Reversible Multi-head Finite Automata Characterize Reversible Logarithmic Space

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Overview

- Motivation
- Informal acronyms: RMFAs and RTMs
- Simulating a logspace RTM with an RMFA
  - Encoding of configurations
  - Simulation of transitions
  - Read/write procedures
- Summing up

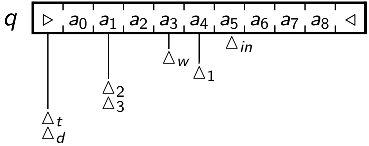
Motivation

Why this characterization?

- Recent open problem stated by Morita
  - Reversible PDAs are *weaker* than DPDAs
  - Reversible LBAs are *equivalent* to DLBAs
  - What about automata between these?
- $\mathcal{L}$(RMFA) is strong: palindromes, unary primes, DYCK(1)
- (D)MFAs characterize logspace very nicely
  - Logspace = constant amount of pointers
- Usual reversibilization trick won't work for RMFAs
  - No space for a history, or other *garbage data*
  - What to do if we *cannot* write at all?
- Nice use case for techniques from recent RTM work

Reversible Multi-head Finite Automata

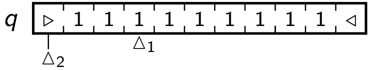
RMFA R

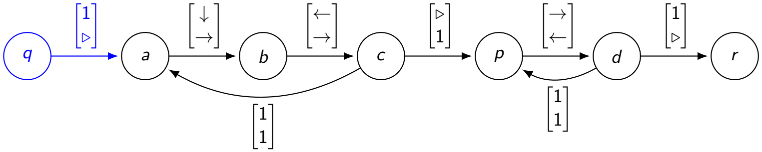


- Input word $w \in \Sigma^*$ in delimited form $\triangleright\, w \,\triangleleft$
- k heads (all start by pointing at $\triangleright$)
- Triple format transitions
  - symbol rules: $(q, [a, b, \triangleright, \cdots, c], p)$
  - move rules: $(q, [\leftarrow, \downarrow, \leftarrow, \cdots, \rightarrow], p)$
- Reversibility = forward + backward determinism
  - Fwd determinsm: $(q, s, \cdot)$ and $(q, t, \cdot)$ imply $s, t \in \Sigma^k$, $s \neq t$
  - Bwd determinsm: $(\cdot, s, p)$ and $(\cdot, t, p)$ imply $s, t \in \Sigma^k$, $s \neq t$

RMFA example

Doubling the unary counter in $\triangle_1$:





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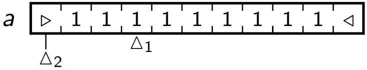

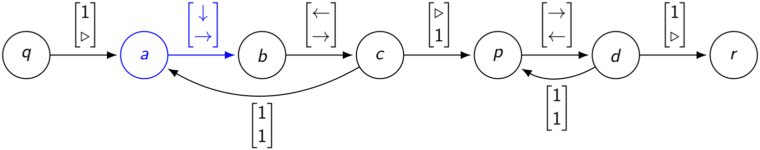

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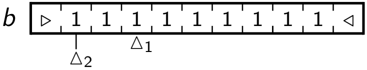

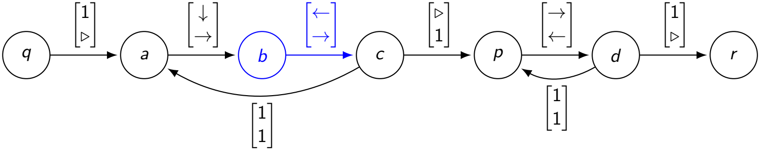

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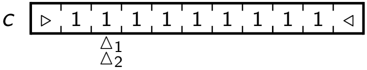

