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Who is the human computer?

Alan Turing (1936, section 9, I) famously provided an analysis of effective computability by formulating a set of restrictive constraints ("axioms") on the computing agent. It is widely agreed that Turing had in mind an idealized human agent, and that he "makes no reference whatsoever to calculating machines" (Gandy 1988: 77; see also Kleene 1952; Sieg 1994, 2008). My aim is to explore two very different understandings of the concept of a human computer; I call them the cognitive and the non-cognitive approaches. According to the cognitive approach, a human computer is restricted by the limitations of certain human cognitive capacities. The claim need not be that these limitations apply to human mental processes *in general*, but to the cognitive abilities involved in calculation. The non-cognitivist, in contrast, thinks that a human computer is restricted to certain finite means, regardless of whether or not these means reflect the limitations of human cognitive capacities. These means are simply part of the concept of effective computation as it is properly used and as it functions in the discourse of logic and mathematics.

The restrictive conditions that Turing formulated can be respectively understood in two ways. The cognitivist might see them as reflections of certain limitations of human cognitive capacities. These limitations *ground* or *justify* the restrictive conditions. According to the cognitive approach, computability is constrained by conditions 1-5 *because* these constraints reflect the limitations of human cognitive capacities. The non-cognitivist thinks that the restrictive conditions do not, and need not, necessarily reflect cognitive limitations. The non-cognitivist offers no other

justification for the conditions. In fact, a call for further justification has no place at all in the analysis of computability, according to the non-cognitivist.

I argue that the founders of computability and their interpreters take a stand between the approaches. Post has one foot, or possibly even both feet, in the cognitive camp, saying that the purpose of his analysis "is not only to present a system of a certain logical potency but also, in its restricted field, of psychological fidelity" (1936: 105). Gandy apparently also belongs to this cognitive standpoint, in declaring that "by considering the limitations of our sensory and mental apparatus Turing arrives at the following restrictions on the actions of a computor [human computer]" (2001: 11). Turing seems to vacillate between the approaches. He says about his definition of computability that "for the present I shall only say that the justification lies in the fact that the human memory is necessarily limited" (1936: 59), which locates him in the cognitive camp. But he later on says that the restriction about states of mind "is not one which seriously affects computation", and that we can always replace states of mind with "writing more symbols on the tape" (p. 76). This indicates that he might have been a non-cognitivist after all.

Kleene follows Turing's suggestion to externalize the computation procedure, saying that "the notion of an 'effective calculation procedure' or 'algorithm' (for which I believe Church's thesis) involves its being possible to convey a complete description of the effective procedure or algorithm by a finite communication, in advance of performing computations in accordance with it. My version of the Church-Turing thesis is thus the '*Public-Processes Version*'" (1987: 493-494). Yet it remains unclear whether the *finiteness*-of-communication requirement is rooted in our cognitive

abilities or not. Sieg, too, says that the finiteness of the procedure is grounded in the "normative requirement on the fully explicit presentation of mathematical proofs in order to insure inter-subjectivity" (2009: 532). But he also argues that "limitations of our sensory apparatus seem to be involved" (1994: 96). So his human computer seems to at least include a cognitive component. Gödel and Church appear much closer to the non-cognitive camp. As early as 1934 Gödel was thinking of a computation procedure as a finite procedure, and at no point did he imply that this reflects, or is justified in terms of, limitations in human cognition. In fact, Gödel made no explicit mention of human computability at all.

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