Broadcast Abstraction in a Stochastic Calculus for Mobile Networks

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Lei Song and Jens Chr. Godskesen

IT University of Copenhagen and MT-Lab Center of Excellence

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MANETs

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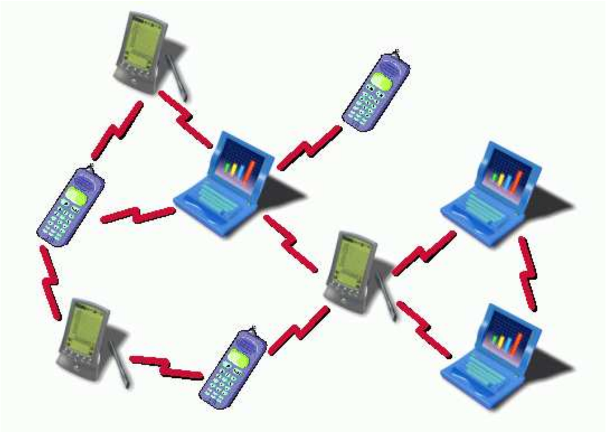

Main Features of MANETs

- Local Broadcast
- Mobility
- Unreliable Connections
- Unidirectional
- Separated Connectivity

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  Broadcast in an MANET is local, meaning that only devices within the transimission range of the emitting device can receive a message.
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Main Features of MANETs

- Local Broadcast
- Mobility

  The devices in an MANET are not stationary and may crash at anytime, therefore the connectivity topology undergoes constant changes. Moreover there is no any pre-installed infrastructure or central control components in an MANET.
- Unreliable Connections
- Unidirectional
- Separated Connectivity

Main Features of MANETs

- Local Broadcast
- Mobility
- Unreliable Connections
  Compared to wired connections, we cannot guarantee that a broadcast message will reach all the devices even if they are within the transmission range.
- Unidirectional
- Separated Connectivity

Main Features of MANETs

- Local Broadcast
- Mobility
- Unreliable Connections
- Unidirectional
  The wireless connection is not bidirectional since two devices may have different transmission ranges.
- Separated Connectivity

Main Features of MANETs

- Local Broadcast
- Mobility
- Unreliable Connections
- Unidirectional
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  A protocol should work properly for any connectivity.

MANETs

All the features make the protocols of MANETs error-prone and it is important and necessary to analyze them thoroughly before they are applied in practice.

Overview

We propose a setting where mobility and connectivity between nodes is probabilistic.

- Stochastically timed behavior of MANETs
- A novel mobility function allowing to change a number of connections at the same time
- Group broadcast
- Flooding avoidance operator
- The semantics gives rise to Markov automata

Overview

We propose a setting where mobility and connectivity between nodes is probabilistic.

- Stochastically timed behavior of MANETs

  Modeling time dependent randomized back-off technique, for instance $p = q + \lambda \cdot p$ means that p may behave as q or it may after some exponential delay distributed by λ back-off and iterate its behavior.
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  We may for instance specify connectivity such that the probability for l or k receiving messages from m is at least 0.9.
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  By group broadcast, we may specify a set of destinations for a message.
- Flooding avoidance operator
- The semantics gives rise to Markov automata

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  Flooding occurs when the same message is broadcasted over and over again during the execution of a protocol, but where it is sufficient to have received and dealt with the message just once.
- The semantics gives rise to Markov automata

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  Thus is a combination of discrete and continuous-time probability, non-determinism, and concurrency.

Syntax

Processes

$$p, q ::= 0 \mid Act \cdot p \mid p + q \mid [x = y]p, q \mid \nu x p \mid A$$

$$Act ::= \lambda \mid \langle x \rhd L^* \rangle \mid (x),$$

Networks

$$E, F ::= 0 \mid \lfloor p \rfloor_l^M \mid \{\mathbb{L} \longmapsto l\} \mid \nu x E \mid E \parallel F$$

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L^* is either a finite set of locations or is equal to the set of all the locations.

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M is a finite memory used to keep track of all the messages received.

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$\{\mathbb{L} \longmapsto l\}$ is used to denote the connectivity information. For example $\{\{(0.5, m), (0.9, n)\} \longmapsto l\}$ means that nodes at m and n can receive messages from l with probabilities 0.5 and 0.9 respectively.

