

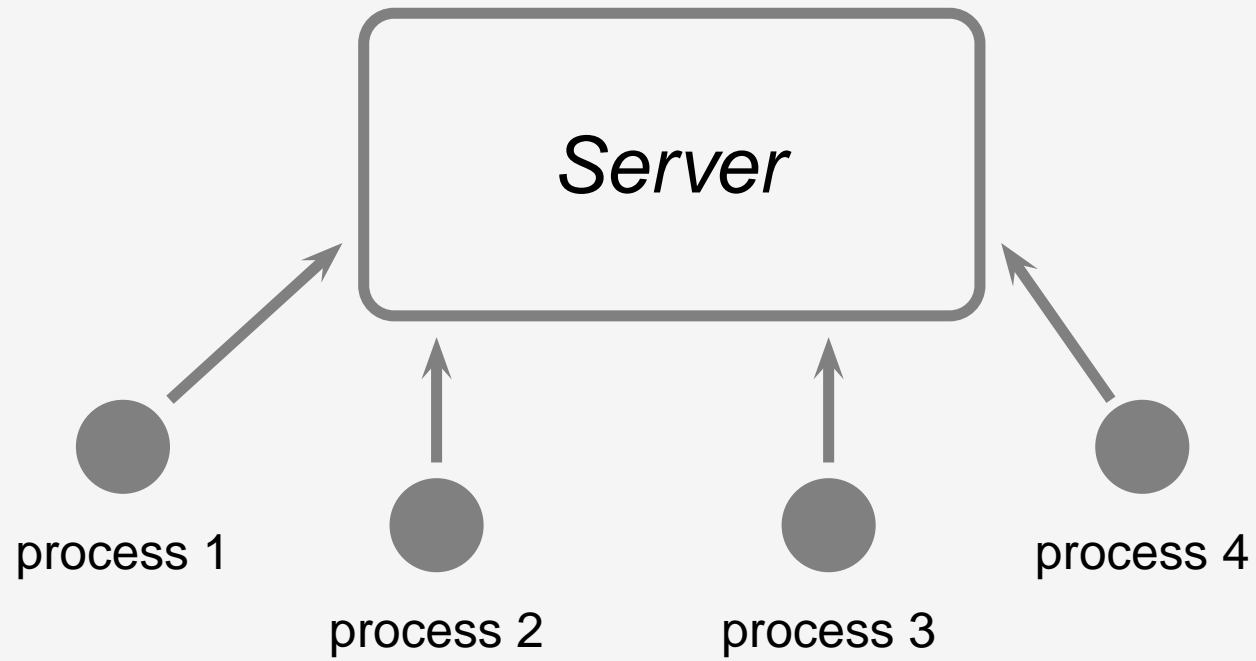
Feasible Automata for Two-Variable Logic with Successor on Data Words

Ahmet Kara, Thomas Schwentick, and Tony Tan

LATA 2012, A Coruña



Systems with Unboundedly Many Processes

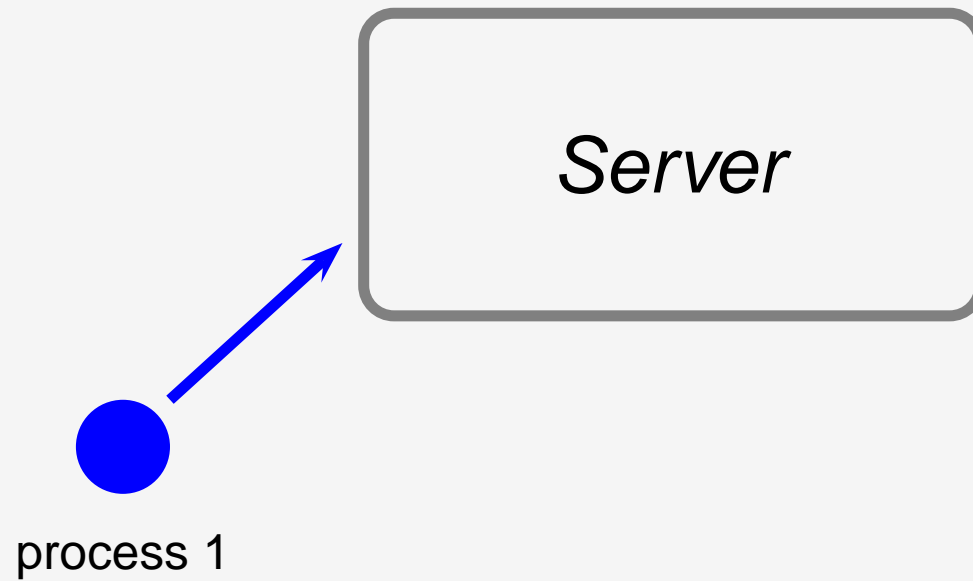


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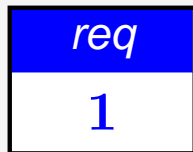