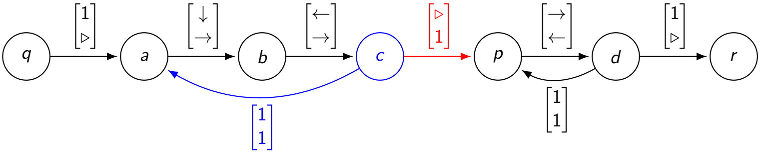

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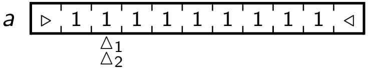

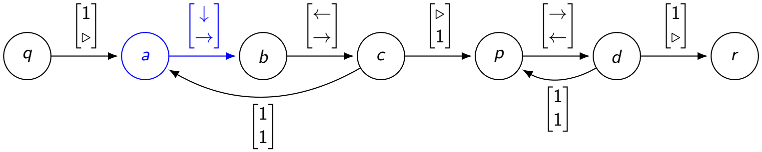

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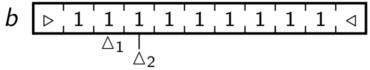

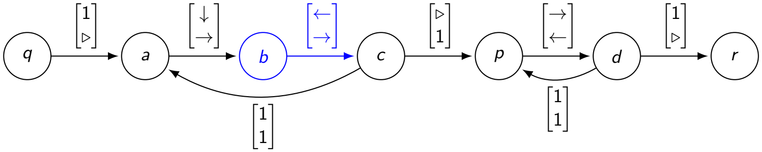

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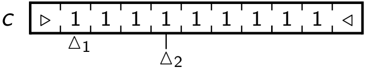

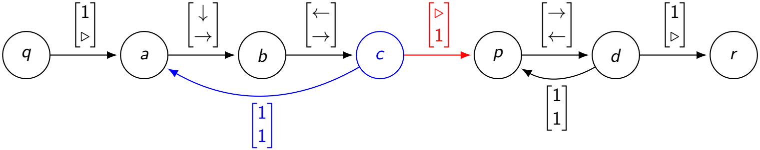

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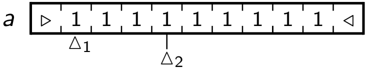

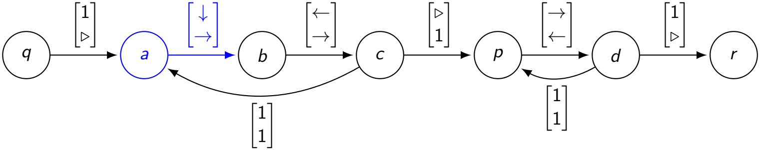

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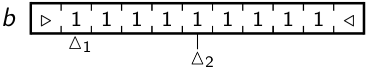

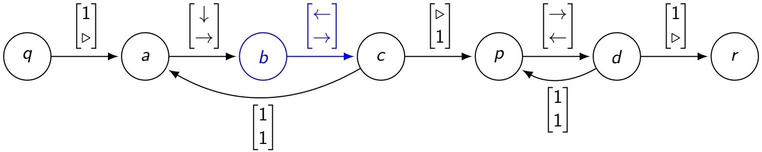

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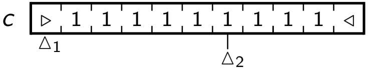

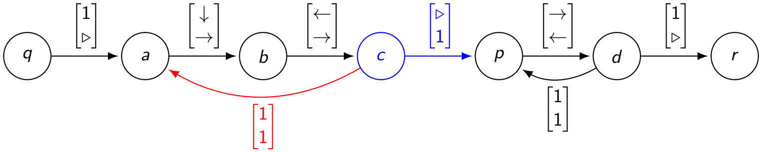

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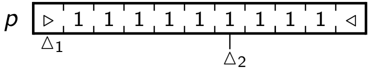

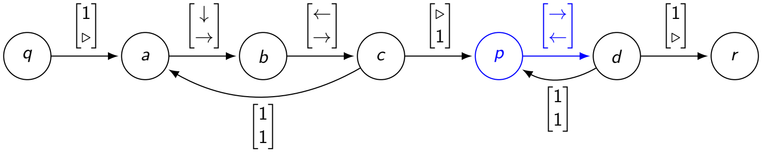

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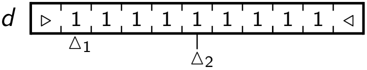

