Catalytic Petri Nets are Turing Complete

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Motivations

Membrane computing and Petri nets have *contacts*

- investigations on translating membrane systems into suitable Petri nets
- what about the vice versa?

The connection between membrane systems and Petri nets has led to new classes of nets

- can the vice versa have similar effects?

Once a strong relation among nets and membrane computing and Petri nets has been established, which results can be transferred?

- many membrane systems are Turing complete, Petri nets are not
- can we transfer some of these results?

Membrane computing

Inspiring metaphor: the living cell

ingredients: compartments, objects, rules

some locality: rules are applied in each compartment in a maximal way fashion

rules may be of a lot of types: classifications based on these

objects may be also a source for classification (*e.g.* catalysts...)

many extensions possible: they are not intended to change the expressiveness

Petri nets

Inspiring metaphor: automata in a non classical (relativistic) setting

ingredients: places, transitions, arcs and tokens

firing a transition is done in a purely local way

monotonic enabling: if a transition is enabled and you add a token to a place, the transition remains enabled

classifications based on firing rules, or places capacity, or auto concurrency, or ...
(some classifications depend on the *static* of the net, others on the *dynamic*)

many extensions possible: most are intended to change the expressiveness (*e.g.* inhibitor arcs), and are applications oriented

Achieving Turing completeness in Petri nets

the problem has remained open for 10 years...

some proposals:

- adding priorities to transitions: losing the locality in firing
- requiring that all the enabled transitions fires: again losing locality
- adding inhibitor arcs: losing monotonicity – enabling is not any longer monotone
- adding reset arcs: again losing monotonicity
- ...

all the proposals are unfaithful to the original intuition...

many of them have origin in the *application world* of Petri nets

What we do?

We introduce a new class of Petri nets: Catalytic Petri nets

- characterized structurally
- with a suitable firing rule using catalysts
- the firing rule retains monotonicity and to some extent locality

We translate a catalytic membrane system into a catalytic Petri net

- showing that computations in the two models correspond

hence if catalytic membrane systems are Turing complete also catalytic Petri nets are Turing complete as well (under the proper firing rule)

(Flat) Membrane Systems

Definition

A *(flat) P system* is the 4-tuple $\Pi_f = (O, w^0, R, O')$ where

- O is a finite set of *objects*, and $O' \subseteq O$ are the *final* objects,
- $w^0 \in \partial V$ is a finite multiset of objects, called the *initial* configuration, and
- R is a finite set of rules of the form $r = u \rightarrow v$, with $u, v \in \partial O$ and $u \neq \mathbf{0}$.

A *configuration* of a membrane system is any finite multiset of objects.

(Flat) Membrane Systems

We can focus on this model without problem:

Oana Agrigoroaiei & Gabriel Ciobanu: "Flattening the transition p systems with dissolution", 11th Int. Conf. on Membrane Computing

Theorem

Each membrane system can be turned into a flat one without losing expressivity.

Objects code the membranes structure, suitable rules may be added to match the evolutions rules of the original membrane system...

(Flat) Membrane Systems: evolution

rule $r = u \rightarrow v$: $lhs(r) = u$ and $rhs(r) = v$

(Flat) Membrane Systems: evolution

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Multiset of rules $\mathcal{R}$ ($\mathcal{R} \in \partial R$) and a configuration w: $w \stackrel{\mathcal{R}}{\Rightarrow} w'$

- $\oplus_{r \in R} \mathcal{R}(r) \cdot lhs(r) \subseteq w$ enabling
- $w' = w \ominus (\oplus_{r \in R} \mathcal{R}(r) \cdot lhs(r)) \oplus (\oplus_{r \in R} \mathcal{R}(r) \cdot rhs(r))$ reached configuration

The transition $w \stackrel{\mathcal{R}}{\Rightarrow} w'$ is called an evolution step.

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A configuration w is reachable iff $w_1 \stackrel{\mathcal{R}_1}{\Rightarrow} w_1 \cdots w_n \stackrel{\mathcal{R}_r}{\Rightarrow} w_{n+1}$ such that $w_1 = w^0$ and $w = w_{n+1}$.