Server

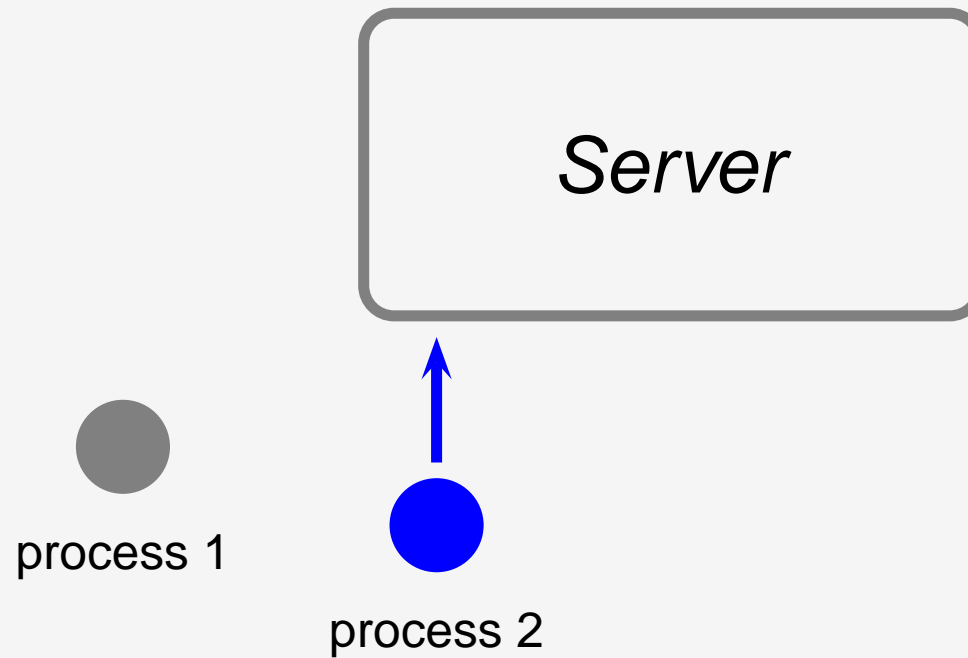
Systems with Unboundedly Many Processes



- A system run



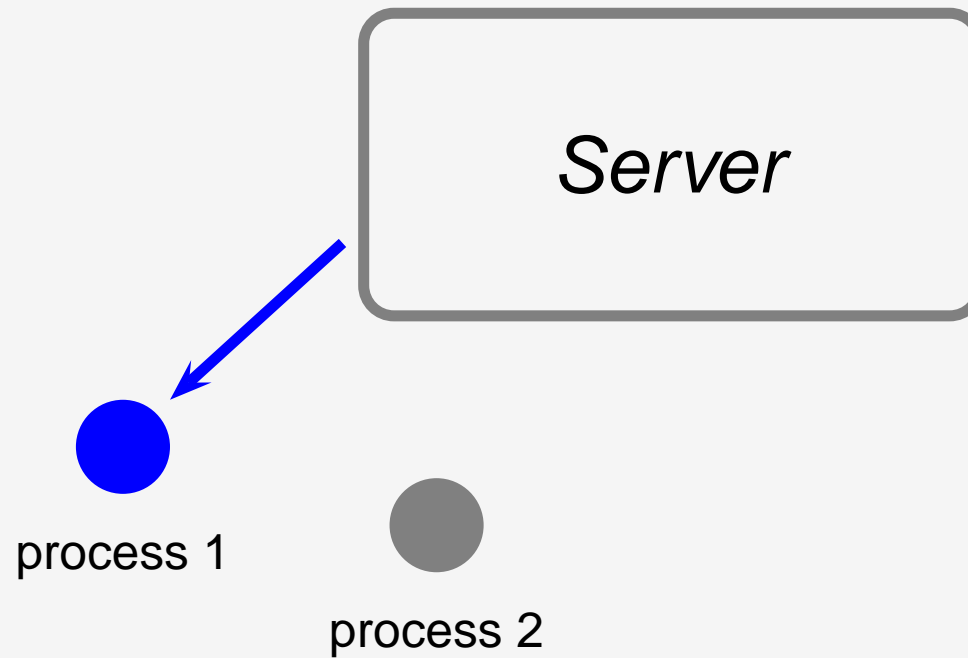
Systems with Unboundedly Many Processes



- A system run

<i>req</i>	<i>req</i>
1	2

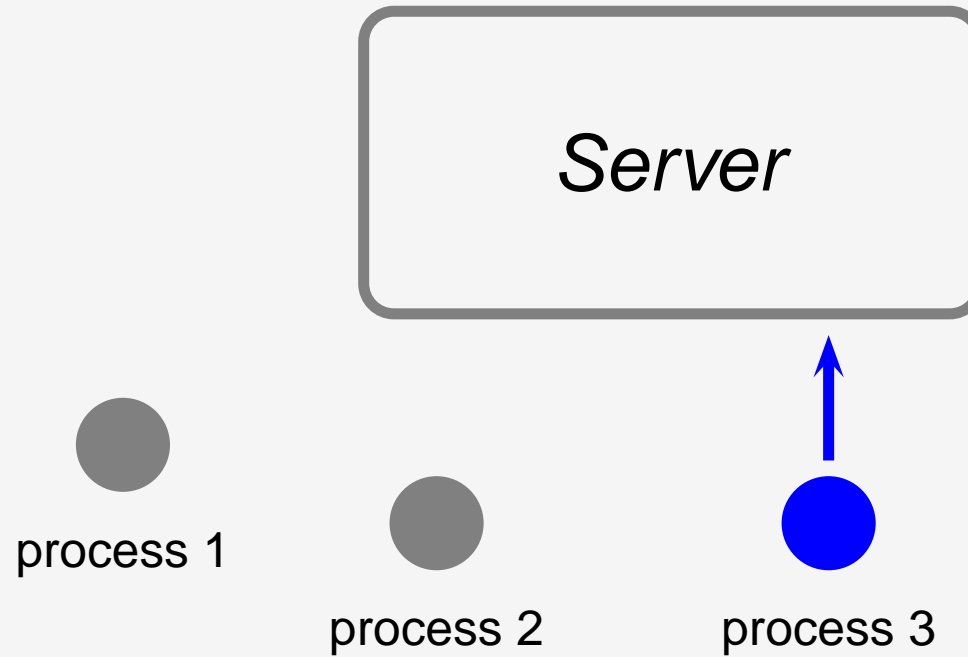
Systems with Unboundedly Many Processes



- A system run

<i>req</i>	<i>req</i>	<i>ack</i>
1	2	1

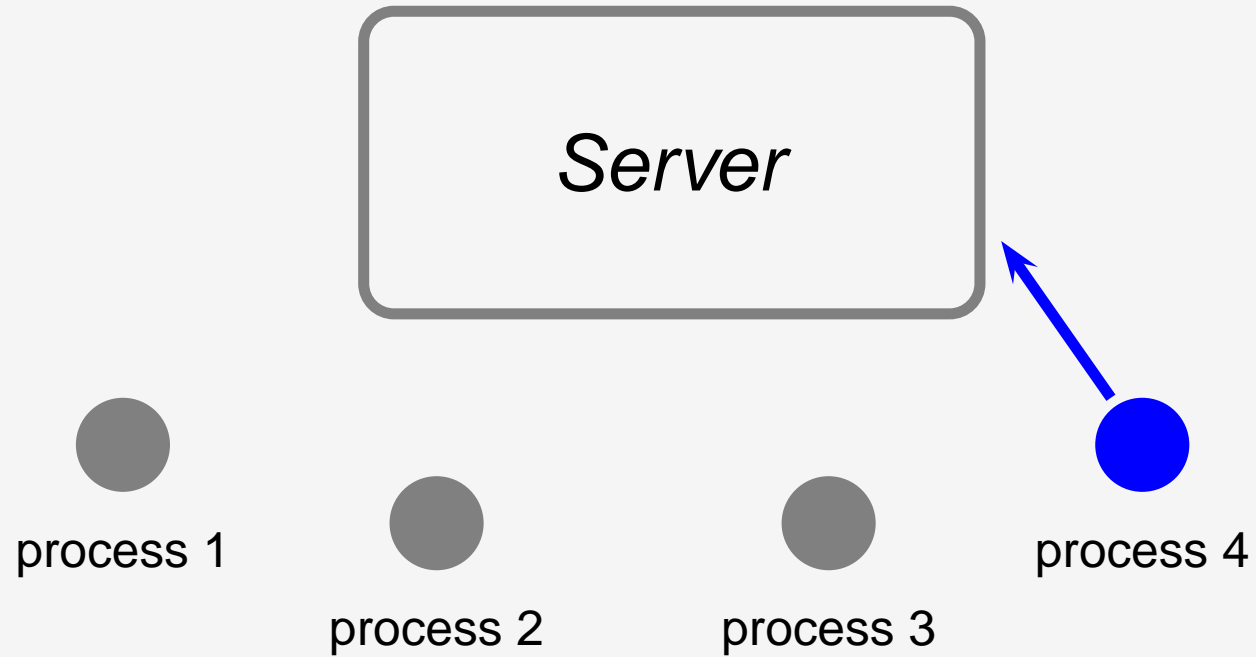
Systems with Unboundedly Many Processes



- A system run

<i>req</i>	<i>req</i>	<i>ack</i>	<i>req</i>
1	2	1	3

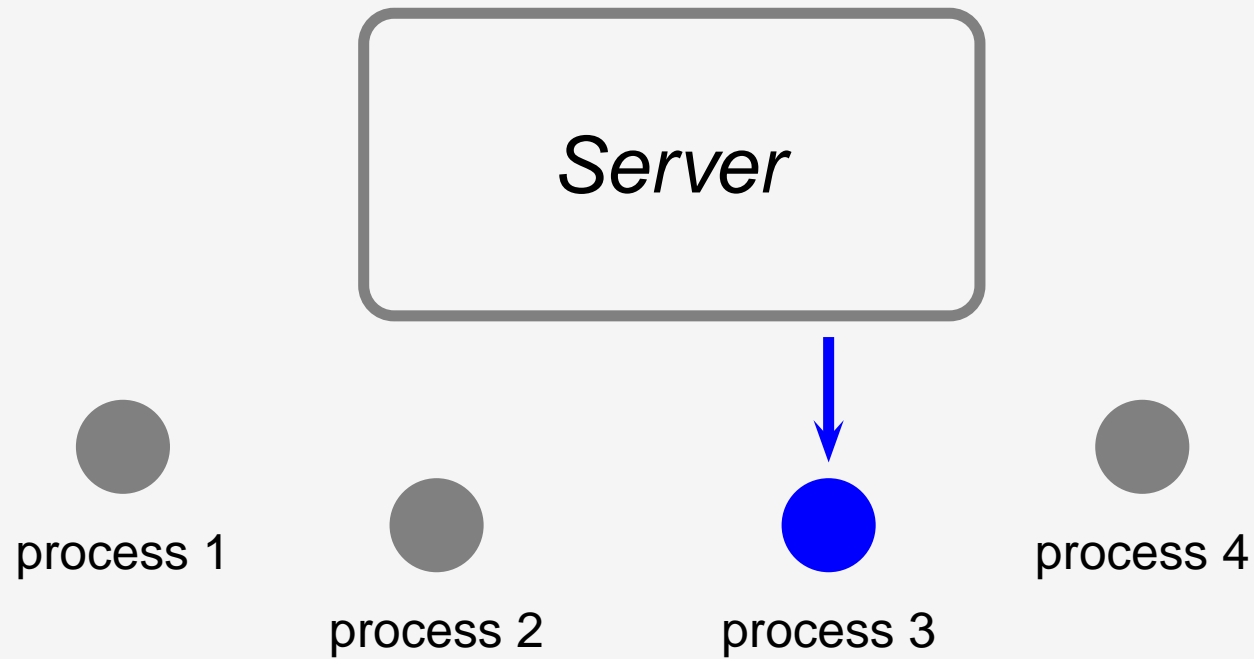
Systems with Unboundedly Many Processes



- A system run

<i>req</i>	<i>req</i>	<i>ack</i>	<i>req</i>	<i>req</i>
1	2	1	3	4

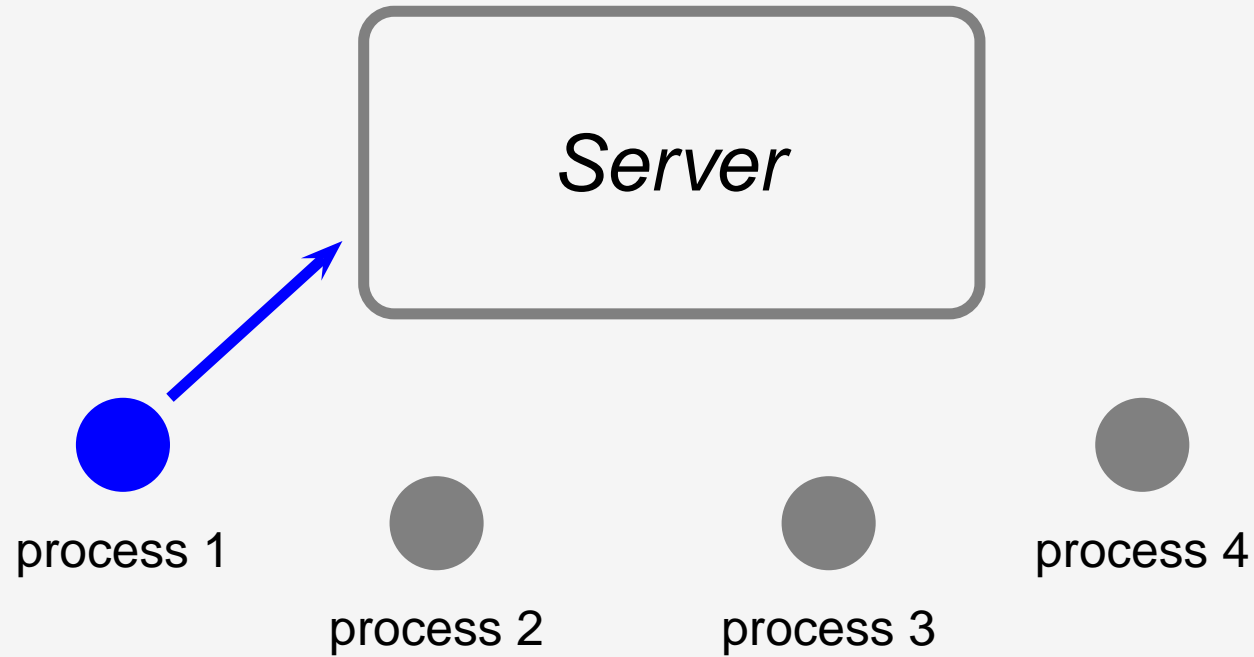
Systems with Unboundedly Many Processes



- A system run

<i>req</i>	<i>req</i>	<i>ack</i>	<i>req</i>	<i>req</i>	<i>ack</i>
1	2	1	3	4	3

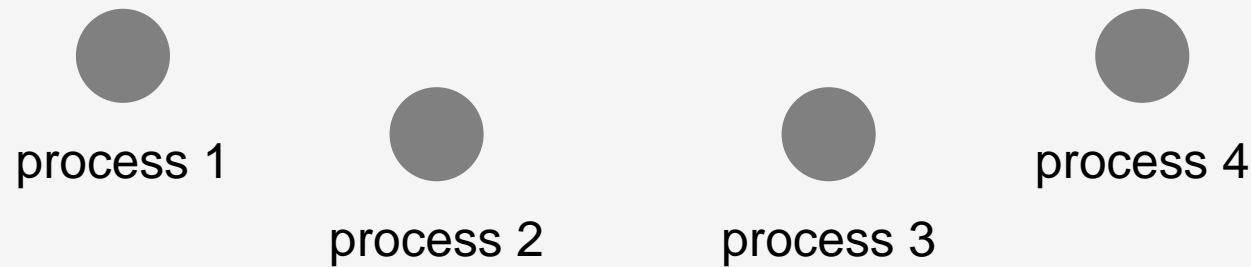
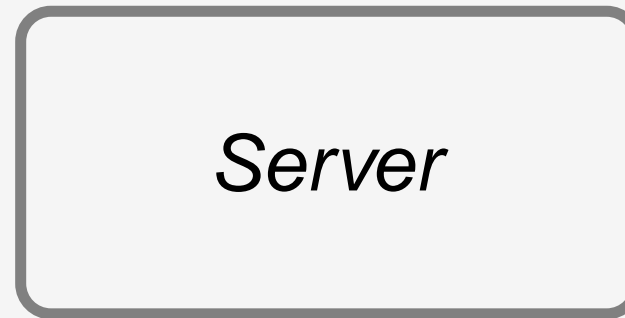
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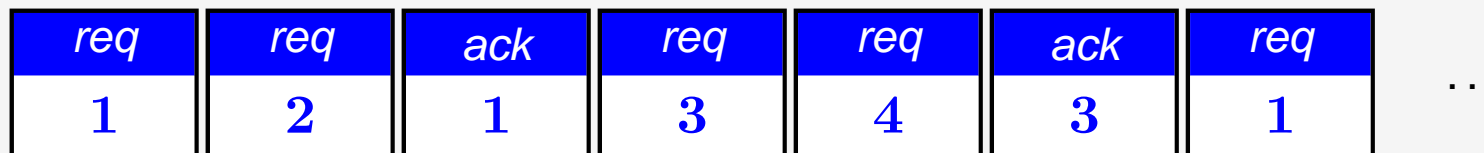
- A system run