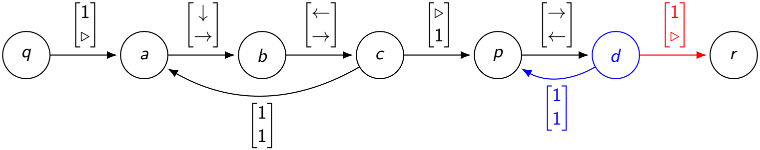

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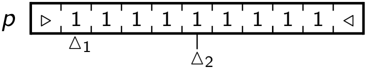

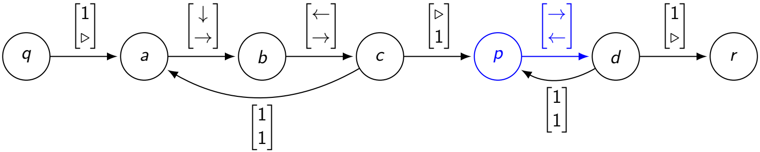

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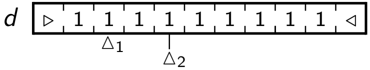

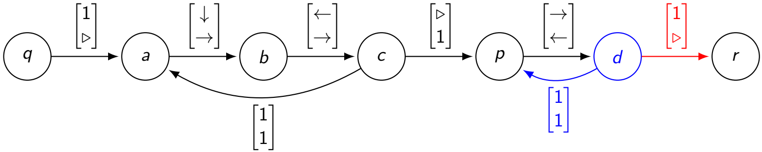

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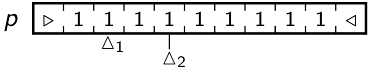

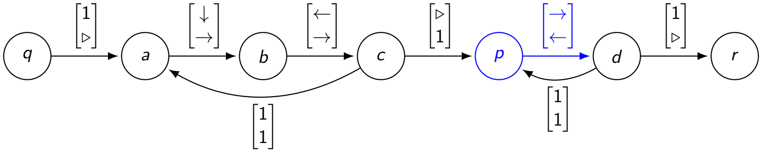

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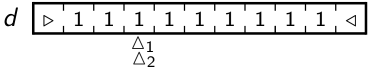

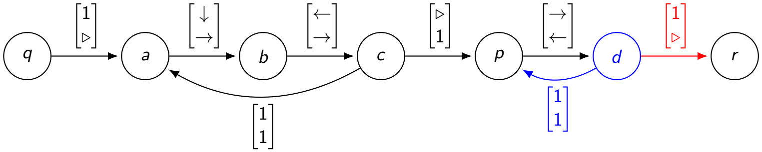

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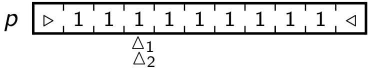

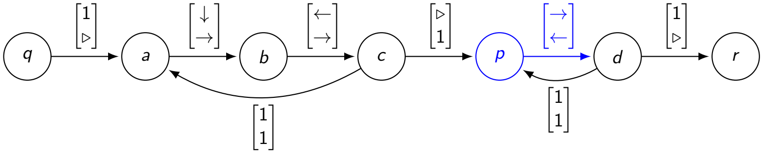

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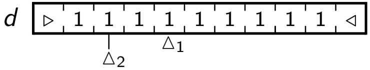

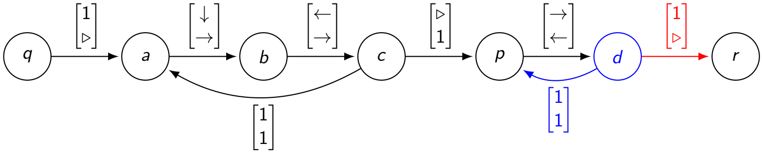

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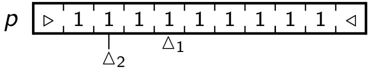