(Flat) Membrane Systems: evolution is maximal

rule $r = u \rightarrow v$: $lhs(r) = u$ and $rhs(r) = v$

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$w \stackrel{\mathcal{R}}{\Rightarrow} w'$ is assumed *maximal*: for each $\mathcal{R}' \supseteq \mathcal{R}$ it holds that $\bigoplus_{r \in R} \mathcal{R}'(r) \cdot lhs(r) \not\subseteq w$.

Catalytic Membrane Systems

Definition

A flat membrane system Π_f is called *catalytic* iff there is a designated subset $O_C \subset O$ of *catalysts* and the rules have the following form:

either $r = a \to v$ with $a \in O \setminus O_C$ and $v \in \partial(O \setminus O_C)$

or $r = ca \to cv$ with $a \in O \setminus O_C$, $v \in \partial(O \setminus O_C)$ and $c \in O_C$.

If all the rules are of the form $r = ca \to cv$ we say that the catalytic P system is *purely* catalytic.

We denote catalytic P systems with $C\Pi$, and purely catalytic ones with $CP\Pi$.

Catalytic Membrane Systems: evolution

catalysts awareness: $w \overset{\mathcal{R}}{\Rightarrow} w'$ is non any longer *maximal*

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but for each catalyst in $c \in O_C$ either there is a rule in $\mathcal{R}$ that uses it or for each $\mathcal{R}' \supseteq \mathcal{R}$ it holds that $(\oplus_{r \in R} \mathcal{R}'(r) \cdot lhs(r))(c) = 0$

Catalytic Membrane Systems: evolution

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maximality is dropped and substituted with a resource awareness...

locality depends much on catalysts

Example

Consider the objects a, b, e and two catalysts c_1 and c_2

rules: $r_1 = c_1 a \longrightarrow c_1 b$, $r_2 = c_2 b \longrightarrow c_2 e$, $r_3 = a \longrightarrow b$ and $r_4 = b \longrightarrow e$

initial configuration $c_1 + c_2 + 3a + 2b$

final configuration $c_1 + c_2 + 5e$

a computation:

$$c_1 + c_2 + 3a + 2b \stackrel{r_1+r_2}{\Longrightarrow} c_1 + c_2 + 2a + 2b + e \stackrel{r_1+r_2}{\Longrightarrow} c_1 + c_2 + a + 2b + 2e \stackrel{r_1+r_2}{\Longrightarrow} c_1 + c_2 + 2b + 3e \stackrel{r_2}{\Longrightarrow} c_1 + c_2 + b + 4e \stackrel{r_2}{\Longrightarrow} c_1 + c_2 + 5e$$

the rules r_3 and r_4 can be applied at some steps, but it may as well not...

$$c_1 + c_2 + 3a + 2b \stackrel{r_1+r_2+2r_3}{\Longrightarrow} c_1 + c_2 + 4b + e \stackrel{r_2+3r_4}{\Longrightarrow} c_1 + c_2 + 5e$$

Computability results

A catalytic P system with a single membrane and only two catalysts has the power of a Turing machine

Theorem

$NO_{min}((cat, n)) = RE$ and $PsO_{min}((cat, n)) = PsRE$, for $n \geq 2$.

Three catalysts are needed in the case of purely catalytic systems.

Theorem

$NO_{min}((p - cat, n)) = RE$ and $PsO_{min}((p - cat, n)) = PsRE$, for $n \geq 3$.

Petri Nets

Definition

A Petri net is a tuple $N = ((S, T, F, m_0), \mathfrak{S})$ where S is a set of *places*, T is a set of *transitions*, $F : (S \times T) \cup (T \times S) \to \mathbb{N}$ is a *flow relation*, $m_0 \in \partial S$ is the initial marking and $\mathfrak{S} \subseteq S$ is a set of *final* places. Furthermore $S \cap T = \emptyset$.

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With ${}^{\bullet}x$ ($x^{\bullet}$, respectively) we indicate the multiset $F(\_, x)$ ($F(x, \_)$, respectively), and they are called the *preset* (*postset*, respectively) of x.

