Ten years of triviality

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K-trivials: synopsis

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Several problems on the relationship of K-trivials to Martin-Löf randoms have been solved recently, or appear close to a solution.
This relies on surprising new connections of Martin-Löf randomness with the analytic concept of density in a closed class.

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There is a universal prefix-free machine $\mathbb{U}$:
for every prefix-free machine M,

$$M(\sigma) = y \text{ implies } \mathbb{U}(\tau) = y \text{ for some } \tau \text{ with } |\tau| \leq |\sigma| + d_M,$$

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The prefix-free Kolmogorov complexity of string y is the length of a shortest $\mathbb{U}$-description of y:

$$K(y) = \min\{|\sigma| : \mathbb{U}(\sigma) = y\}.$$

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An infinite sequence of bits A is K-trivial if, for some $b \in \mathbb{N}$,

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$$\begin{array}{lllll} Z \text{ is} & \text{random} & \Leftrightarrow & \forall n \, [K(Z\restriction_n) > n & -O(1)] \\ A \text{ is} & K\text{-trivial} & \Leftrightarrow & \forall n \, [K(A\restriction_n) \leq K(n) & +O(1)] \end{array}$$

Thus, being K-trivial means being far from random.

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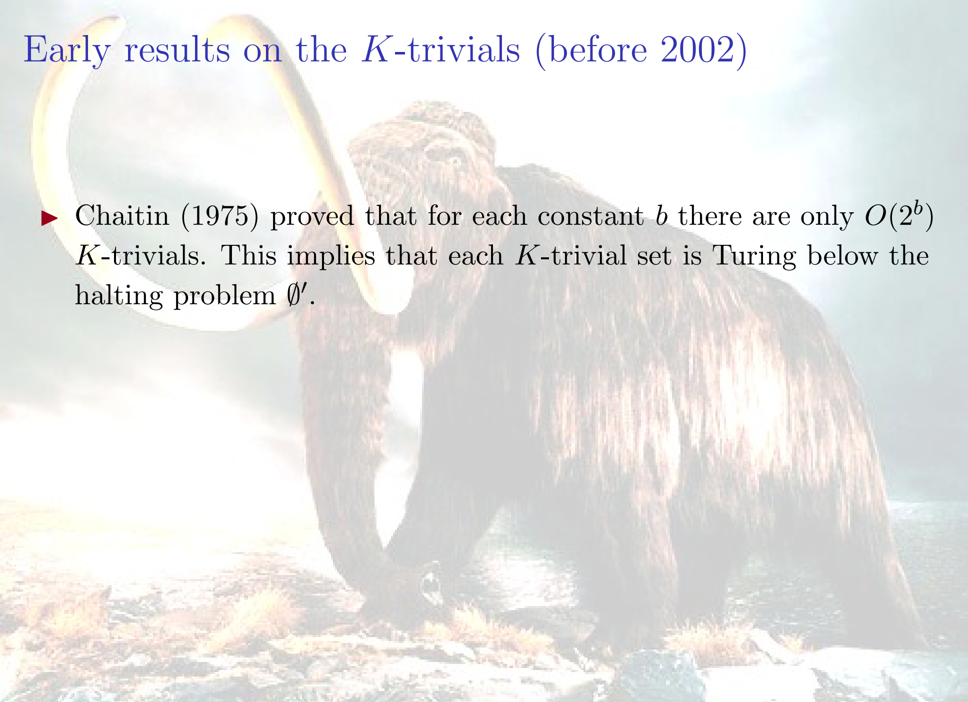
- $\forall n\,[C(Z\restriction_n) > n - O(1)]$: there is no such Z (Katzev).
- $\forall n\,[C(A\restriction_n) \leq C(n) + O(1)]$: the only such A are the computable sets (Chaitin, 1976).

Early results on the K-trivials (before 2002)



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- Solovay (1976) built an incomputable K-trivial set.
- Zambella (1990 ILLC technical report), Kummer (unpubl), and Calude-Coles (1999) all gave examples of such sets that were computably enumerable.

The awakening of triviality (2002-2004)

- Downey, Hirschfeldt, Nies, and Stephan[1] gave a simple "cost function" construction of an incomputable c.e. K-trivial set. The construction was similar to the construction of a low for random set due to Kučera and Terwijn (1999).

[1] Trivial reals. Proceedings of the 7th and 8th Asian Logic Conferences, Singapore University Press (2003), 103-131.

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- Downey et al. also showed that no K-trivial is Turing equivalent to the halting problem $\emptyset'$. For this they introduced what was later called the decanter method [2].

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For each e, eventually we can choose a number x so large that

$$\sum_{w=x+1}^{s} 2^{-K_s(w)} \leq 2^{-e-2}.$$

Because $\sum_e 2^{-e-2} = 1/2$, we will never run out of measure for new descriptions in the prefix-free machine we are building.

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- Kučera and Terwijn (1999) built a c.e. incomputable set of this kind.

Far from random $=$ close to computable

Theorem (N, Adv. in Math., 2005)

Let $A \subseteq \mathbb{N}$. Then

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- The proof combined the decanter with the "golden run" method, where an oracle $\emptyset''$ would be needed to find the node in a tree of runs that satisfies the conditions required.

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- Since low for ML-randomness is closed downward under $\leq_{\mathrm{T}}$, this implies that the K-trivials are closed downward under $\leq_T$.
- Nies also used the golden run method to show that each K-trivial A is superlow ($A' \leq_{\mathrm{tt}} \emptyset'$), and truth-table below a c.e. K-trivial.

