Weak Synchronization and Synchronizability of Multi-tape Automata and Machines

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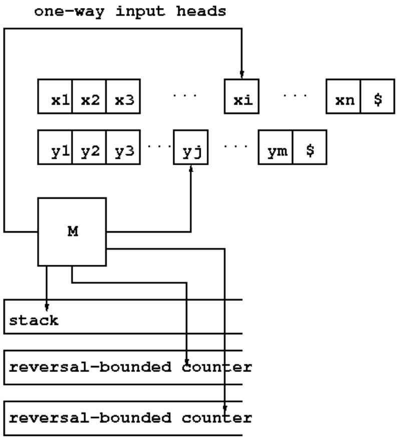
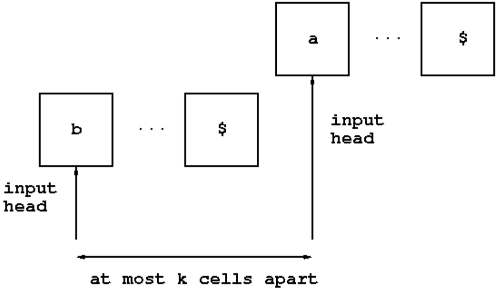
Language and Automata Theory and Applications, 2012

Outline

Basic Model: Multi-tape PDA + Reversal-bounded Counters



Problem I: Is machine *weakly k-synchronized*?



weakly k-synchronized: If $(x_1, x_2, \ldots, x_l)$ is accepted, then there is an *accepting* computation in which no two input heads *not reading $* are ever more than k cells apart

Problem II: Is machine *weakly synchronizable* ?

weakly synchronizable: Given a machine M, is there an M' such that $L(M) = L(M')$ and M' is weakly k-synchronized ?

Motivation: SQL Injection Attack

```
<?php
$query = 'SELECT * FROM users WHERE
         email = "' . $_POST['email'] . '" ' .
         ' AND pwdhash = "' .
         hash('sha256', $_POST['password']) . '"';
?>
```

Input:

```
email:  "OR 1=1;--
password:  anything
```

Output:

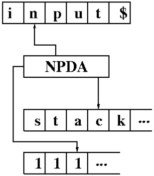
```
SELECT * FROM users WHERE email="" OR 1=1;-- "
AND pwdhash="..."
```

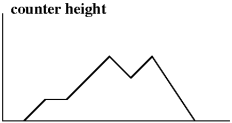
Previous Works

- *single-track* DFA to represent individual string variables [Xu et al.], [Shannon et al.]
- *multi-track* DFA to represent groups of string variables [Yu et al.]
- *strongly synchronized multi-tape* and multi-head FA [Ibarra et al.]
- *weakly synchronized multi-tape* FA [Egecioglu et al.]

Variants of Basic Model

- *k-ambiguous*: at most k accepting computations per input
- *unambiguous*: 1-ambiguous
- *bounded input*: input $x \in a_{i1}^* a_{i2}^* \ldots a_{ik_i}^*$ (a_{ij}'s distinct letters)
- *NCM*: NPDA with unary stack alphabet
- *reversal-bounded counters*: the stack height function has a bounded number of local maxima/minima





Main Results: multi-tape PDA

2-ambiguous 2-tape 1-reversal NCM (I, II)

2-ambiguous 2-tape 3-reversal NPDA over $\{a, b\}^* \times c^*$ (I, II)

2-tape 1-reversal NCM over $\{a, b\}^* \times c^*$ (I, II)

3-ambiguous 2-tape NCM over $\{a, b\}^* \times c^*$ (I, II)

undecidable

decidable

Unambiguous n-tape NPDA + reversal-bounded counters (I)

n-tape NPDA + reversal-bounded counters over bounded inputs (I)

Technique 1: Infiniteness of one-tape NPDA

Theorem

It is decidable whether an unambiguous n-tape NPDA M with reversal-bounded counters is weakly k-synchronized for some k.

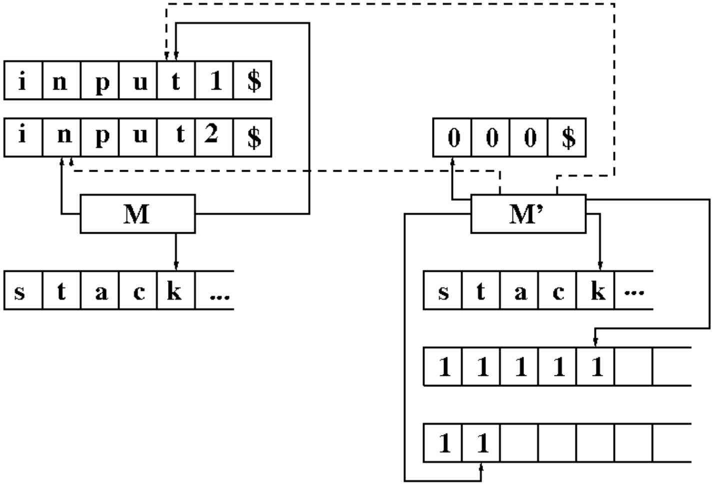
Proof.

M is *not* weakly synchronized iff two of its heads get separated by more than k cells for infinitely many k's on accepting computations.