We assume that for each transition t, $dom({}^{\bullet}t) \neq \emptyset$

Petri Nets

Given a net $N = ((S, T, F, m_0), \mathfrak{S})$, we say that N is a *state machine* iff $\forall t \in T.\ |dom({}^{\bullet}t)| = |dom(t^{\bullet})| = 1$

each transition has just one incoming arc and one outgoing arc

N is an *input state machine* iff $\forall t \in T.\ |dom({}^{\bullet}t)| = 1$ and ${}^{\bullet}t = [\![{}^{\bullet}t]\!]$.

each transition has just one incoming arc and its weight is 1

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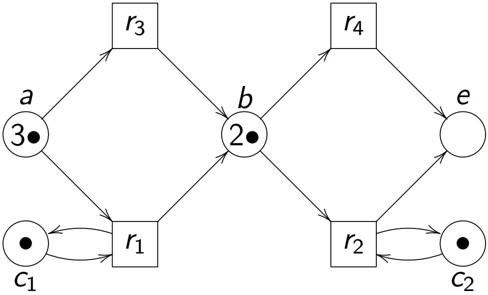
Subnet:

Let $S' \subseteq S$ be a subset of places, the *subnet* of N generated by S' is the net defined as follows: $N@S' = ((S', T@S', F@S', m_0@S'), \mathfrak{S} \cap S')$ where $T@S' = \{t \in T \mid \exists s \in S'$ such that either $F(s,t) > 0$ or $F(t,s) > 0\}$, $F@S'$ is the restriction of F to S', and $m_0@S' = m_0|_{S'}$.

Petri Nets: examples



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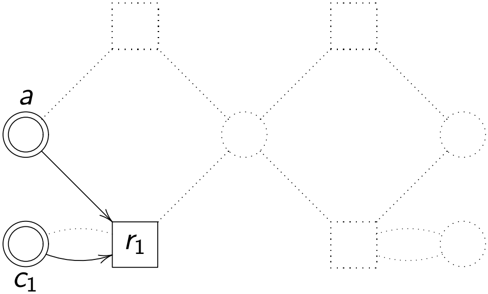
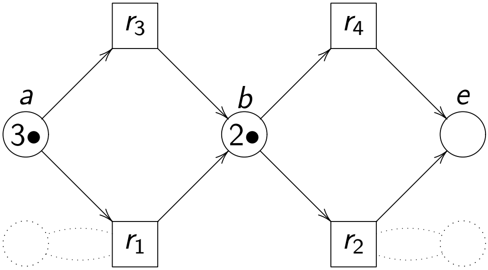


preset of r_1

Petri Nets: examples



the subnet generated by a, b and e

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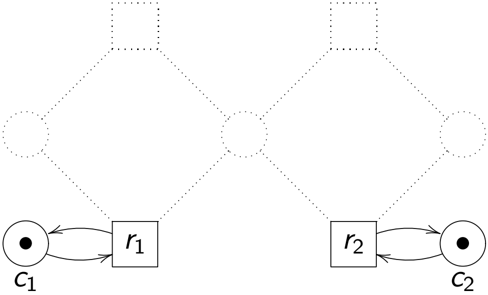


the subnet generated by c_1 and c_2

Petri nets dynamic: classical firing rule

A multiset $U \in \partial T$ of transitions is *enabled* under m if for all $s \in S$ $\sum_{t \in T} U(t) \cdot F(s,t) \leq m(s)$, and it is written as $m\,[U\rangle_{st}$

If a finite multiset $U \in \partial T$ is *enabled* at a marking m, then U may *fire* reaching a new marking m' defined as, for all $s \in S$:

$$m'(s) = m(s) + \sum_{t \in T} U(t) \cdot (F(t,s) - F(s,t))$$

in deciding which transitions fire, no structural control on the marking is done: locality

if a token is added to a place the transition remains enabled: monotonicity

Petri nets dynamic: other firing rules

subset of places $\mathcal{S} \subseteq S$

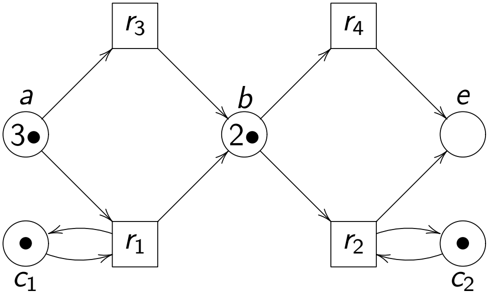
$m\,[U\rangle_{\mathcal{S}}$: A step U enabled at a marking m is $\mathcal{S}$-enabled iff for all $s \in \mathcal{S}$ either there exists a $t \in dom(s^\bullet)$ and $U(t) \neq 0$, or for all $t' \in dom(s^\bullet)$ it holds that $\neg m\,[t'\rangle$