<i>req</i>	<i>req</i>	<i>ack</i>	<i>req</i>	<i>req</i>	<i>ack</i>	<i>req</i>
1	2	1	3	4	3	1

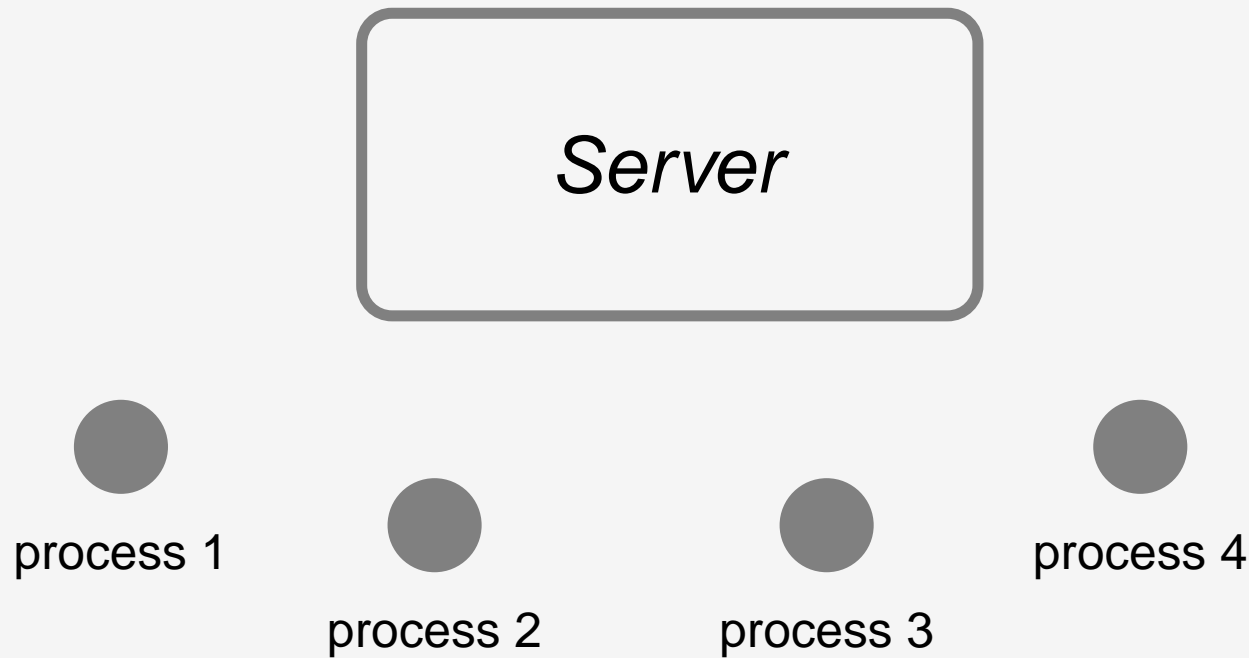
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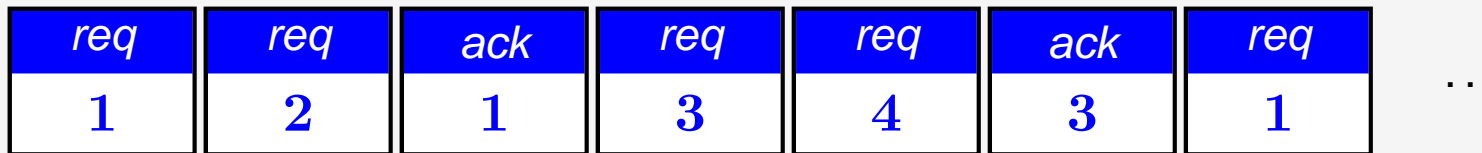
- A system run



Systems with Unboundedly Many Processes



- A system run



- A system property

„Every *req* of a process is followed by some *ack* for the same process.”

$$\forall x \exists y (req(x) \rightarrow (x < y \wedge ack(y) \wedge x \sim y))$$

Words and Data Words

A Word over $\Sigma = \{a, b, c\}$

c *c* *a* *c* *a* *b* *c* *b*

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<i>c</i>	<i>c</i>	<i>a</i>	<i>c</i>	<i>a</i>	<i>b</i>	<i>c</i>	<i>b</i>
----------	----------	----------	----------	----------	----------	----------	----------

A Data Word over $\Sigma = \{a, b, c\}$

<i>c</i>	<i>c</i>	<i>a</i>	<i>c</i>	<i>a</i>	<i>b</i>	<i>c</i>	<i>b</i>
1	4	3	2	2	3	7	2

Definition: Data Words

- Let
 - ▶ Σ be a **finite** alphabet
 - ▶ \mathcal{D} be an **infinite** set of data values
- $w \in (\Sigma \times \mathcal{D})^*$ is a **data word** over Σ

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<i>c</i>	<i>c</i>	<i>a</i>	<i>c</i>	<i>a</i>	<i>b</i>	<i>c</i>	<i>b</i>
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- $\mathcal{D} = \{1, 2, 3, \dots\}$

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Logics on Data Words – Data Logics

- Even very weak logics on data words have an undecidable satisfiability problem.
 - ▶ **LTL** in general not decidable [Demri et al. 06]
 - ▶ **FO³** (with only three variables) not decidable [Bojańczyk et al. 06]
 - ▶ **FO²** decidable but exact complexity not known [Bojańczyk et al. 06]

Known Results

DECIDABLE

$\text{EMSO}^2(\sim, \text{Suc}, \leq)$

.....

Known Results

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$\text{EMSO}^2(\sim, \text{Suc}, \leq)$

is captured



Data Automata



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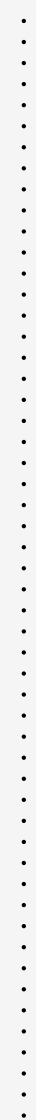
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Known Results

2NEXPTIME

$\text{EMSO}^2(\sim, \text{Suc})$

DECIDABLE

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Data Automata

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Existential Monadic Second Order Logic on Data Words

Existential Monadic Second Order Logic with two position variables ($\mathbf{EMSO}^2(\sim, \mathbf{Suc}, \leq)$):

- formulas are of the form
 $\exists M_1 \dots \exists M_n \varphi$
where φ is a first order formula

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- ▶ uses only two **position variables**

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Example

„Every a is followed by some b with the same data value.”

c	c	a	c	a	b	c	b
1	4	3	2	2	3	7	2

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$\forall x$

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y

$\forall x \exists y$

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		x			y		

$$\forall x \exists y (a(x))$$

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$x \longrightarrow y$

$$\forall x \exists y (a(x) \rightarrow (x < y))$$

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„There are two neighbouring positions with different data values.”

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c	c	a	c	a	b	c	b
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$x \quad y$

$$\exists x \exists y$$

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$x \quad y$

$$\exists x \exists y (\text{Suc}(x, y))$$

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$x \quad y$

$$\exists x \exists y (\text{Suc}(x, y) \wedge \neg(x \sim y))$$

Data Automata

Data Automata (DA) [Bojańczyk et al. 06]

A **data automaton** $\mathcal{D} = (\mathcal{B}, \mathcal{C})$ over an alphabet Σ consists of

- a base automaton \mathcal{B} : a nondeterministic letter-to-letter string transducer with input alphabet Σ and output alphabet Γ
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$(\mathcal{B}, \mathcal{C})$

σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7	σ_8
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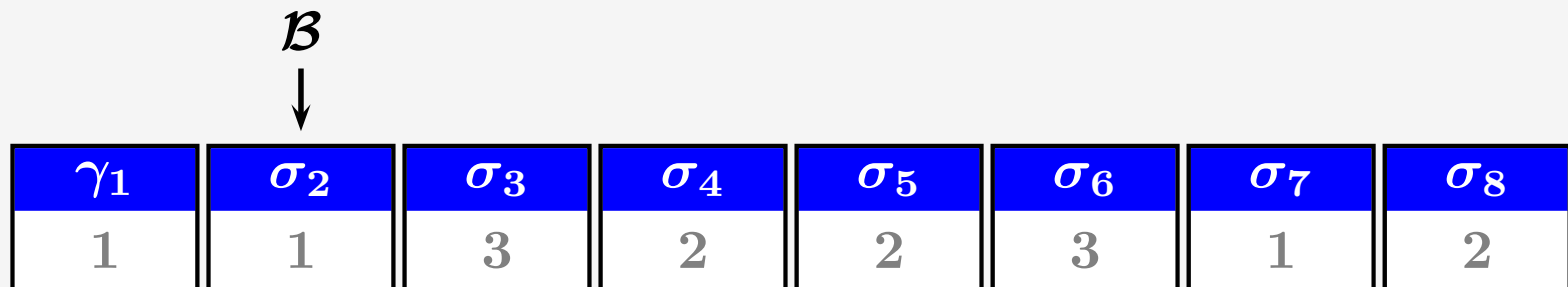
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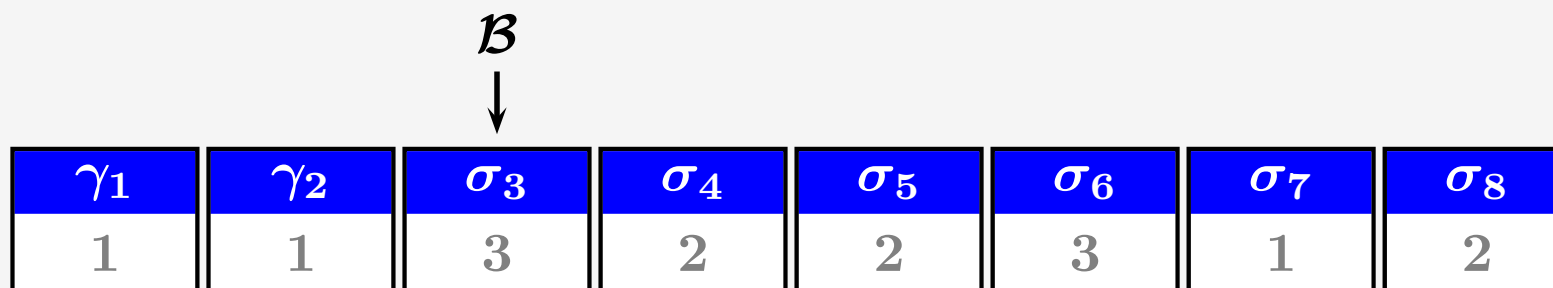
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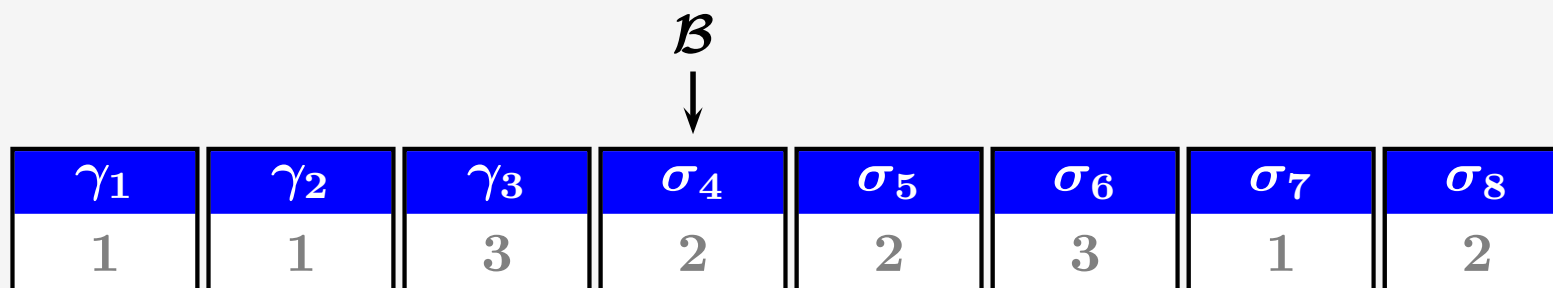
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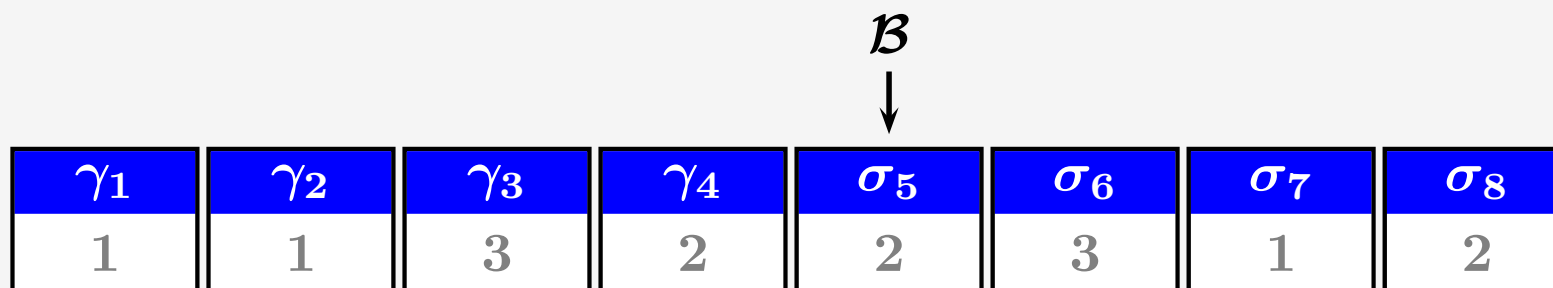
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$(\mathcal{B}, \mathcal{C})$