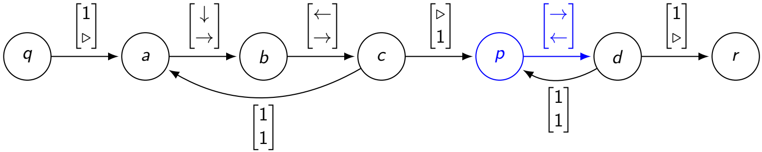

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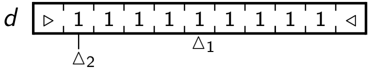

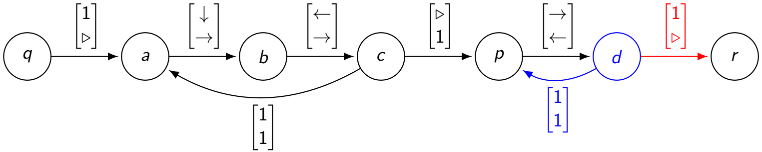

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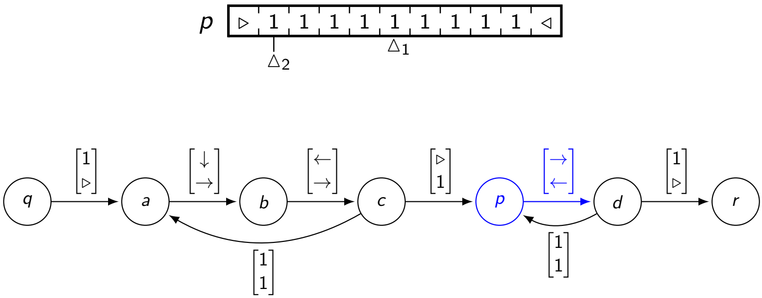

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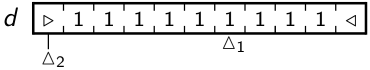

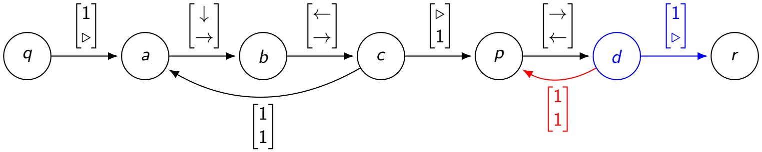

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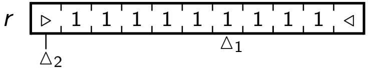
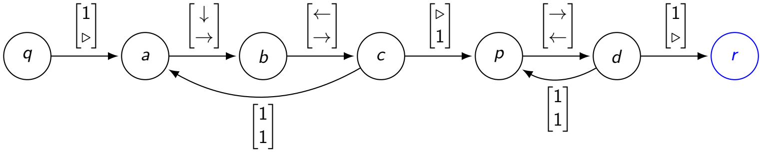
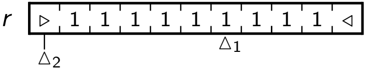




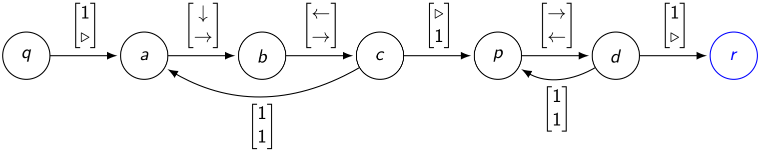
RMFA example: inversion

RMFAs are *invertible*: swap direction of transitions, swap $\leftarrow$ and $\rightarrow$

Doubling the unary counter in $\triangle_1$



Invert:



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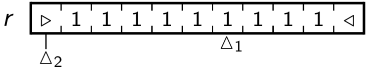