the evolution is $m\,[U\rangle_{\mathcal{S}}\, m'$ and m' is obtained as before

in deciding which transitions fire, no structural control on the whole marking is done, but a limited one yes: a kind locality

if a token is added to a place the transition remains enabled: monotonicity

Petri nets dynamic: examples



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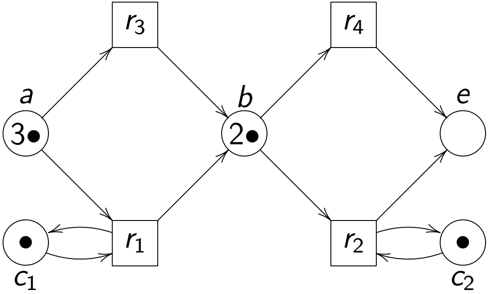
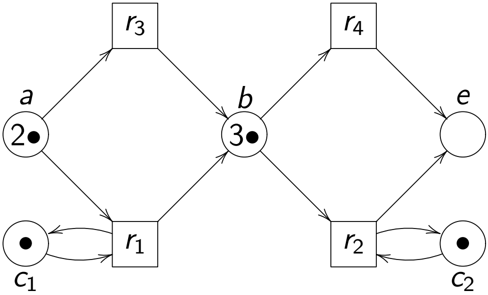


firing r_1

Petri nets dynamic: examples



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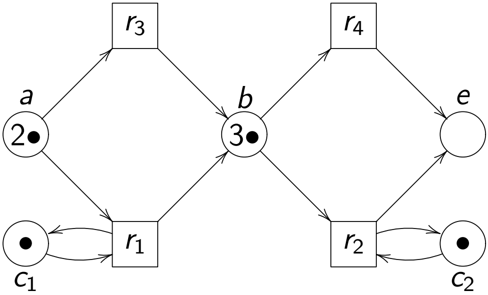
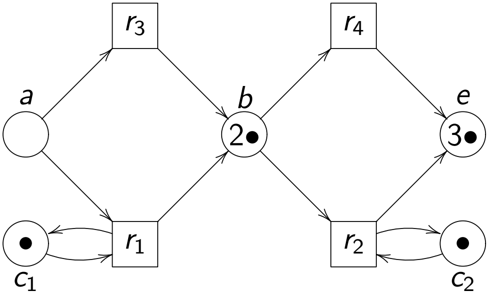


maximal firing: $r_1 + r_3 + r_2 + 2r_4$

Petri nets dynamic: examples



Catalytic Petri nets

Places in a catalytic net are partitioned into two subsets, the catalytic places ($\mathcal{C}$) and the non catalytic ones ($S \setminus \mathcal{C}$).

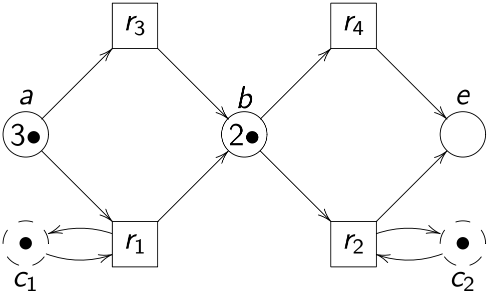
Definition

Let $N = ((S, T, F, m), \mathfrak{S})$ be a Petri net. N is *catalytic* iff the set of places S is partitioned into two disjoint sets $\mathcal{C}$ and $\mathcal{V}$ such that

1. the subnet $N@\mathcal{V}$ is an input state machine, and
2. the subnet of $N@\mathcal{C}$ is a state machine, and for all $t \in T@\mathcal{C}$ we have ${}^{\bullet}t = t^{\bullet}$ and $\#({}^{\bullet}t) = 1$.

A net $N = ((S, T, F, m), \mathfrak{S})$ is said to be *purely catalytic* iff $T = T@\mathcal{C}$.