Semantics by an Example

Let $E = \{\{(0.6, l), (0.7, k)\} \mapsto m\}$, then

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$$\lfloor \langle y \rangle \cdot p \rfloor_m \parallel \lfloor (x) \cdot q \rfloor_l \parallel \lfloor (x) \cdot q' \rfloor_k \parallel E \xrightarrow{\langle y, \{(0.6,l),(0.7,k)\}\rangle @ m}$$

$$\begin{cases} 0.6 \cdot 0.7 & : \lfloor p \rfloor_m \parallel \lfloor q\{y/x\} \rfloor_l \parallel \lfloor q'\{y/x\} \rfloor_k \parallel E \\ 0.6 \cdot (1 - 0.7) & : \lfloor p \rfloor_m \parallel \lfloor q\{y/x\} \rfloor_l \parallel \lfloor (x) \cdot q' \rfloor_k \parallel E \\ (1 - 0.6) \cdot 0.7 & : \lfloor p \rfloor_m \parallel \lfloor (x) \cdot q \rfloor_l \parallel \lfloor q'\{y/x\} \rfloor_k \parallel E \\ (1 - 0.6) \cdot (1 - 0.7) & : \lfloor p \rfloor_m \parallel \lfloor (x) \cdot q \rfloor_l \parallel \lfloor (x) \cdot q' \rfloor_k \parallel E \end{cases}$$

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- $$\{\{(0.5, m)\} \longmapsto n\} \xrightarrow{2} \{\{(1, m)\} \longmapsto n\}$$

Broadcast Abstraction (1)

Motivations

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- Abstract from the locations of the emitting nodes.
- Abstract from the steps of broadcast.
- Two networks are equivalent if they can deliver the same messages to the same locations with the same probabilities.

Broadcast Abstraction (2)

$$E \xRightarrow{\langle x \triangleright L, \mathbb{L} \rangle @ l} \mu \text{ iff } E \xRightarrow{\alpha_1} \xRightarrow{\alpha_2} \ldots \xRightarrow{\alpha_n} \mu \text{ with}$$

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$\alpha_1 \otimes \alpha_2 = \alpha$ where

$$\alpha = \langle x \triangleright \{m, n, l\}, \{(0.7, m), (0.76, n), (0.8, l)\}\rangle @ k,$$

where

$$\alpha_1 = \langle x \triangleright \{m, n\}, \{(0.7, m), (0.4, n)\}\rangle @ k_1,$$

$$\alpha_2 = \langle x \triangleright \{n, l\}, \{(0.6, n), (0.8, l)\}\rangle @ k_2.$$

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$$0.76 = 1 - (1 - 0.4) \cdot (1 - 0.6)$$

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- If the node at l cannot receive, then

$$\lfloor \langle x \triangleright l \rangle \rfloor_k \parallel \lfloor 0 \rfloor_l \ \approx \ \lfloor 0 \rfloor_k \parallel \lfloor 0 \rfloor_l$$

no matter whether l is connected to k or not.

Broadcast Abstraction (4)

Bisimulation over Distributions

$$E = \lfloor \langle y \triangleright k \rangle \cdot \langle y \triangleright k \rangle \rfloor_l \parallel \lfloor (x) \cdot p \rfloor_k^{\emptyset} \parallel \{\{(0.5, k)\} \longmapsto l\},$$

$$F = \lfloor \langle y \triangleright k \rangle \rfloor_l \parallel \lfloor (x) \cdot p \rfloor_k^{\emptyset} \parallel \{\{(0.75, k)\} \longmapsto l\}$$

Broadcast Abstraction (4)

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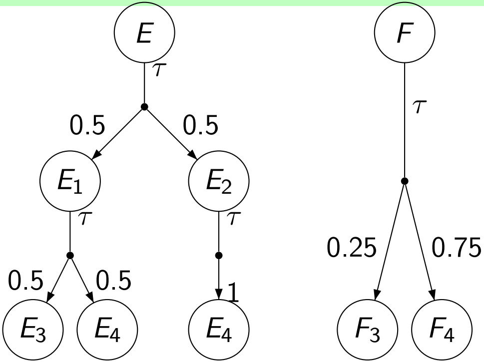

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- Novel Broadcast Abstraction based on Group Broadcast and Flooding Avoidance Operators
- A Weak Bisimulation Congruence
- Case Study — Leader Election Protocol

Future Work

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- Model Checking Algorithm and Tool for Markov Automata

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Thank you for your attention.
Q&A