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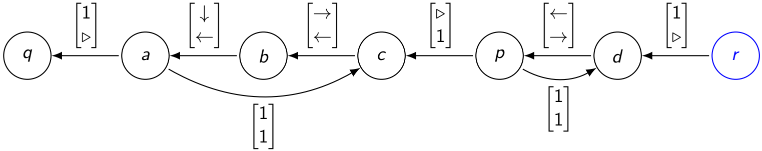
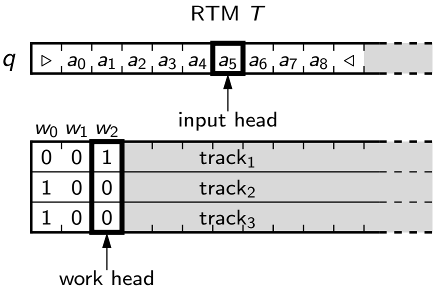


This is an RMFA for halving the unary counter in $\triangle_1$

Reversible Turing Machines



- Input tape as before, but only one input head.
- Read/write work tape: one head, k binary tracks.
  - Space usage measured on the work tape.
- Triple format transitions, but symbol rules can now write on the work tape: $(q, [a, 100 \mapsto 011], p)$.
- Reversibility as before.

What do we know & what shall we prove

$$
\begin{array}{ccc}
\mathcal{L}(\text{RMFA}) & \overset{\text{Morita}}{=\!=} & \mathcal{L}(\text{DMFA}) \\
\| \text{ Here} & & \| \text{ Hartmanis'72} \\
\text{RevSPACE}(\log n) & \overset{\text{Lange } et\ al.\text{'00}}{=\!=} & \text{DSPACE}(\log n)
\end{array}
$$

Proving $\mathcal{L}(\text{RMFA}) = \text{RevSPACE}(\log n)$

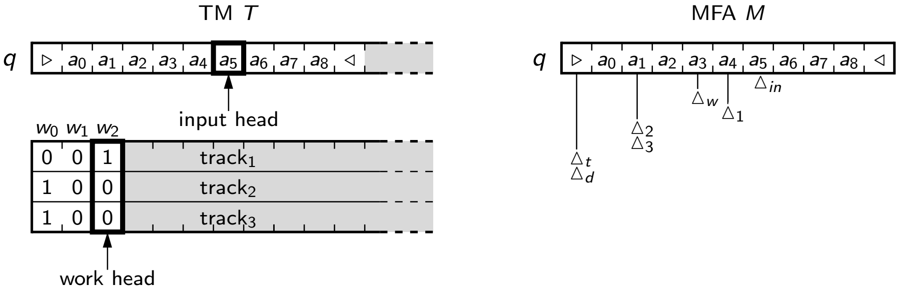
Will show $\text{RevSPACE}(\log n) \subseteq \mathcal{L}(\text{RMFA})$.

Proof: Simulate a logspace RTM with an RMFA.

- Encoding of RTM configurations
- Transition simulation
- Tape read/write simulation

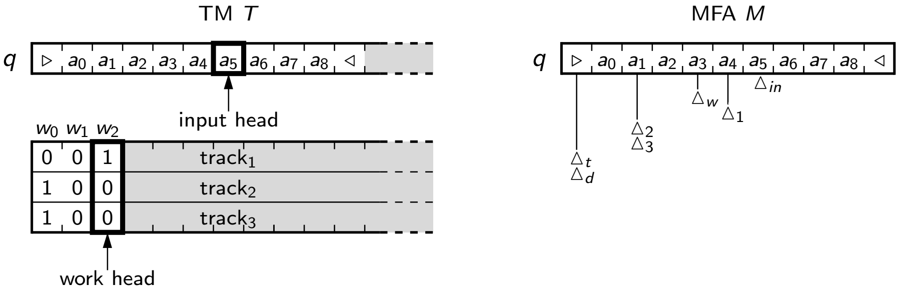
Encoding logspace TM configurations in MFAs

TMs use k binary tracks, strictly $\log n$ tape bounded.