γ_1	γ_2	γ_3	γ_4	γ_5	σ_6	σ_7	σ_8
1	1	3	2	2	3	1	2

\mathcal{B}
↓

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$(\mathcal{B}, \mathcal{C})$

γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	σ_7	σ_8
1	1	3	2	2	3	1	2

\mathcal{B}
↓

Data Automata

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1	1	3	2	2	3	1	2

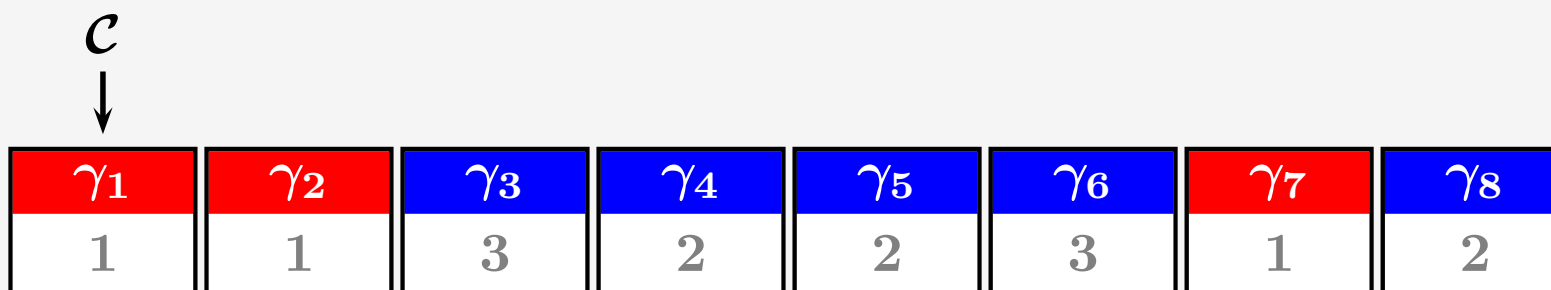
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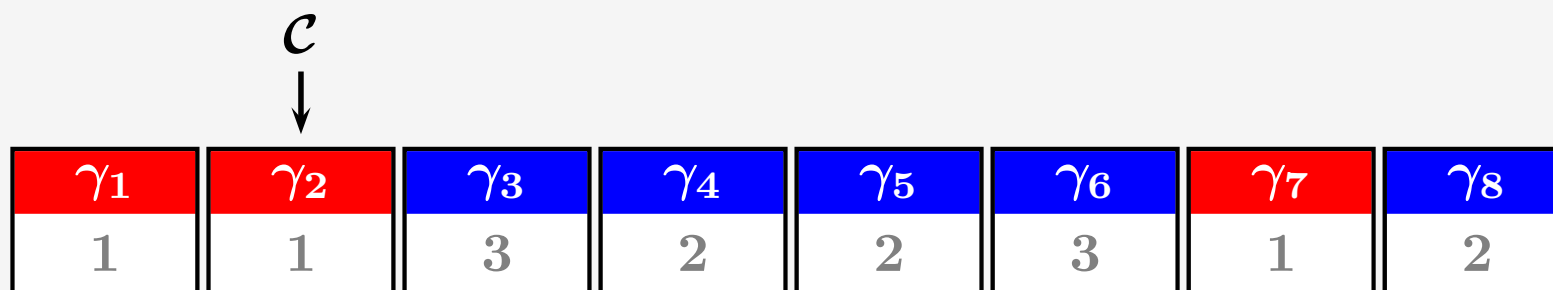
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$(\mathcal{B}, \mathcal{C})$

γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7	γ_8
1	1	3	2	2	3	1	2

\mathcal{C}
↓

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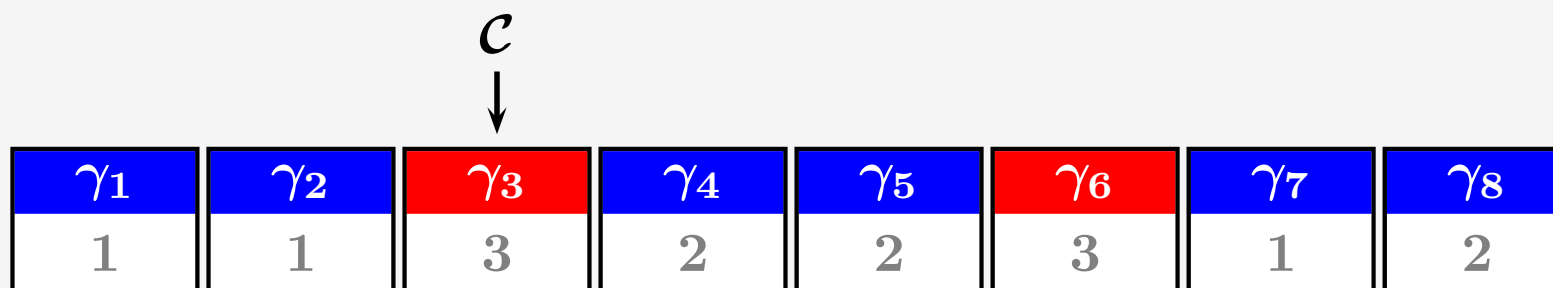
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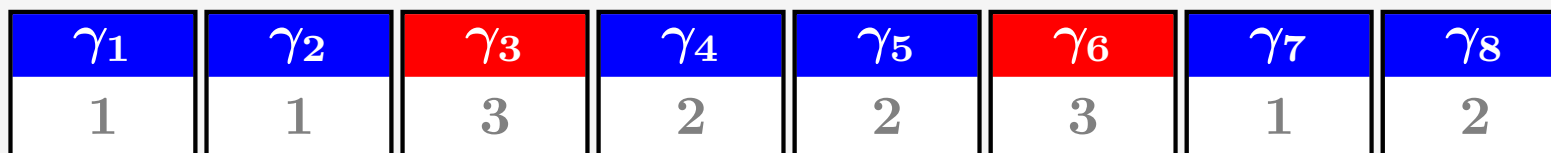
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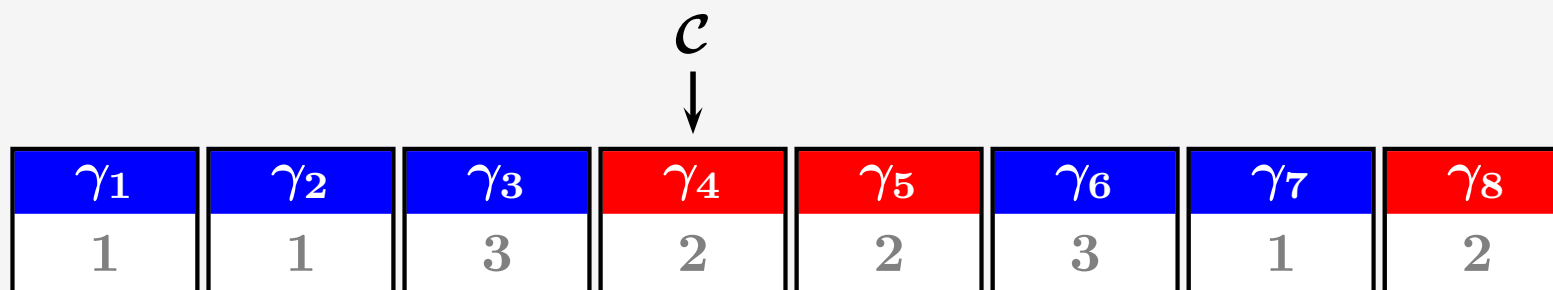
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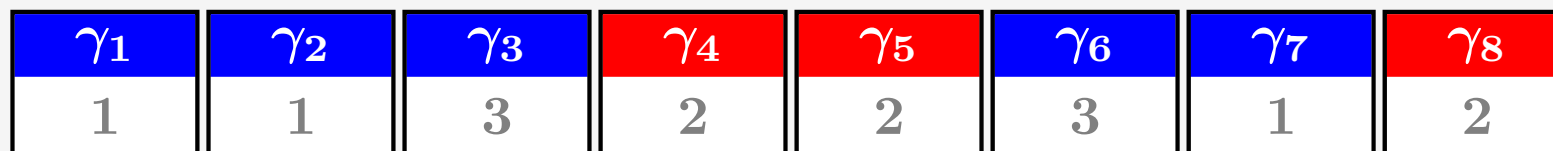
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↓

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Example

„There is an a which is followed by some b with a different data value.”