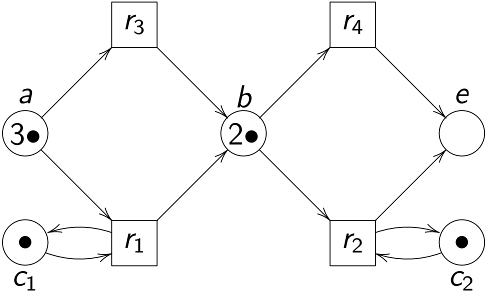
Catalytic Petri nets



places c_1 and c_2 are catalytic places...

Catalytic firing rule

All the transitions using catalysts, if enabled, must fire:



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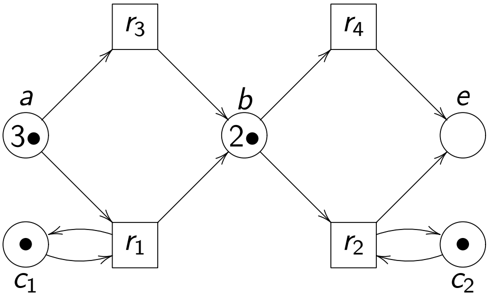

$r_1 + r_2$ is a step

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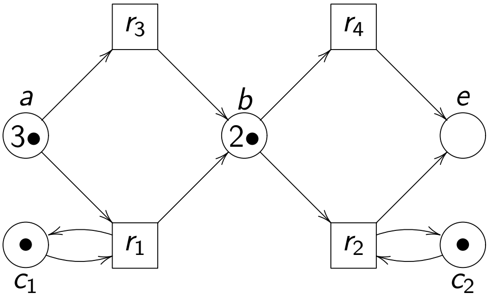

$r_1 + r_2$ is a step, $r_1 + 2r_4$ no

From flat membrane system to Petri nets

Definition

From $\Pi = (O, w^0, R, O')$ to $\mathcal{F}(\Pi) = ((S, T, F, m), \mathfrak{S})$:

- $S = O$ and $T = \{t^r \mid r \in R\}$
- for all transitions $t = t^r \in T$ and all the places $s \in S$ with $r = u \to v$, we define

$$F(s,t) = \begin{cases} u(a) & \text{if} \quad s = a \\ 0 & \mathit{otherwise} \end{cases} \text{ and } F(t,s) = \begin{cases} v(a) & \text{if} \quad s = a \\ 0 & \mathit{otherwise} \end{cases}$$

- $m(s) = \begin{cases} w^0(a) & \text{if } s = a \\ 0 & \text{otherwise} \end{cases}$
- $\mathfrak{S} = O'$

From flat membrane system to Petri nets

w is a configuration of Π: $\nu(w)$ is the marking defined by $\nu(w) = w$

$C \stackrel{\vec{R}}{\Longrightarrow} C'$ be an evolution step of Π: $\sigma(\vec{R})$ is the multiset defined by $\sigma(\vec{R})(t^j) = \vec{R}(r^j)$, for all $t^j \in T$

Theorem

Let $\Pi = (O, w^0, R, O')$ be a membrane system, and $\mathcal{F}(\Pi) = ((S, T, F, m), \mathfrak{S})$ be the associated Petri net. $w \stackrel{\vec{R}}{\Longrightarrow}_{fs} w'$ iff $\nu(w)\, [\sigma(\vec{R})\rangle_{fs}\, \nu(w')$ with $fs \in \{step, max, \mathcal{S}\}$.

(in Kleijn & Koutny & Rozenberg: "Towards a Petri net semantics for membrane systems", in Proceedings of the 6th International Workshop on Membrane Computing)

From flat membrane system to Petri nets

$$\Pi = (\{a, b, e, c_1, c_2\}, 3a + 2b, \{r_1 = c_1 a \rightarrow c_1 b,$$
$$r_2 = c_2 b \rightarrow c_2 e,\ r_3 = a \rightarrow b, r_4 = b \rightarrow e\}, \{e\})$$

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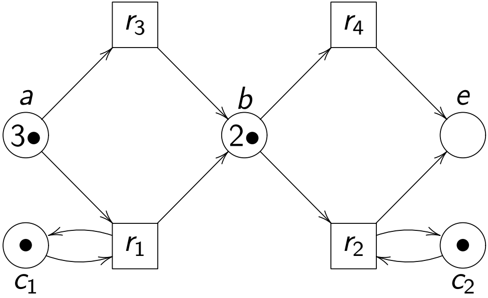
$\mathcal{F}(\Pi) =$



and vice versa!