- State q (in the TM) maps to q (in the MFA)
- $\triangle_{in}$ mirrors *input head* position
- $\triangle_w$ is $2^{\text{work head position}}$ places from $\triangleright$
- $\triangle_1, \triangle_2, \triangle_3, \ldots, \triangle_k$ simulates work tape *content* by track
- $\triangle_t, \triangle_d$ are ancillary heads for book-keeping

Encoding track content by head position



Track encoding: $\triangle_i$ points at a_n means track$_i$ contains $(\mathrm{bin}(n))^R$:

- track$_1$ contains '001', in reverse binary $100 = 4$,
- so $\triangle_1$ points at a_4, encoding 4 in unary.

To read work cell w_2 we must consult all of $\triangle_1, \triangle_2, \triangle_3$

Simulating RTM transitions

By determinism, the transitions going out of source state q are

- all symbol rules, or
- a single move rule.

We'll tackle these in turn.

Simulating symbol transitions $(q, [\cdot, \cdot \mapsto \cdot], \cdot)$

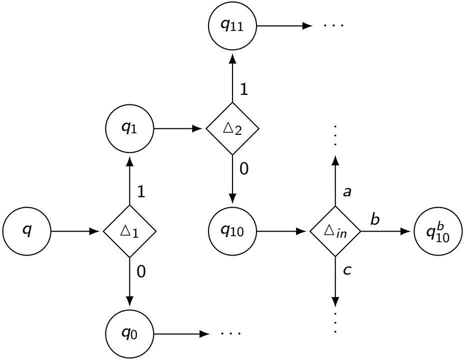
To simulate a symbol transition $(q, [\cdot, \cdot \mapsto \cdot], \cdot)$ we need

- to read the input tape symbol (easy)
- to read the work tape symbol (not so easy)
- to write a new work tape symbol (easy)
- to go into target state (not so easy)

Work tape symbol is encoded across all track heads by their *position alone*, not the local symbol.

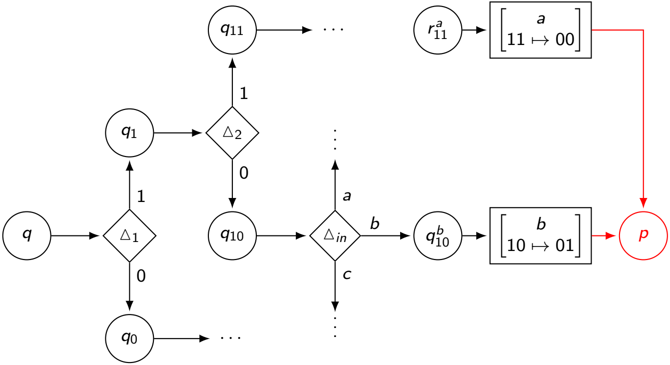
Use a *staged reading* of the symbols, store in the state.

Simulating RTM symbol rules $(q, [\cdot, \mapsto \cdot], \cdot)$



(Assuming we have a read procedure for track heads $\triangle_i$.)

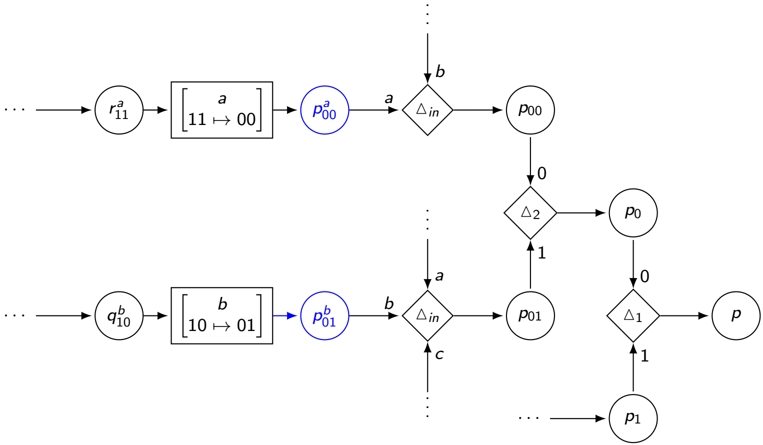
Simulating RTM symbol rules $(q, [b, 10 \mapsto 01], p)$



Doesn't work: there can be *other* symbol transitions targeting p, say, $(r, [a, 11 \mapsto 00], p)$. Direct links to p break reversibility.