$(\mathcal{B}, \mathcal{C})$

c	c	a	c	a	b	c	b
1	1	3	2	2	3	1	2

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$(\mathcal{B}, \mathcal{C})$

\mathcal{B}



c	c	a	c	a	b	c	b
1	1	3	2	2	3	1	2

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$(\mathcal{B}, \mathcal{C})$

\mathcal{B}
↓

—	c	a	c	a	b	c	b
1	1	3	2	2	3	1	2

Data Automata

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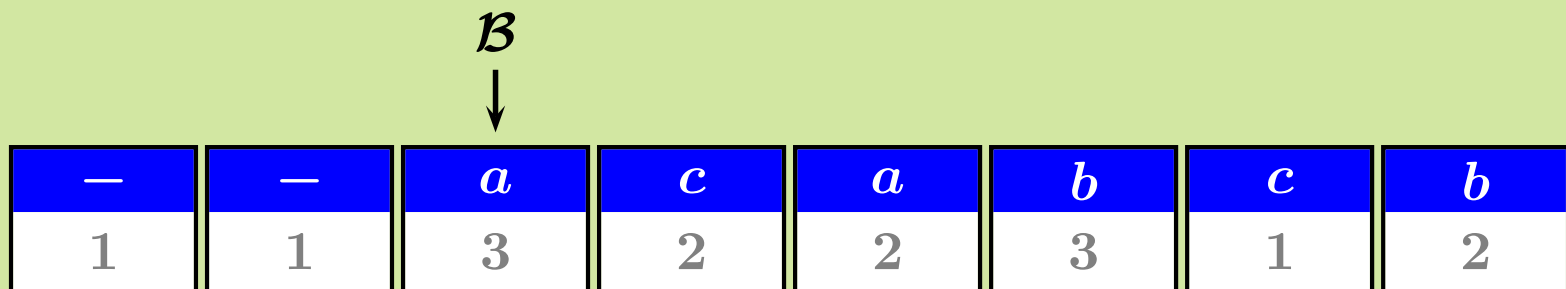
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Data Automata

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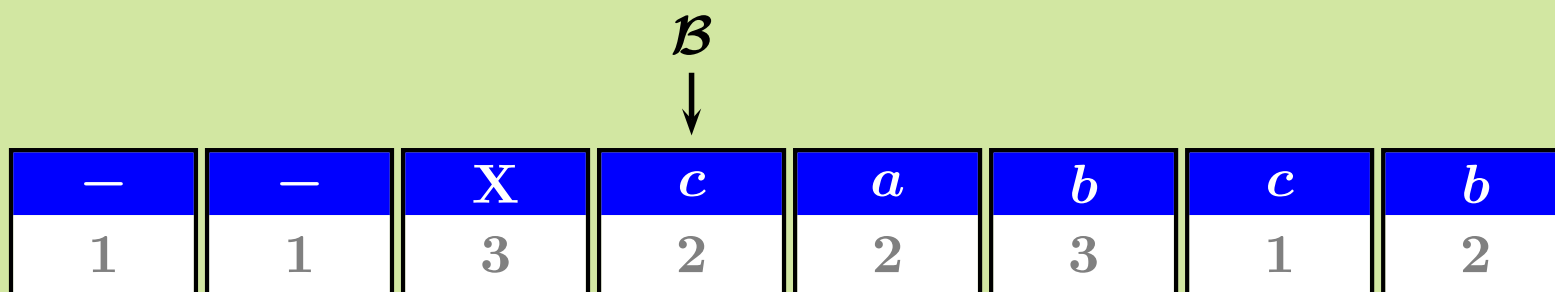
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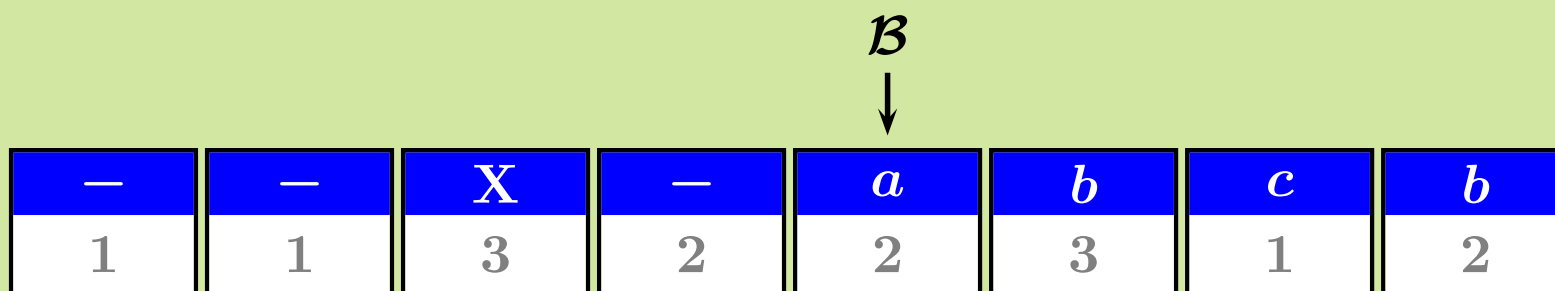
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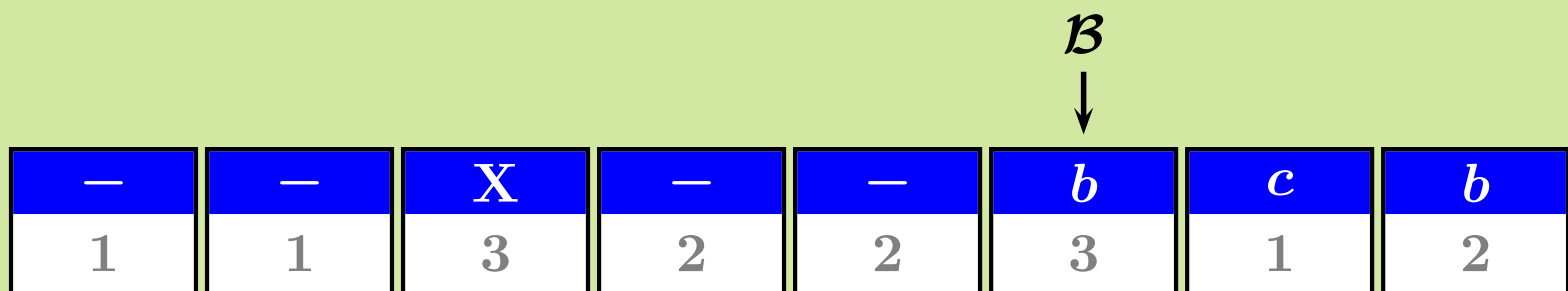
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$(\mathcal{B}, \mathcal{C})$

—	—	X	—	—	—	c	b
1	1	3	2	2	3	1	2

\mathcal{B}
↓

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$(\mathcal{B}, \mathcal{C})$

—	—	X	—	—	—	—	X
1	1	3	2	2	3	1	2

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1	1	3	2	2	3	1	2

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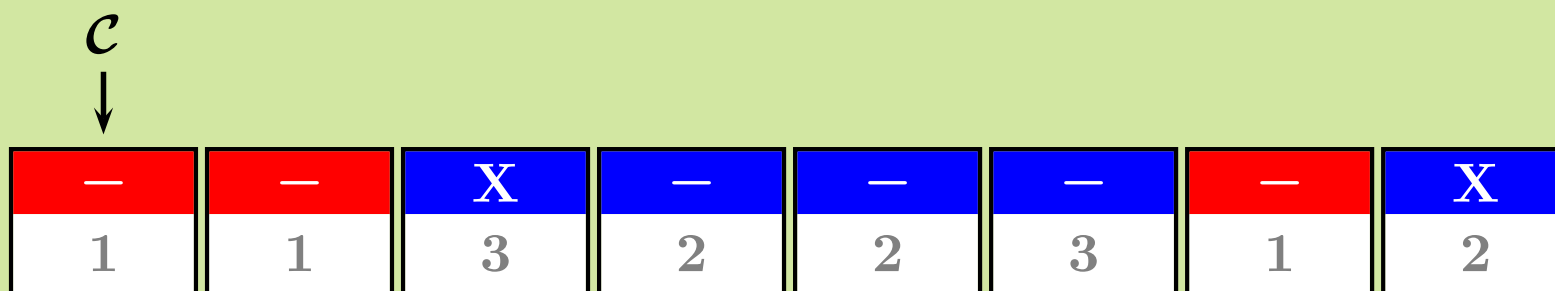
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$(\mathcal{B}, \mathcal{C})$



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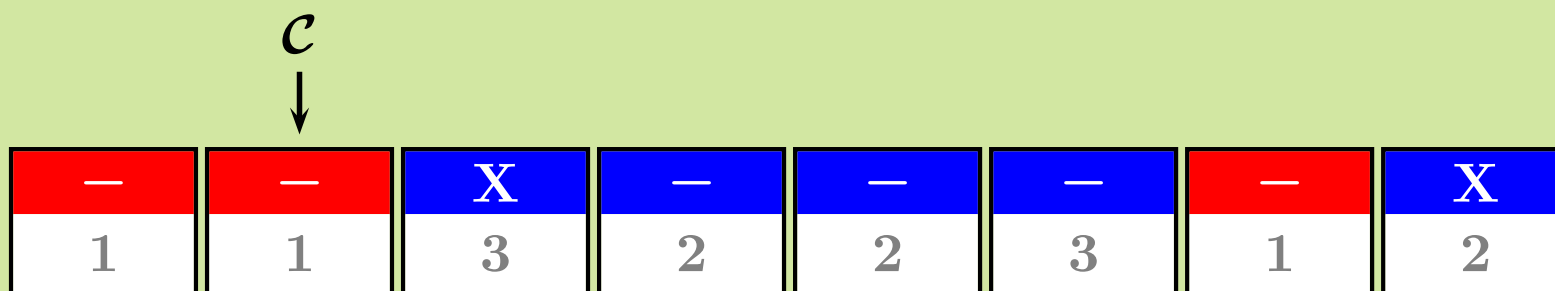
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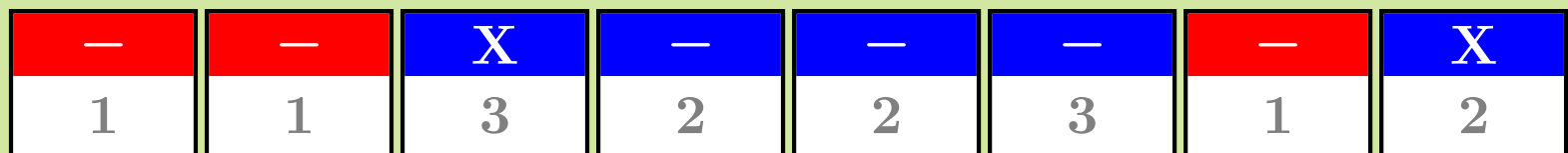
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$(\mathcal{B}, \mathcal{C})$

—	—	X	—	—	—	—	X
1	1	3	2	2	3	1	2

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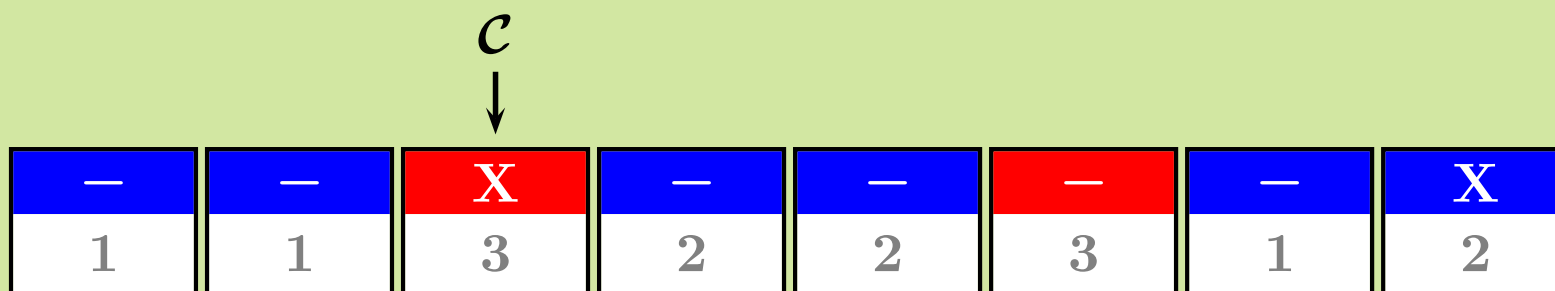
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$(\mathcal{B}, \mathcal{C})$