Construct a one-tape NPDA M' with 2 counters to catch these offending *accepting* computations.

It is decidable whether $L(M')$ is infinite! □

Guessing offending accepting computations



Technique 2: Post Correspondence Problem

Theorem

Given a (2-ambiguous) 2-tape 1-reversal NCM M, it is undecidable whether there is a 2-tape 0-synchronized NPDA M' such that $L(M) = L(M')$.

Proof.

Reduction from Post Correspondence Problem:

INPUT: $I = \{x_1, \ldots, x_m\}, \{y_1, \ldots, y_n\}$

OUTPUT: yes iff $x_{i_1} \ldots x_{i_k} = y_{i_1} \ldots y_{i_k}$ for some indices $i_1, \ldots, i_k$.

Reduction function: $f(I) = M$, which accepts $L_1 \cup L_2$:

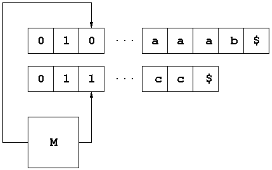
$$L_1 = \{(xa^i b^j, yc^k) : i, j, k > 0, x \neq y\}$$
$$L_2 = \{(xa^{3i} b^{2i}, xc^i : i > 0, x \text{ encodes a solution of } I\}$$

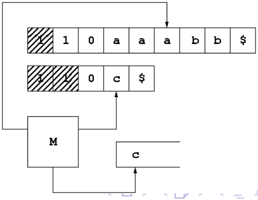
Construction of 2-tape 1-reversal NCM M

$$
\begin{aligned}
L \quad = \quad & \{(xa^i b^j, yc^k) : i, j, k > 0, x \neq y\} \cup \\
& \{(xa^{3i} b^{2i}, xc^i : i > 0, x \text{ encodes a solution of } I\}
\end{aligned}
$$

On input $xa^r b^s, yc^t$, M nondeterministically checks:

- $x \neq y$ (0-synchronized DFA);
- $x = x_{i_1} \ldots x_{i_k}$, $y = y_{i_1} \ldots y_{i_k}$ for some $i_1, \ldots i_k$, and $r = 3t$, $s = 2t$





I has a solution iff L is accepted by a 0-synchronized 2-tape NCM

- I has no solutions:

  $L = L_1$ and hence L is accepted by a 0-synchronized 2-tape DFA

- I has a solution $x = i_1, i_2, \ldots, i_k$:

  if L is accepted by a 0-synchronized 2-tape NCM M, convert M into a one-tape two-track NCM M'

  apply Ogden's lemma to $\binom{xa^{3i}b^{2i}}{xc^i}$ to obtain a string not in L, contradiction.

Technique 3: Halting Problem

Theorem

Given a 2-tape 1-reversal NCM M over $\Sigma^* \times c^*$, where $|\Sigma| \geq 2$, it is undecidable whether M is weakly k-synchronized for some k.

Proof.

Reduction from the halting problem for one-tape Turing machines T on blank input.

Define $H(T) = I_1 \# I_2 \# \ldots \# I_m \#$ to be an encoding of the halting computation of T on blank input, if exists.

Reduction function: $f(T) = M$, a 2-tape NCM that accepts:

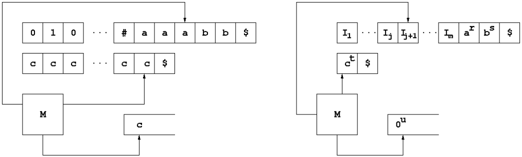
$$L = \{(xa^i b^j, c^{|x|+k}) : x \in \{0,1,\#\}^* \wedge (i,j,k > 0) \wedge [x \neq H(T) \vee (i = 3k \wedge j = 2k)]\}$$

Construction of 2-tape 1-reversal NCM M

$$L = \{(xa^ib^j, c^{|x|+k}) : x \in \{0,1,\#\}^* \wedge (i,j,k > 0) \wedge [x \neq H(T) \vee (i = 3k \wedge j = 2k)]\}$$

On input xa^rb^s, c^t, M nondeterministically checks:

- $r = 3k$ and $s = 2k$, where $k = t - |x|$ (one reversal)
- $x \neq H(T)$: 0-synchronized DFA



T halts on λ iff M is a 0-synchronized 2-tape NCM

- T does not halt on λ:

  $L = \{(xa^i b^j, c^k) : x \in \{0,1,\#\}^*, i,j,k > 0\}$ so there is a 0-synchronized accepting computation for all $x \in L$

- T halts on λ:

  if M is 0-synchronized, convert M into a one-tape two-track NCM M'

  apply Ogden's lemma to $\binom{H(T)a^{3i}b^{2i}}{c^{|H(T)|+i}}$ to obtain a string not in L, contradiction.

Summary of Selected Results

- studied synchronization and synchronizability (motivated by web security)
- main result: synchronization is undecidable for 2-ambiguous, 2-tape, 1-reversal NCM
- becomes decidable for unambiguous or bounded input (even with additional tapes, reversal bounded counters and one pushdown stack)
- when one tape is unary, synchronization is undecidable for ambiguous NCM with one reversal, 3-ambiguous NCM, 2-ambiguous NPDA with three reversals