Bases for randomness (2005-2006)

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Let Y be an incomplete Martin-Löf random. Let $A \leq_{\mathrm{T}} Y$ be a c.e. set. Then A is K-trivial.

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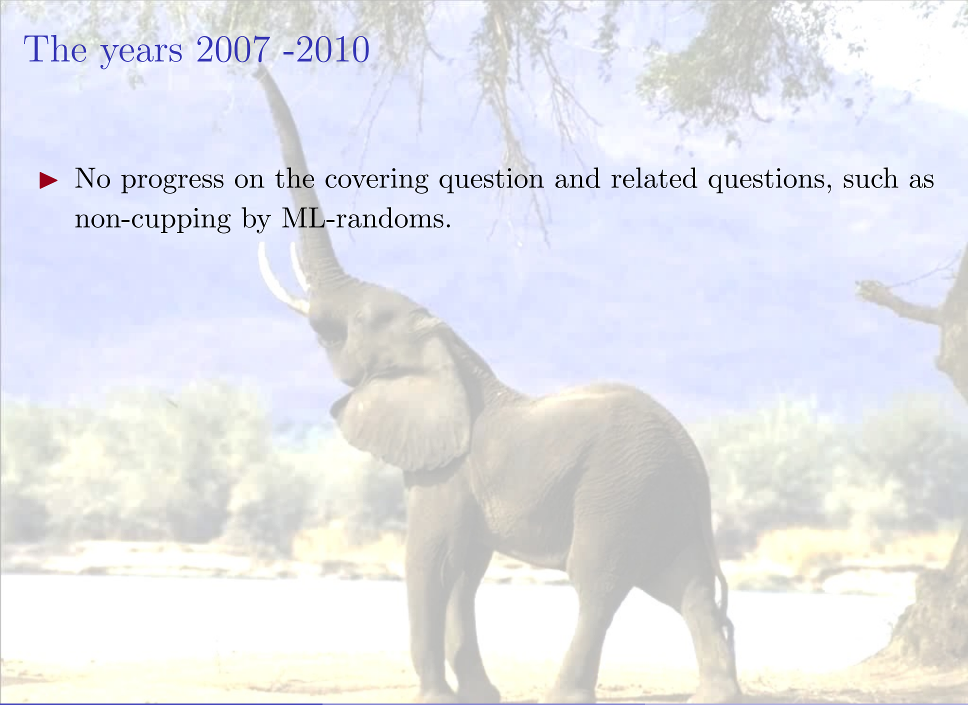
Converse?

Covering question

Is every (c.e.) K-trivial below an incomplete ML-random?

The years 2007 -2010

- No progress on the covering question and related questions, such as non-cupping by ML-randoms.



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- Bienvenu and Downey (STACS 2009),
  Bienvenu, Merkle and N (STACS 2011): Solovay functions.

Solovay functions

Definition (going back to Solovay, 75)

A computable function $f\colon \mathbb{N} \to \mathbb{N}$ is a Solovay function if $\forall n\, K(n) \le f(n)$, and infinitely often equal (all within constants).

Theorem (Bienvenu, Downey, Nies, Merkle)

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Program: redo theorems on K-trivials using Solovay functions. E.g, there is a new proof that every K-trivial is Turing below a c.e. K-trivial due to Bienvenu, 2010, unpubl.

The non-cupping question

Theorem (N, 2007; improved by N and Hirschfeldt)

There is an incomputable c.e. A such that

$$(*)\ A \oplus Y \geq_T \emptyset' \text{ for ML-random } Y \text{ implies } Y \geq_T \emptyset'.$$

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Does the property $(*)$ hold for every K-trivial set A?

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- Bienvenu, Greenberg, Kučera, N and Turetsky (2012) build a "smart" K-trivial A:

  every ML-random $Y \geq_T A$ is LR-hard (close to T-complete).

Positive density in a Π^0_1 class

Let $z \in \mathcal{P} \subseteq [0,1]$, where $\mathcal{P}$ is a Π^0_1 class. Let $\beta \in (0,1]$.
We say that z has (lower) density β in $\mathcal{P}$ if the portion of $\mathcal{P}$ around z as we zoom in is eventually β:

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Day-Miller used this analytic understanding of Turing incompleteness among the ML-randoms for the non-cupping question.

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We build a "smart" K-trivial A:

There is K-trivial A such that no $Y \geq_T A$ is Oberwolfach random.

Recall that Y is LR-hard if every Y-random set is random relative to $\emptyset'$. Random incomplete LR-hard sets exist, but are very close to Turing complete. We improved a result of BHMN 2012:

If Y is ML-random but not OW-random, then Y is LR-hard.

More developments

Bienvenu, Greenberg, Kučera, Miller, N and Turetsky (2012):

If a Martin-Löf random z has lower density < 1 in some Π^0_1 class $\mathcal{P} \ni z$, then z computes every K-trivial set.

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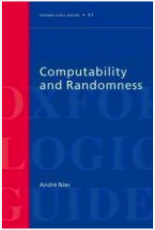
Melnikov and N (Proc. AMS, in press) extended K-triviality to functions, and then to points in a computable metric space.

Recent references

- "The Denjoy alternative for computable functions", with Bienvenu, Hoelzl, and Miller, STACS 2012. "K-triviality, Oberwolfach randomness, and differentiability". Oberwolfach preprint by BGKNT.
- 

My book "Computability and Randomness". Updated and affordable paperback version has appeared (March 2012).
- Book by Downey and Hirschfeldt (2010).
- These slides and the logic blog, on my web page.