \otimes p_2 \otimes \cdots \otimes p_n \leq s$$

is

$$\overrightarrow{w_1 \cdots w_n} := F(\leq)(\overrightarrow{w_1} \otimes \cdots \otimes \overrightarrow{w_n}).$$

Concrete Example

Type	Functor F	Vectors
n	\Uparrow	$\overrightarrow{\text{vampire, blood}} \in V$ with fixed basis.
n^r, n^l	\Uparrow	$V^* \cong V$
s	\Uparrow	$\overrightarrow{\text{Vampires drink blood}} \in V$
$n^r \otimes s \otimes n^l$	\Uparrow	$\overrightarrow{\text{drink}} \in V \otimes V \otimes V$???

Concrete Example

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$n^r \otimes s \otimes n^l$	\Rightarrow	$\overrightarrow{\text{drink}} \in V \otimes V \otimes V$???

$$\begin{aligned}
 \overrightarrow{\text{Vampires drink blood}} &= F(\leq)(\overrightarrow{\text{vampire}} \otimes \overrightarrow{\text{drink}} \otimes \overrightarrow{\text{blood}}) \\
 &= \sum_{ijk} C_{ijk} \langle \overrightarrow{v_i} | \overrightarrow{v_j} \langle \overrightarrow{v_k} | \text{blood} \rangle \rangle
 \end{aligned}$$

Frobenius Embeddings

Abstractly: words with complex types live in tensor spaces.

Transitive verbs have type $n^r \otimes s \otimes n^l$, hence live in $V \otimes V \otimes V$.

Concretely: the degrees according to which the verb has related its subjects to its objects:

$$\overline{\text{drink}} = \sum_i (\text{sub} \otimes \text{obj})_i \in V \otimes V$$

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Frobenius Algebras: $(V, \sigma, \iota); \quad \sigma: V \rightarrow V \otimes V$

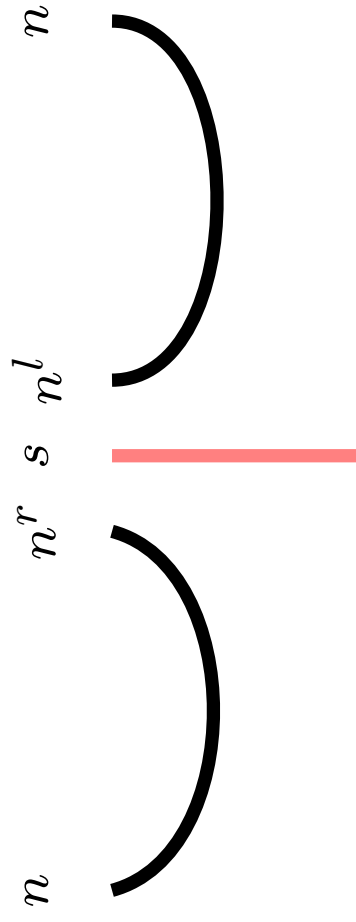
Vectors Spaces: Copies lower dim vectors to higher ones: diag.

Result:

$$\begin{aligned} \overline{\text{Vampires drink blood}} &= (\sigma(\overline{\text{drink}}) \times \overline{\text{blood}}) \times (\overline{\text{vampire}}) \\ &= \overline{\text{drink}} \odot (\overline{\text{blood}} \otimes \overline{\text{vampire}}) \end{aligned}$$