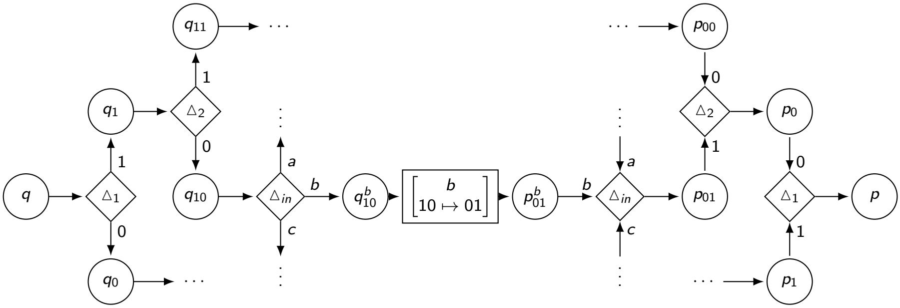
Solution: Exploit reversibility of the RTM. *No other transition targeting p can write both b and 01 to the tapes.*

Simulating RTM symbol rules $(\cdot, [\cdot, \cdot \mapsto \cdot], p)$



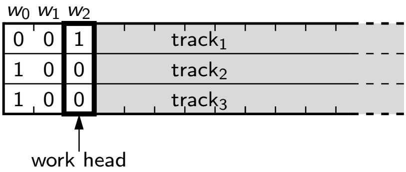
Intuition: Link up the appropriate leaves of decision trees of *source* states (r, q) with leaves of inverse decision tree for *target* state p.

Simulating RTM symbol rules $(q, [b, 10 \mapsto 01], p)$



Subpart of symbol rule simulation for $(q, [b, 10 \mapsto 01], p)$.

Reading procedure



1. Each track head encodes just 1 bit from each work tape cell.
2. If the RTM work head points to cell w_p, then the RMFA work head $\triangle_w$ is 2^p cells to the right of $\triangleright$.

Idea: The pth bit of n in binary is

$$(n \text{ div } 2^p) \bmod 2\,.$$

Reading procedure 2

Idea: The pth bit of n in binary is

$$(n \text{ div } 2^p) \bmod 2\,.$$

most sign.	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
least sign.	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

Reading procedure: Read off pth bit from track i, by repeatedly 'subtracting' 2^p ($\triangle_w$'s offset) from $\triangle_i$. $\triangle_i$'s original position is conserved in $\triangle_t$, and the original position of $\triangle_w$ in $\triangle_d$.

Reading procedure 2

Idea: The pth bit of n in binary is

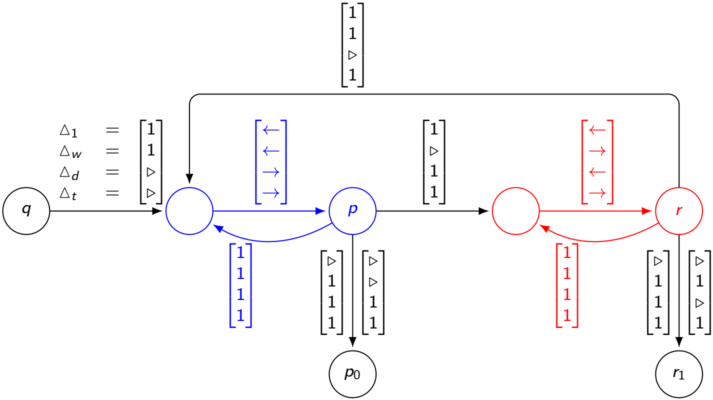
$$(n \text{ div } 2^p) \bmod 2\,.$$

most sign.	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
least sign.	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
$\triangleright$											$\triangle_2$					

Reading procedure: Read off pth bit from track i, by repeatedly 'subtracting' 2^p ($\triangle_w$'s offset) from $\triangle_i$. $\triangle_i$'s original position is conserved in $\triangle_t$, and the original position of $\triangle_w$ in $\triangle_d$.