Definition

From $N = ((S, T, F, m_0), \mathfrak{S})$ to $\mathcal{K}(N) = (O, w^0, R, O')$:

- $O = S$ and $w^0(s) = m(s)$,
- for all $t \in T$ define a rule $r^t = u \longrightarrow v$ in R where $u = {}^\bullet t$ and $v = t^\bullet$, and
- $O' = \mathfrak{S}$.

and vice versa!

$\xi(s) = s$ and $\eta(t) = r_i^t$

$\xi(m)$ is the configuration defined by $\xi(m)(s) = m(s)$, for all $s \in S$

$m\,[U\rangle\, m'$ be a step of N. Then $\eta(U)$ is the evolution step defined by $\eta(U)(r^t) = U(t)$, for all $r^t \in R$

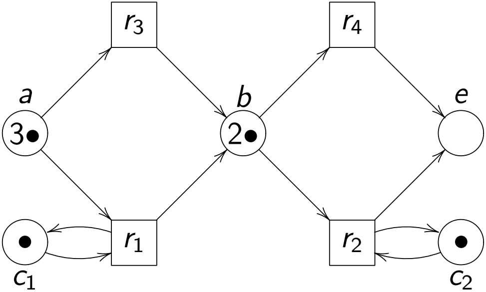
Theorem

Let $N = ((S, T, F, m_0), \mathfrak{S})$ be a Petri net and $\mathcal{K}(N)$ be its associate membrane system. For $fs \in \{step, max, \mathcal{S}\}$, we have

$$m\,[U\rangle_{fs}\, m'' \quad \textit{iff} \quad \xi(m) \stackrel{\eta(U)}{\Longrightarrow}_{fs} \xi(m').$$

and vice versa!

$N =$



$$\mathcal{K}(N) = (\{a, b, e, c_1, c_2\}, 3a + 2b, \{r_1 = c_1 a \rightarrow c_1 b, \\ r_2 = c_2 b \rightarrow c_2 e,\ r_3 = a \rightarrow b, r_4 = b \rightarrow e\}, \{e\})$$

Catalytic membrane system and catalytic Petri nets

Proposition

Let $\Pi = (O, O_C, w^0, R, O')$ *be a catalytic P system, and* $\mathcal{F}(\Pi)$ *be its associated structure. Then* $\mathcal{F}(\Pi)$ *is a catalytic net.*

Proposition

Let $N = ((S, T, F, m_0, \mathcal{C}), \mathfrak{S})$ *be a catalytic Petri net. Then* $\mathcal{K}(N)$ *is a flat catalytic P system.*

$$\mathcal{F}(\mathcal{K}(N)) = N \text{ and } \mathcal{K}(\mathcal{F}(\Pi)) = \Pi$$

Expressiveness

$CPN(n)$: catalytic Petri nets with n catalytic places

$PCPN(n)$: purely catalytic Petri nets with n catalytic places

Theorem

$$\{\mathfrak{F}_{\mathcal{C}}^{\#\mathfrak{S}}(N) \mid N \in CPN(n)\} = RE \quad \textit{and}$$
$$\{\mathfrak{F}_{\mathcal{C}}^{\mathfrak{S}}(N) \mid N \in CPN(n)\} = PsRE, \textit{ for } n \geq 2.$$

Theorem

$$\{\mathfrak{F}_{\mathcal{C}}^{\#\mathfrak{S}}(N) \mid N \in PCPN(n)\} = RE \quad \textit{and}$$
$$\{\mathfrak{F}_{\mathcal{C}}^{\mathfrak{S}}(N) \mid N \in PCPN(n)\} = PsRE \textit{ for } n \geq 3.$$

Conclusions

the firing in catalytic Petri nets is still local: you check only catalytic places

the Turing completeness is achieved respecting as much as possible the original intuition

Conclusions and future works

the firing in catalytic Petri nets is still local: you check only catalytic places

the Turing completeness is achieved respecting as much as possible the original intuition

other classes of Petri nets may be introduced using suitable membrane systems

a canonical labeling for nets mimicking the membrane stucture

behavioral equivalence on membrane systems inspired by Petri nets

...