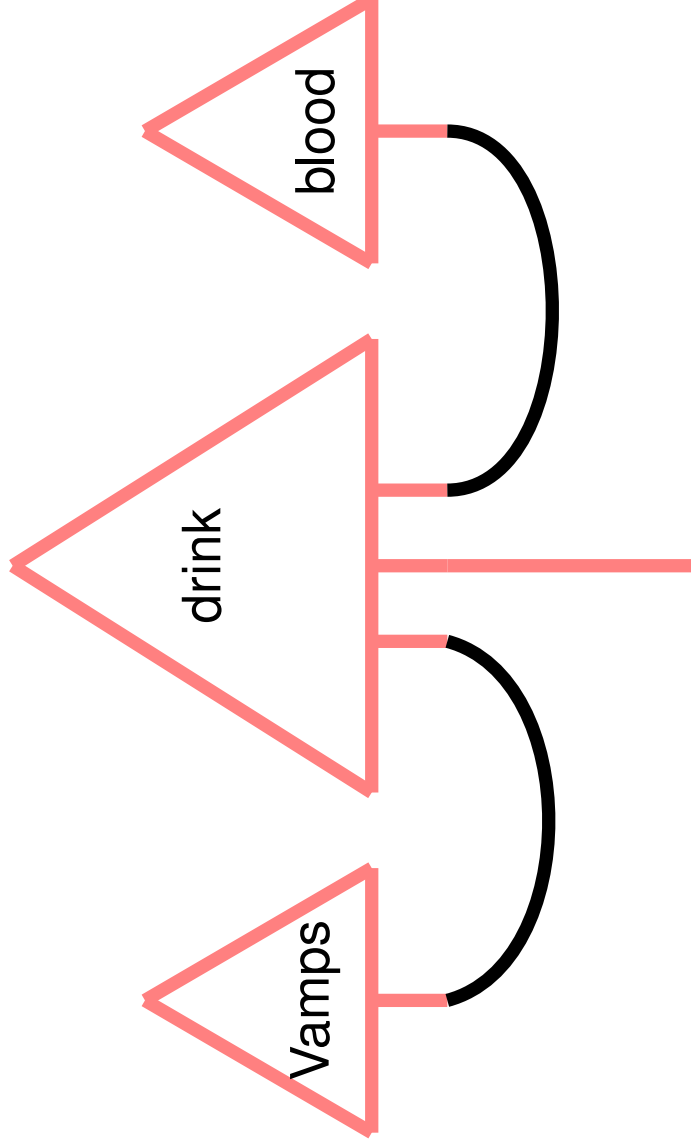
Interaction Diagrams

Only Types



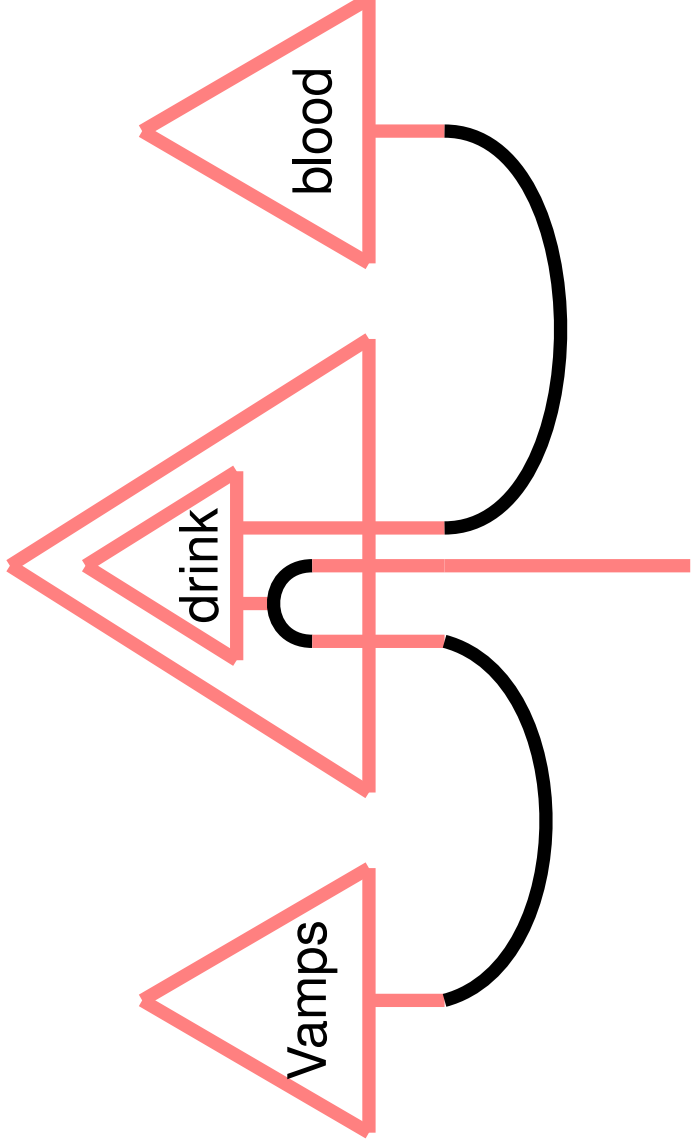
Vector Interaction Diagrams

Abstract Vectors



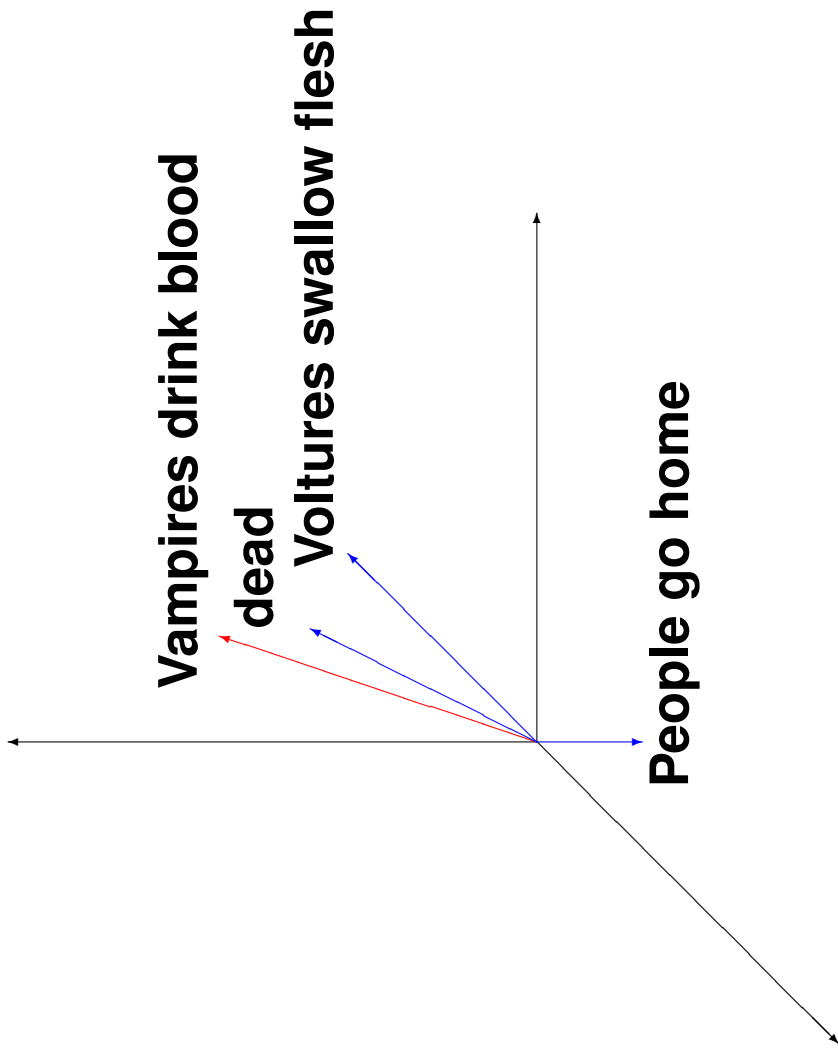
Vector Interaction Diagrams

Frobenius Anatomy



Meaning of Sentences as Vectors

- Build vectors for sentences.
- Measure sentence/phrase/word synonymy.



Evaluation

Like Curran, we asked, this is all nice, but does it work?

But sentence synonymy is much harder to verify.

It does not make sense to do an automated thesaurus for sentence.

Application: disambiguation, term/definition classification

Disambiguation

Extension of Mitchel & Lapata 2009

Some verbs have more than one meaning.

Their context can help disambiguate them.

file (1) smooth (2) register
woman filed her nails. woman filed her records.

#	Sen. 1	Sen. 2	Deg. of Syn.	Human Rank
1	woman filed nail	woman smooth nail	0.765	6
2	woman filed nail	woman registered nail	0.224	3
:				
400				

Experimental Results

Model	High	Low	ρ
Baseline	0.47	0.44	0.16