Example: $\triangle_2$ point at a_{10}, which encodes 1010. $\triangle_w$ points at w_2, so repeatedly subtract $2^2 = 4$ from 10, until we hit $\triangleright$.

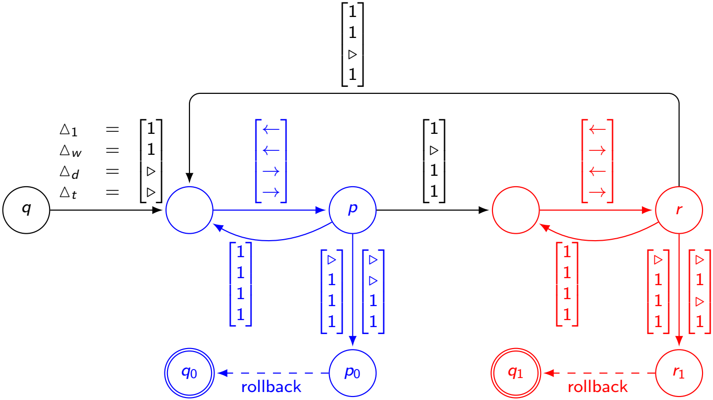
Reversible reading procedure for $\triangle_1$



Blue loop: subtract $\triangle_w$ from $\triangle_1$, believing the bit is 0.

Red loop: subtract $\triangle_w$ from $\triangle_1$, believing the bit is 1.

Reversible reading procedure for $\triangle_1$



Problem: head positions are not conserved.

Solution: *rollbacks* are *inverse* copies of the reading procedure, restoring all heads to their previous positions, *reversibly!*

Reversible writing procedure

An RTM symbol rule such as $(q, (a, 1010 \mapsto 0011), p)$ define *explicit* bit flips. Flipping the pth bit encoded on track i is simple:

- $0 \mapsto 1$: move $\triangle_i$ 2^p positions to the *right*
- $1 \mapsto 0$: move $\triangle_i$ 2^p positions to the *left*

Example: Let $\triangle_2$ point at a_{10}, which encodes 1010. To flip the 3rd bit from 0 to 1 we move $\triangle_2$ 2^2 right. It then points at a_{14}, encoding 1110.

Although overwriting symbols *in general* is irreversible, overwriting a *specific* known symbol is not.

Simulate move rules

Simulating an RTM move $(q, [\leftarrow, \rightarrow], p)$ is considerably easier.

The RMFA does as follows.

- From q,
- $\triangle_{in}$ mirrors input head,
- if work tape head moves:
  - right, then *double* $\triangle_w$
  - left, then *halve* $\triangle_w$ (with inverse doubling procedure)
- go into state p.

Note: We do *not* have to worry about about other transitions targeting p, because by reversibilty there *are no others!*

Summary

We now have:

- Encoding from RTM to RMFA configuration
- Simulation of RTM transitions in the RMFA

So we're done with showing $\mathrm{RevSPACE}(\log n) \subseteq \mathcal{L}(\mathrm{RMFA})$.

The reverse inclusion, $\mathcal{L}(\mathrm{RMFA}) \subseteq \mathrm{RevSPACE}(\log n)$ is considerably easier, so

$$\mathrm{RevSPACE}(\log n) = \mathcal{L}(\mathrm{RMFA})\,.$$

Future work: Other reversible automata models, 2-way vs. 1-way.

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Workshop Organizer

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