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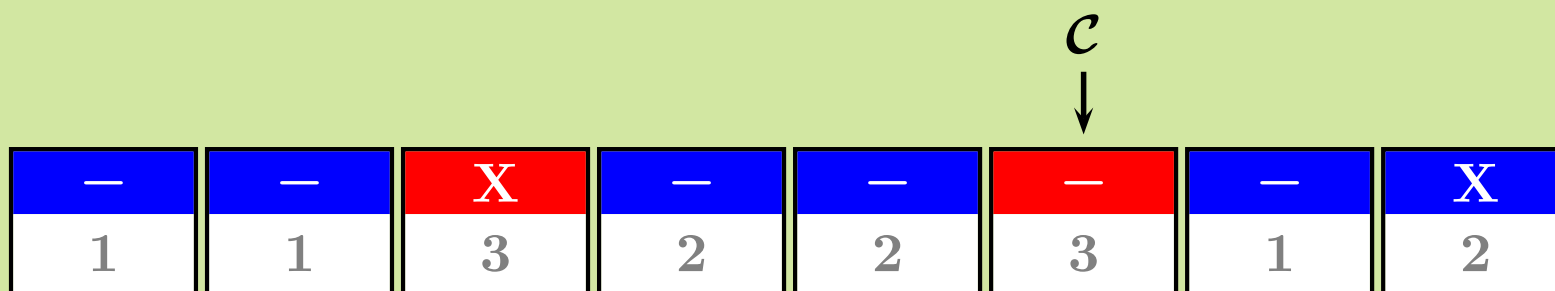
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Example

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$(\mathcal{B}, \mathcal{C})$

—	—	X	—	—	—	—	X
1	1	3	2	2	3	1	2

Data Automata

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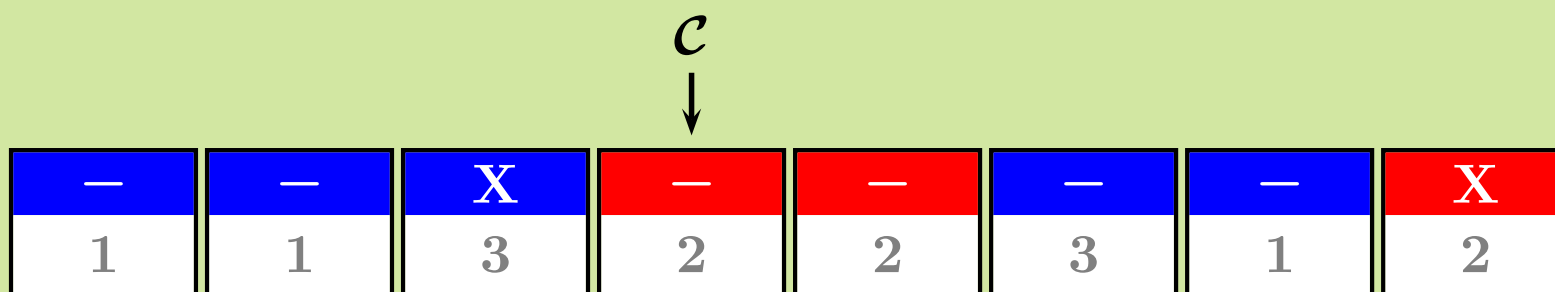
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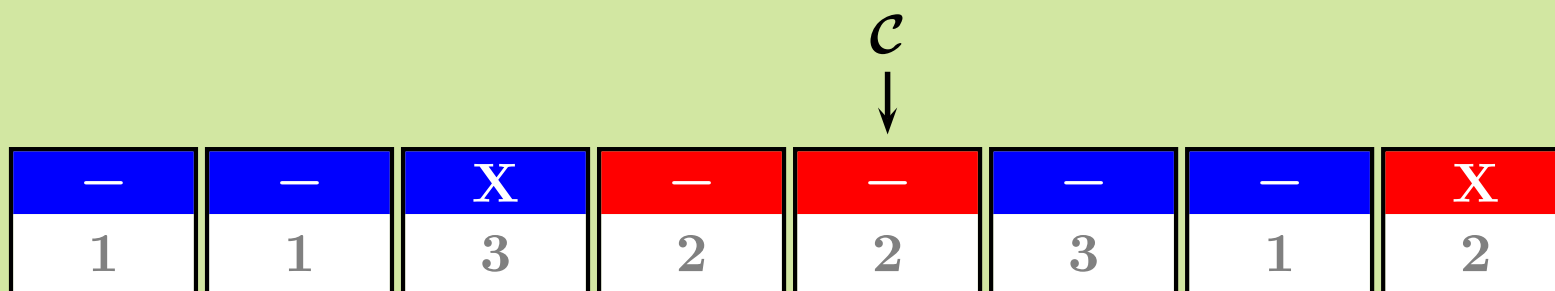
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$(\mathcal{B}, \mathcal{C})$

—	—	X	—	—	—	—	X
1	1	3	2	2	3	1	2

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Example

„Every a is followed by some b with the same data value.”

$(\mathcal{B}, \mathcal{C})$

c	c	a	c	a	b	c	b
1	1	3	2	2	3	1	2

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Example

„Every a is followed by some b with the same data value.”

$(\mathcal{B}, \mathcal{C})$

\mathcal{B}
↓

c	c	a	c	a	b	c	b
1	1	3	2	2	3	1	2

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Example

„Every a is followed by some b with the same data value.”

$(\mathcal{B}, \mathcal{C})$

\mathcal{B}
↓

c	c	a	c	a	b	c	b
1	1	3	2	2	3	1	2

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$(\mathcal{B}, \mathcal{C})$

\mathcal{B}
↓

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1	1	3	2	2	3	1	2

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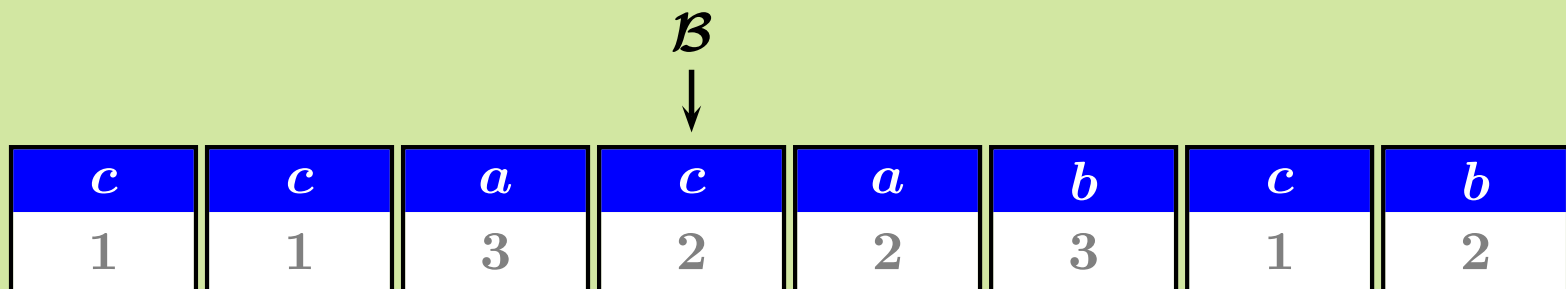
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Example

„Every a is followed by some b with the same data value.”

$(\mathcal{B}, \mathcal{C})$



Data Automata

Data Automata (DA) [Bojańczyk et al. 06]