Add	0.90	0.90	0.05
Multiply	0.67	0.59	0.17
Ours	0.34	0.26	0.28
UpperBound	4.80	2.49	0.62

Grefenstette-S., EMNLP 2011

Term/Definition Classification

Term	Definition
blaze	large strong fire
husband	married man
vampire	creature that drinks blood
bleach	make something white

Experimental Results

	Rank	Count	%
Term	1	25	34.7
	2-5	22	30.6
	6-10	5	6.9
	11-72	20	27.8

Mistakes

Term	Original definition	Assigned definition
rod	fishing stick	round handle
jacket	short coat	waterproof cover
husband	married man	bad man

Summary

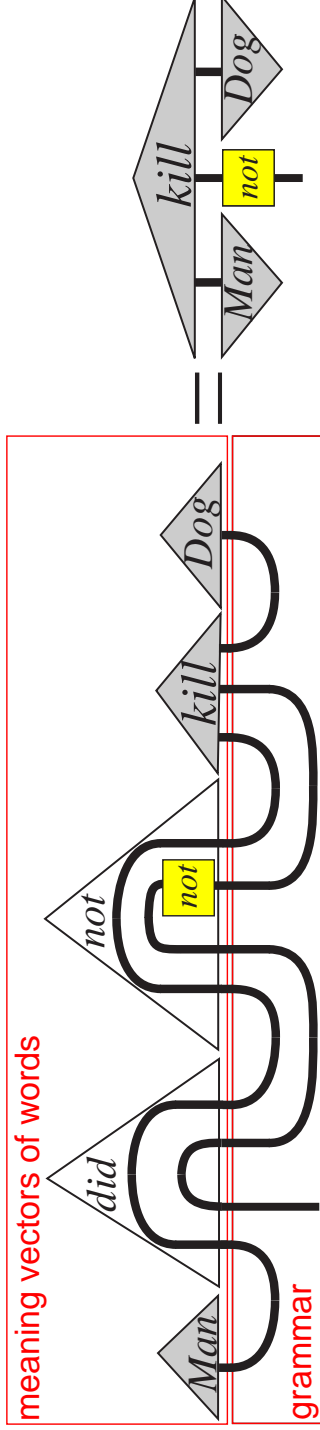
- 0- Identified a real-world problem
- 1- Developed high-level theory, unified two orthogonal models
 - Inspired by high-level model of quantum Abramsky-Coecke

Quality and Quantity: Quantization Passage

- 2- Produce concrete constructions
- 3- Evaluation datasets and Experimental results
- 4- These were well-received by linguistics

Future Work

- Abstract Logical Words:



- Concrete Anatomy: Probabilistic/Fuzzy logic gates, Sheaves.
- NewScientist, Dec 2010.