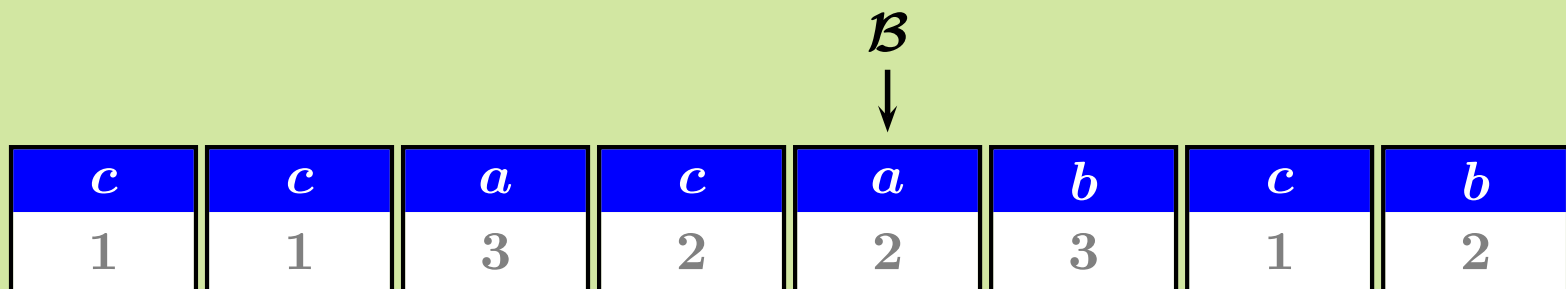
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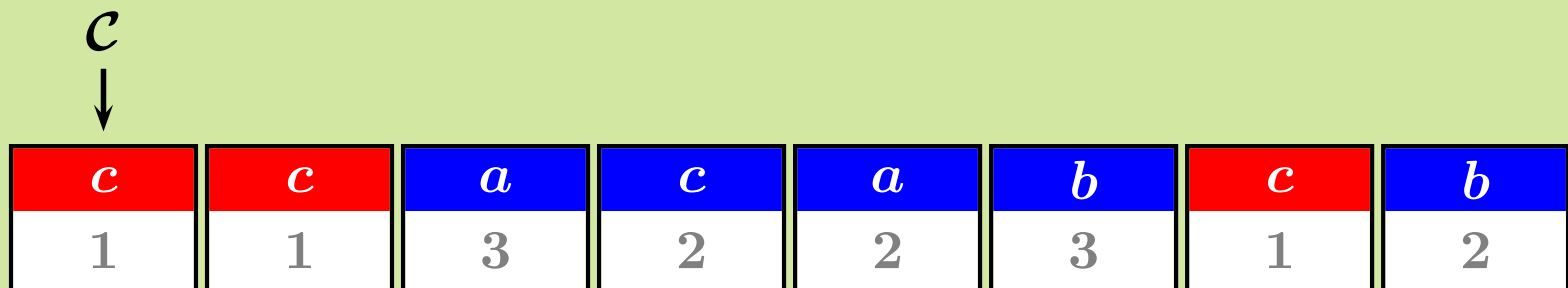
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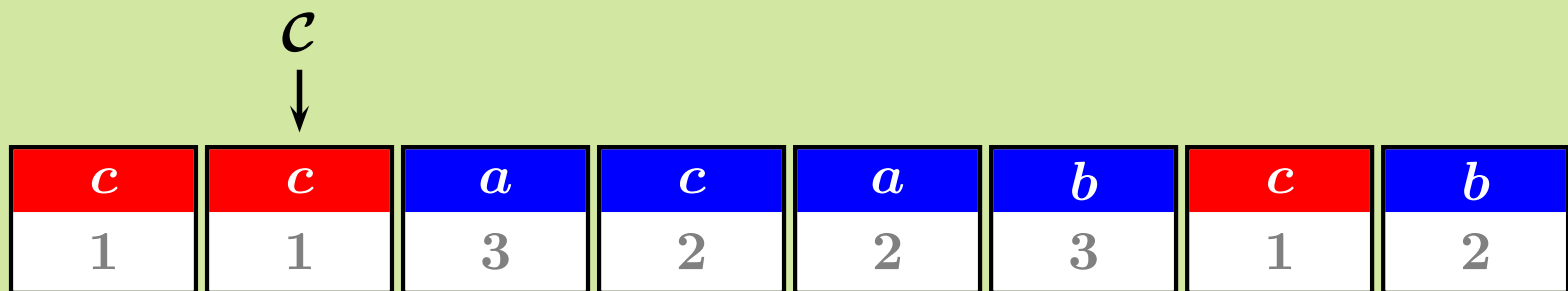
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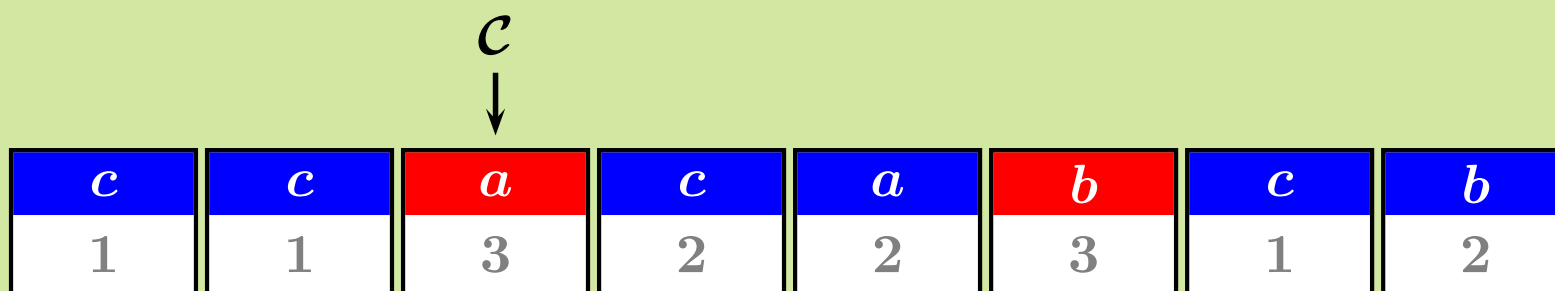
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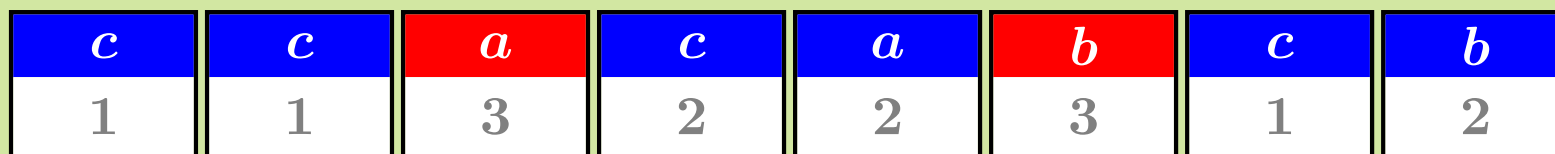
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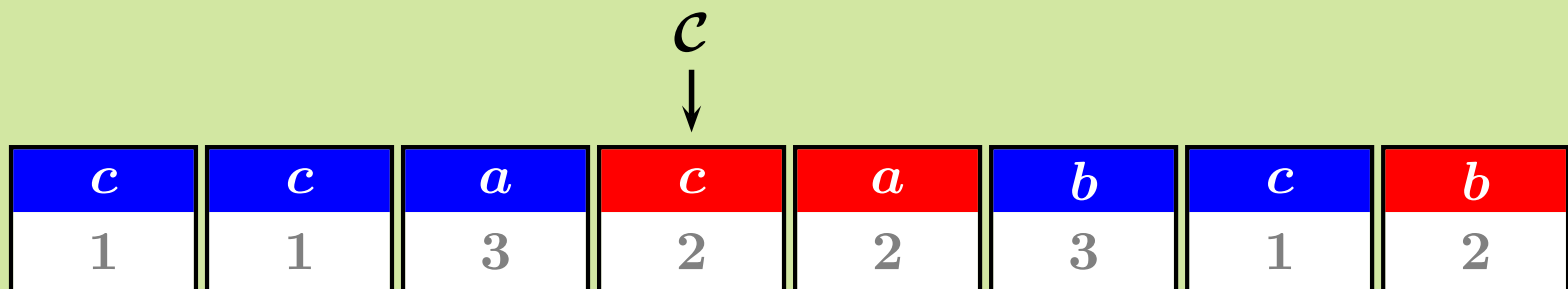
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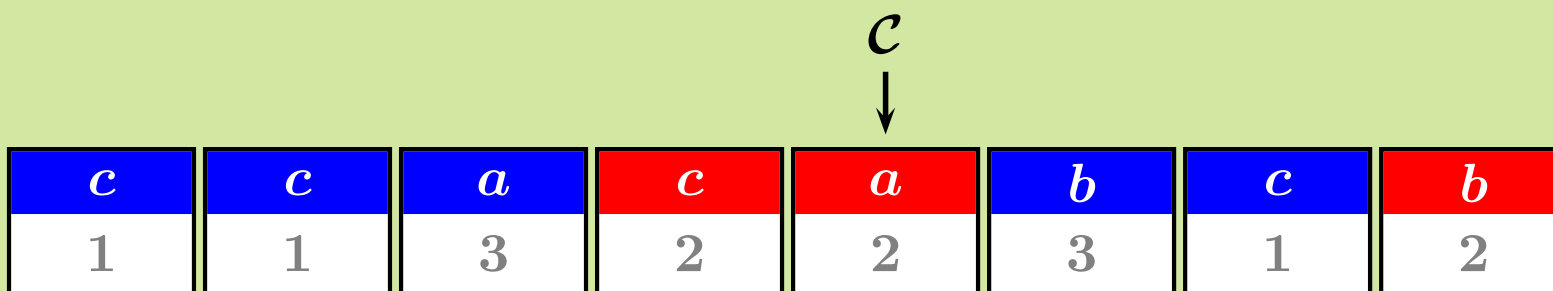
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Known Results

2NEXPTIME

DECIDABLE

$\text{EMSO}^2(\sim, \text{Suc})$



$\text{EMSO}^2(\sim, \text{Suc}, \leq)$

is captured

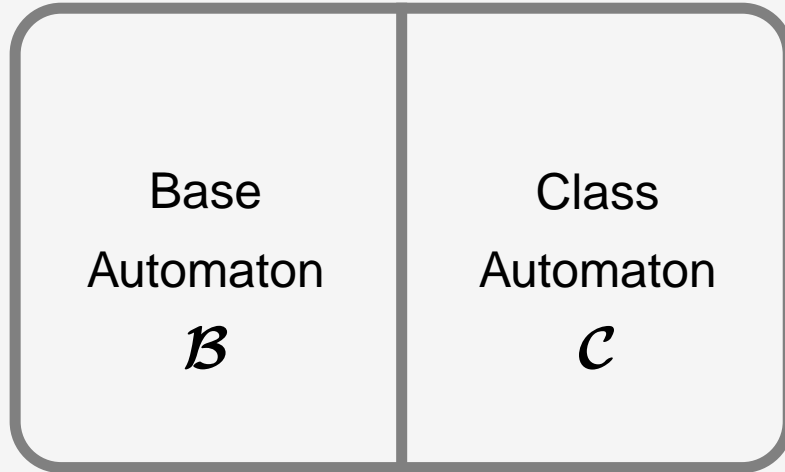


Data Automata

?

Weak Data Automata

Data Automata



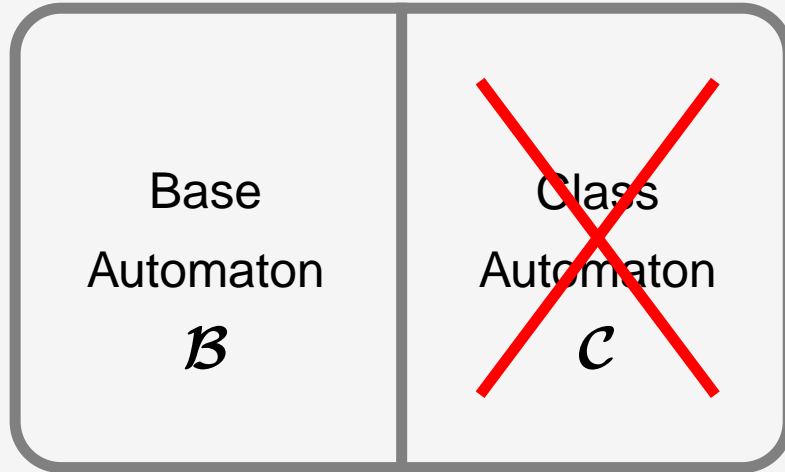
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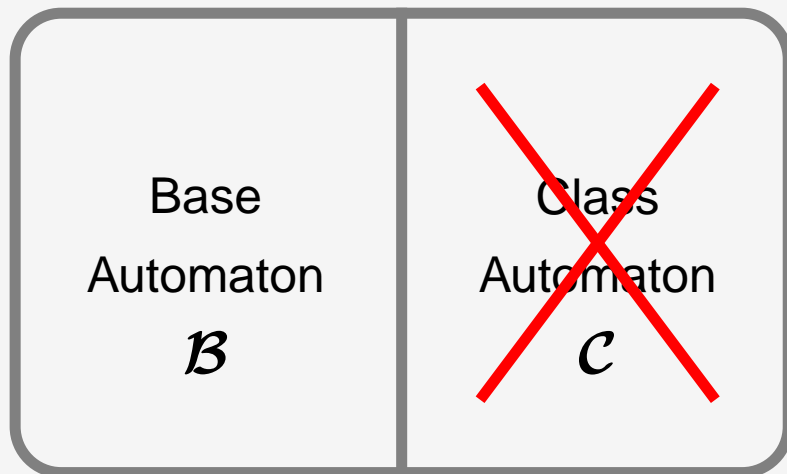
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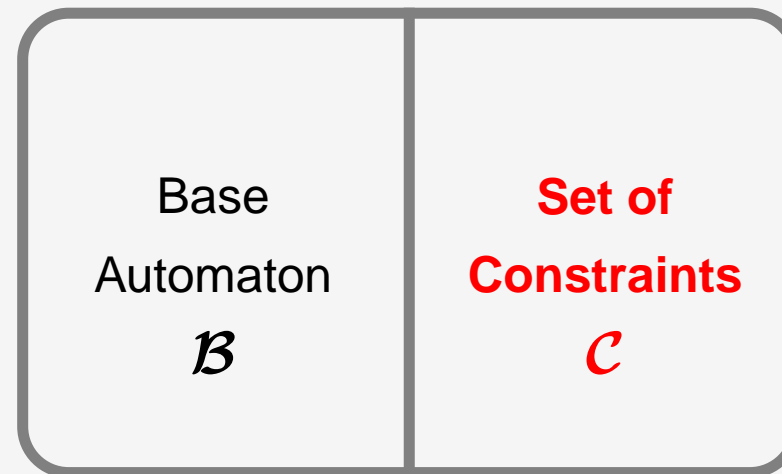
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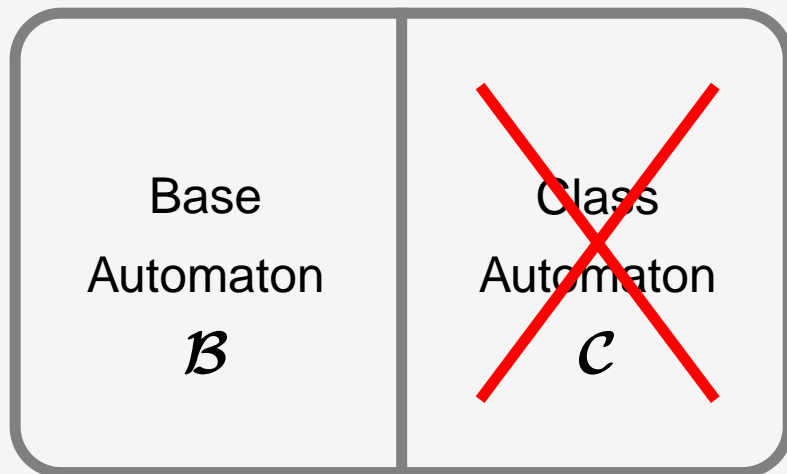
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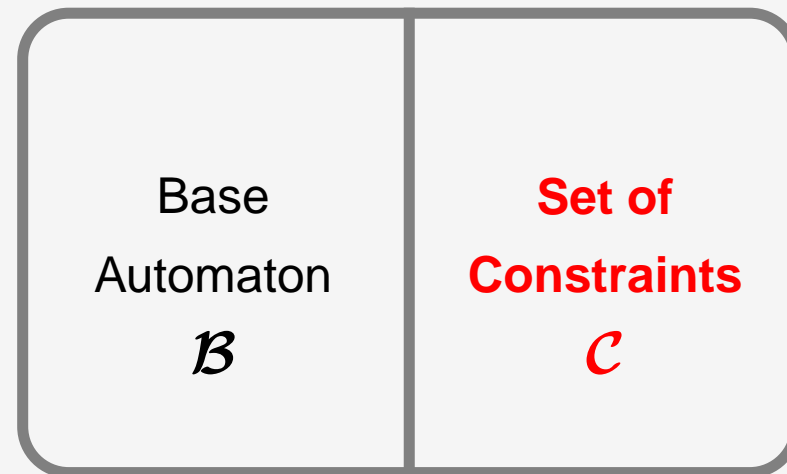


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Constraints on Data Words

Constraints on Data Words

- *key constraints*
 $\text{key}(\sigma)$: all σ -positions have different data values

Example

$w =$

<i>c</i>	<i>c</i>	<i>b</i>	<i>c</i>	<i>a</i>	<i>a</i>	<i>c</i>	<i>b</i>
1	1	4	2	2	3	3	1

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„There is an a and a b with different data values.”

$(\mathcal{B}, \mathcal{C})$

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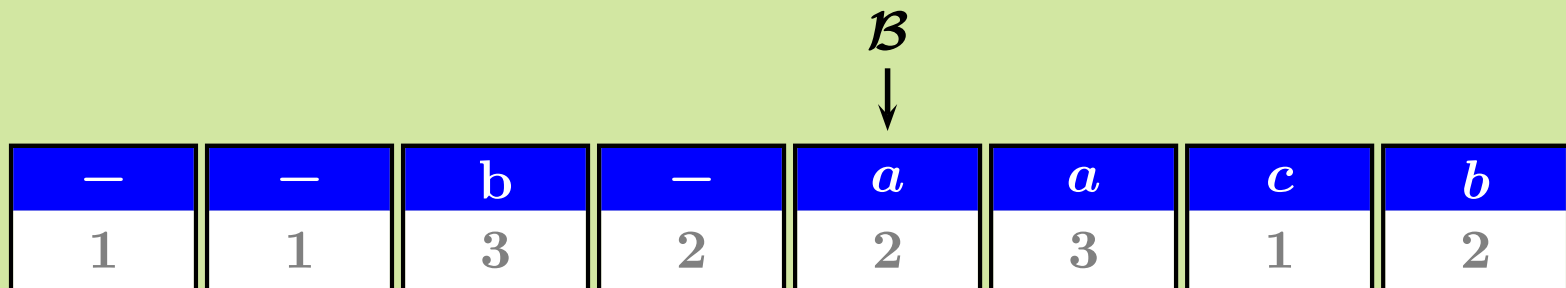
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—	—	b	—	a	—	—	\mathcal{B} ↓ b
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A **weak data automaton** $\mathcal{D} = (\mathcal{B}, \mathcal{C})$ over an alphabet Σ consists of

- a base automaton \mathcal{B} : a nondeterministic letter-to-letter string transducer with input alphabet Σ and output alphabet Γ
- a set \mathcal{C} of constraints over Γ

Example

„For every a there exists some b with the same data value.”

$(\mathcal{B}, \mathcal{C})$

\mathcal{B}



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- $\mathcal{C} = \{V(a) \subseteq V(b)\}$

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- Main difference between class automata and constraints:
constraints cannot access the order of positions

Our Results

2NEXPTIME

DECIDABLE

$\text{EMSO}^2(\sim, \text{Suc})$



$\text{EMSO}^2(\sim, \text{Suc}, \leq)$

is captured



Data Automata

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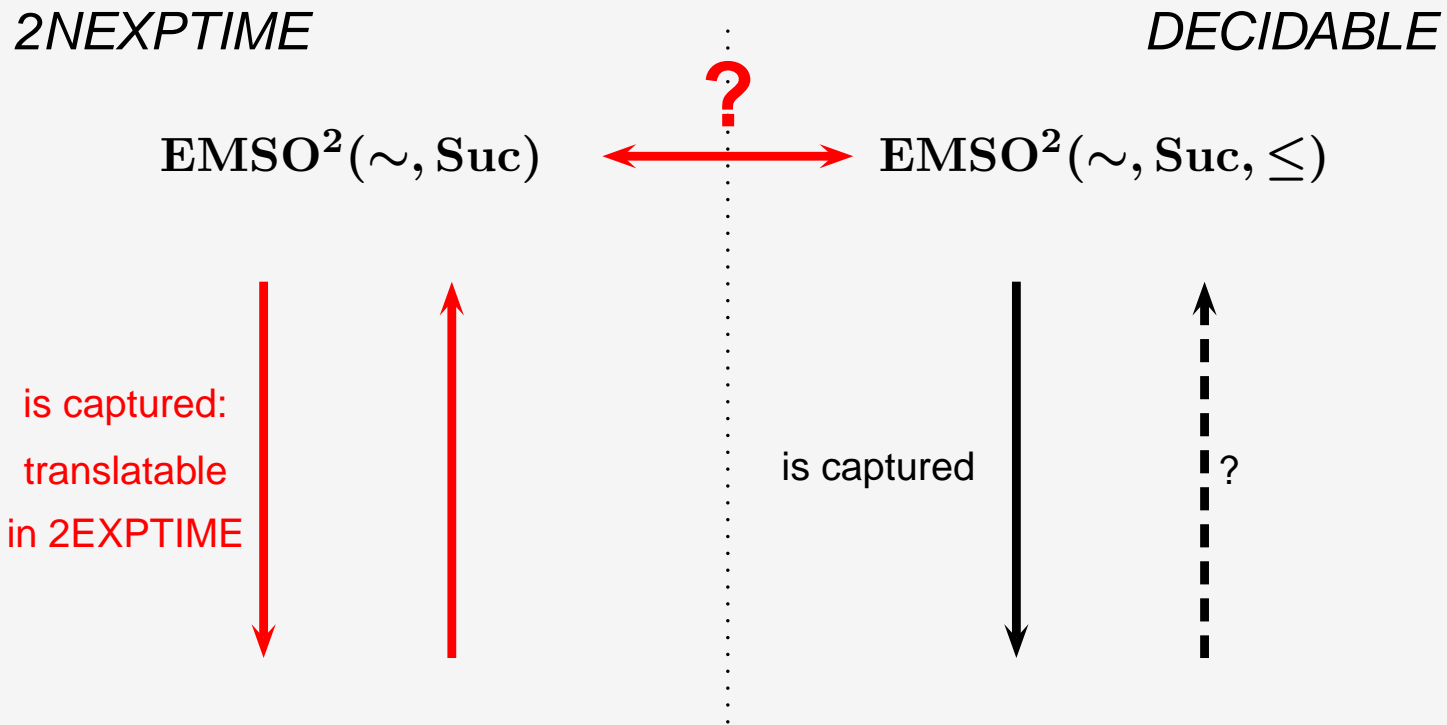
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Data Automata



Our Results



Theorem

WDA < DA

- inclusion: the class automaton can check all constraints
 - ▶ e.g. $\text{key}(a)$: class automaton checks that all class strings contain at most one a
- separating property: „Every a is followed by some b with the same data value.”

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Data Automata



Weak ω -Data Automata

Data ω -words

- Let
 - ▶ Σ be a finite alphabet
 - ▶ \mathcal{D} be an infinite set of data values
- $w \in (\Sigma \times \mathcal{D})^\omega$ is a **data ω -word** over Σ

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$E_\infty\text{MSO}^2(\sim, \text{Suc})$

Extension of $\text{EMSO}^2(\sim, \text{Suc})$:

- quantified sets can be restricted to be bound to infinite sets only

Our Results on Data ω -Words

2NEXPTIME

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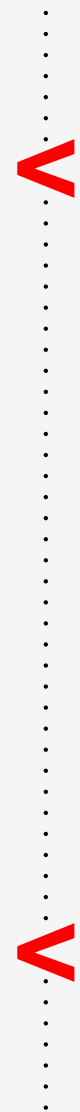
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Data ω -Automata



Conclusion

- **WDA**: automata model for $\mathbf{EMSO}^2(\sim, \mathbf{Suc})$ (and $\mathbf{E}_\infty\mathbf{MSO}^2(\sim, \mathbf{Suc})$) with reasonable complexity

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 - ▶ Are **WDA** the best automata model for these logics?

Conclusion

- **WDA**: automata model for $\mathbf{EMSO}^2(\sim, \mathbf{Suc})$ (and $\mathbf{E}_\infty\mathbf{MSO}^2(\sim, \mathbf{Suc})$) with reasonable complexity
- Open Questions:
 - ▶ Are **WDA** the best automata model for these logics?
 - ▶ What is the exact complexity of the emptiness problem for weak data automata?
 - The 2NEXPTIME upper bound yields only a 3NEXPTIME upper bound for the satisfiability of $\mathbf{EMSO}^2(\sim, \mathbf{Suc})$
 - which is in 2NEXPTIME [Niewerth et al. 09]